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THE

PHYSICAL REVIEW

A *Journal of Experimental and Theoretical Physics*

CONDUCTED BY

EDWARD L. NICHOLS

AND

ERNEST MERRITT

VOL. I

PUBLISHED FOR CORNELL UNIVERSITY
MACMILLAN AND COMPANY
NEW YORK LONDON
BERLIN: MAYER AND MUELLER
1894

L'93-9-12

1899, May 26.
Jefferson Physical Laboratory.
Joseph Lovering fund.

Copyright, 1894,
By MACMILLAN AND CO.

Norwood Press:
J. S. Cushing & Co. — Berwick & Smith.
Boston, Mass., U.S.A.

Volume I. *July–August, 1893.* *Number 1.*

THE

PHYSICAL REVIEW.

A STUDY OF THE TRANSMISSION SPECTRA OF CERTAIN SUBSTANCES IN THE INFRA-RED.

BY ERNEST F. NICHOLS.

WITHIN a few years the study of obscure radiation has been greatly advanced by systematic inquiry into the laws of dispersion of the infra-red rays by Langley,[1] Rubens,[2] Rubens and Snow,[3] and others. Along with this advancement has come the more extended study of absorption in this region. The absorption of atmospheric gases has been studied by Langley[1] and by Ångstrom.[4] Ångstrom[5] has made a study of the absorption of certain vapors in relation to the absorption of the same substances in the liquid state, and the absorption of a number of liquids and solids has been investigated by Rubens.[6]

In the present investigation, the object of which was to extend this line of research, the substances studied were: plate glass, hard rubber, quartz, lamp-black, cobalt glass, alcohol, chlorophyll, water, oxyhæmoglobin, potassium alum, ammonium alum, and ammonium-iron alum.

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[1] Report on Mt. Whitney Expedition, Profess. Papers, U. S. Signal Service, XV.

[2] Annalen der Physik und Chemie, N. F. XLV., p. 238.

[3] Annalen der Physik und Chemie, N. F. XLVI., p. 529.

[4] Bihang till K. Svenska Vet.-Akad. Handlingar, Band 15, Afd. 1, No. 9.

[5] Ofversigt af Kongl. Vetenskaps-Academiens Forhandlingar, 1890, No. 7, Stockholm.

[6] Annalen der Physik und Chemie, N. F. XLV., p. 258.

Apparatus Used.

The galvanometer used was similar in many respects to the instrument described by B. W. Snow,[1] though designed and built independently and used several months before the description referred to appeared. The aggregate weight of the moving parts of the galvanometer was 48 mgs. Throughout the work the four coils were connected in multiple with a resistance of 9.3 ohms. The sensitiveness was varied from time to time for convenience, but for the measurement of energy in the weaker parts of the spectrum deflections corresponding to currents of 10^{-10} ampère were read.

In circuit with the galvanometer was a thermopile, made up of ten bismuth-antimony junctions arranged in a line 15 mm. long, the line being filed until the exposed surface was less than a millimeter in width. The arrangement of the entire apparatus for the study of the infra-red spectrum is given in Fig. 1. The whole is a modified spectrometer. L is a fifty-volt Edison incandescent lamp, the spectrum of which was used as a basis for determining the transmission of the various substances; *f's* are diaphragms, *D* the prism, *ABC* a device for automatically maintaining minimum deviation, *E* a track on which layers of the substances under examination could be passed between the lamp and the first lens, and *T* is the thermopile, interchangeable with the cross-wires of the eyepiece of the observing telescope. The apparatus was arranged in two rooms; the lamp *L* and the track *E* in one, and the remaining parts of the spectrometer in the other. A small hole in the partition, which could be closed at will by means of a shutter, served to connect the two rooms. The room containing the thermopile and prism

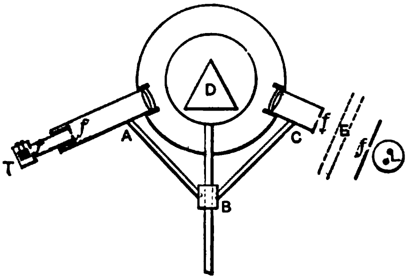

Fig. 1.

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[1] Annalen der Physik und Chemie, N. F. XLVII., p. 213.

was kept dark, and all draughts excluded. It was entered only once or twice a day to change the spectrometer setting. The glass lenses of the spectrometer were retained. It is easily seen that their absorption could not enter as an error in the present investigation; for if the energy of the lamp at any wave-length be taken as unity, and the lamp be allowed to shine on the thermopile so that it receives this wave-length only, with nothing between but a system of plates transmitting a percentage a of the energy they receive, the galvanometer deflection will then be proportional to a or ka. If now another body, which transmits a percentage b of the energy received by it, be placed between the lamp and the plates, then the energy which reaches the thermopile will be ab, and the galvanometer deflection kab. The ratio of the second deflection to the first is b, the percentage of transmission of the body in question.

A carbon bisulphide prism served for dispersion; that substance was selected because it is known that carbon bisulphide at a temperature of fifteen degrees follows the well-known Cauchy formula for dispersion: $N = a + \frac{\beta}{\lambda^2}$ where N is the refractive index and λ the corresponding wave-length.

Rubens[1] has investigated experimentally the law of dispersion of bisulphide of carbon out to wave-length 2 μ, and has found it to agree very closely with this law, and further, from the near agreement between the square root of the dielectric constant and the index of refraction for infinite wave-lengths computed from the Cauchy formula, it is probable that the dispersion follows Cauchy's law throughout the whole spectrum. The carbon bisulphide used in the present study was obtained by double distillation, and the variation of its indices of refraction with temperature was carefully studied at various wave-lengths. The results were found to agree very closely with similar results obtained by Wüllner.[2]

A comparison of the two is given in Table I.

---

[1] Annalen der Physik und Chemie, N. F. XLV., 1892, p. 256.

[2] Annalen der Physik und Chemie, Band CXXXIII., 1868, p. 16.

TABLE I.

C. LINE.			F. LINE.		
Temperature.	**Wüllner.**	**Nichols.**	**Temperature.**	**Wüllner.**	**Nichols.**
18.2		1.6202	19.25		1.65335
18.25	1.6198		19.3	1.6530	
18.55	1.6196		19.42		1.6532
18.58		1.6198	19.6	1.6530	
18.6	1.6196		19.70	1.6529	
18.91		1.6197	19.76		1.6530
19.0	1.6193		20.18		1.6528
19.10	1.6192		20.25	1.6525	
19.14		1.6194	20.5	1.6522	
19.45	1.6189		20.54		1.6524
19.46		1.6192	21.08		1.6520
20.15	1.6183	1.6186	21.1	1.6517	
20.55	1.6181		25.27		1.6485
21.80		1.6181			
22.1	1.6169				
22.32		1.6170			
22.83		1.6166			
23.02		1.6164			
23.4	1.6159				
24.20		1.6156			
25.24		1.6147			
26.08		1.6142			

The temperature change in the indices of refraction is seen to be sufficiently large to make it necessary to correct all observations for temperature. The temperature coefficients of indices of refraction for the various Fraunhofer lines are given in Table II.

If N_t is the index of refraction for a wave-length λ at a temperature t°, and a is the temperature coefficient for that wave-length, then the index of refraction $N_{t'}$ for t'° will be $N_{t'} = N_t - a(t' - t)$.

The temperature coefficients gradually diminished as the wave-length increased through the visible spectrum, and a similar decrease was assumed through the infra-red. In the temperature corrections made to observations, $t' - t$ was never greater than 3°. The temperature taken for the standard to which all observations were reduced was $23^\circ.5$. The prism was calibrated by the

TABLE II.

	A		C		D	
	Range	a	**Range**	a	**Range**	a
Wüllner[1]			$t = 7^\circ$ $t' = 23^\circ$	.00078		
Nichols	$t = 20^\circ$ $t' = 25^\circ.4$	.000759	$t = 18^\circ.2$ $t' = 26^\circ$	.000763	$t = 19^\circ.8$ $t' = 26^\circ.1$	.000768

	F		G		LINE BEYOND G, N $23^\circ.5 = 1.6782$	
	Range	a	**Range**	a		
Wüllner	$t = 7^\circ$ $t' = 23^\circ$	.00082	$t = 7^\circ$ $t' = 23^\circ$	.00085		
Nichols	$t = 19^\circ.2$ $t' = 26^\circ.2$	.000805			$t = 21^\circ.2$ $t' = 26^\circ.2$	.000858

Fraunhofer lines, and the Cauchy constants computed. The equation to the calibration curve of the prism was $N = 1.5772 + \frac{.01681}{\lambda^2}$. The values $\alpha = 1.5772$ and $\beta = .01681$ were obtained by observing the indices of refraction for the Fraunhofer lines *b*, *C*, *B*, and *A*, and substituting these indices in turn with their corresponding wave-lengths into the general formula $N = \alpha + \frac{\beta}{\lambda^2}$.

The indices of refraction observed and computed were: —

TABLE III.

	OBSERVED.	COMPUTED.
b.	1.6398	1.6398
C.	1.6156	1.6162
B.	1.6123	1.6128
A.	1.6063	1.6063

The prism on two angles of its base carried conical points. The point beneath the refracting angle fitted into a corresponding pit; the other point into a groove in the table of the spectrometer. This device made it possible to take the prism off the spectrometer to allow a zero reading at any time, after which the prism could be accurately replaced. For calibration the ordinary spectrometer slit was used with an eye-piece cross-wire, but in the energy measurements, in order to get an intense source, a straight portion of the filament of the incandescent lamp was substituted for the slit, and the rest of the filament screened by diaphragms. To change from the slit to the filament, and from the cross-wire in the eye-piece to the thermopile, without disturbing the calibration, the spectrometer was set at zero reading, the eye-piece carefully focussed on the slit and the observing telescope clamped. The slit tube was then removed and the lamp filament put in the exact place where the slit had been, as determined by its image in the eye-piece. Then the eye-piece was taken out and a slit a millimeter wide was put in the plane of the cross-wires and moved from side to side until the image of the filament fell in the middle of the slit. The line of junctions of the thermopile was fitted accurately to this opening. After these changes the prism could be replaced on the spectrometer, the telescope carrying the thermopile could be unclamped, and by means of the calibration curve the thermopile could be set to receive radiations of any desired wave-length.

---

[1] Annalen der Physik und Chemie, Band CXXXIII., 1868, p. 16.

The width of the line of thermo-junctions in angular measure was 10′. Because of this width and the increasing condensation of rays by the prism as the wave-length increased, measurements beyond $\lambda = 2.^{\mu}6$ were of little value. The wave-lengths given in the tables correspond to the wave-lengths received by a line through the middle of the face of the thermopile.

Method of Observation.

Before beginning a set of readings, the galvanometer circuit was closed once for all. When the galvanometer needle was quiet, the shutter between the lamp and prism was opened, and the lamp allowed to shine on the thermopile until the needle had reached the end of its first swing: the shutter was then closed and the return swing observed. The mean of the reading from which the needle started and the reading to which it returned was taken as the zero reading. Any uniform change in the zero point of the galvanometer during the ten or twelve seconds required to get the two swings was thus eliminated.

The percentage of energy of a given wave-length transmitted by any one of the substances was estimated as follows: 1st, The energy of the bare lamp was measured. 2d, The lamp energy transmitted through the substance. 3d, The energy of the bare lamp again. The ratio of the second reading to the mean of the first and third was taken as a transmission percentage. The use of the mean of the first and third for the lamp reading obviates the error which might otherwise enter from a uniform change in the constant of the galvanometer. Since the glow lamp was fed from a storage battery, the source of radiant energy was very nearly constant. From three to five determinations of this transmission percentage were made for each of the substances at all observed points. The mean of these readings is in each case the transmission percentage given in the tables. The probable error in the transmission percentages in no case exceeds ± 4 %, and in most cases is less than ± 1 %.

The Results.

Table IV. contains the transmission percentages in the infra-red spectra of a piece of French plate glass 8.3 mm. thick, and of a sheet of black hard rubber .2 mm. thick. The transmission spectra of these two substances were examined at nine different points and the corresponding wave-lengths are given in 10^{-6} meters at the heads of the columns.

TABLE IV.

λ	$0^{\mu}.775$	.862	.940	1.19	1.41	1.58	1.85	2.37	2.90
Plate glass . . .	82.4	76.9	77.5	71.7	83.7	82.1	82.5	75.1	76.7
Hard rubber . .	30.7	54.1	58.9	65.0	58.4	58.9	58.1	52.0	56.6

The results in this table are shown graphically in Fig. 2, in which wave-lengths are plotted as abscissæ, and the corresponding percentages of transmission as ordinates. The parallelism of the curves beyond $\lambda = 0^{\mu}.862$ seems to show that if the rubber had been thinner or the glass thicker, the two spectra beyond this point might have been identical within the limits of probable error.

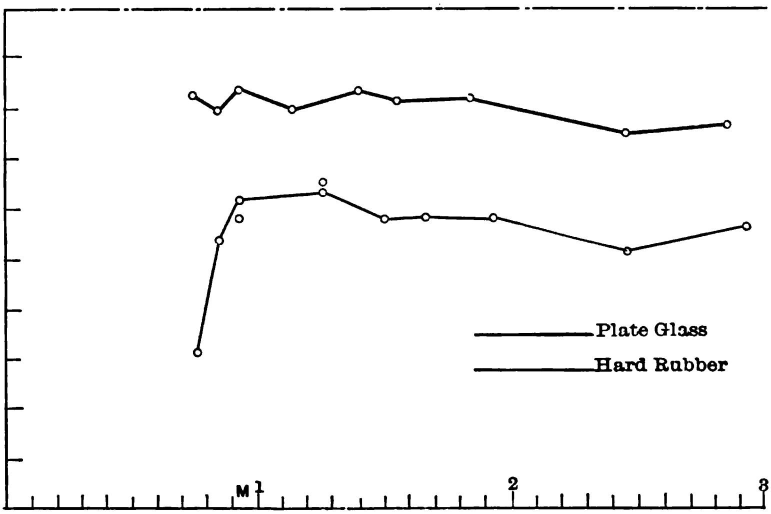

Fig. 2.

Figure 3 shows the same results in a slightly different way. The upper curve in this figure shows the distribution of energy in the normal spectrum of the glow-lamp used.[1] Wave-lengths are plotted as abscissæ, and corresponding intensities of radiant energy as ordinates. The distribution of energy in the prismatic spectrum was obtained by the same apparatus as that used in getting the transmission percentages, consequently the radiant energy before reaching the thermopile passed through two glass lenses, the glass sides of the prism, and the bisulphide of carbon. Because of the lack of selective absorption in glass in this region of the spectrum and the almost perfect diathermancy of carbon bisulphide

---

[1] Obtained by a method to be described in an article on Distribution of Energy in the Spectrum of the Glow-Lamp, by Dr. E. L. Nichols.

(Rubens,[1] Knut Ångstrom[2]), the only effect of this transmission is that all the ordinates of the curve in Fig. 3 are less than the true values by a constant percentage.

The distribution of energy in the normal spectrum is obtained from the distribution in the prismatic spectrum by correcting for the increasing condensation of rays in the direction of the longer wave-lengths.[3]

The ordinates of the middle curve are obtained by taking the product of the transmission percentage of glass for each wave-length and the ordinate of the energy curve of the lamp at that wave-length. The lower curve shows the distribution of energy in the transmission spectrum of hard rubber. From the way in which these curves are plotted the total infra-red energy transmitted by the substances varies as the areas included between the curves and the base line. In the case of glass and hard rubber the percentages of transmission for the different wave-lengths observed

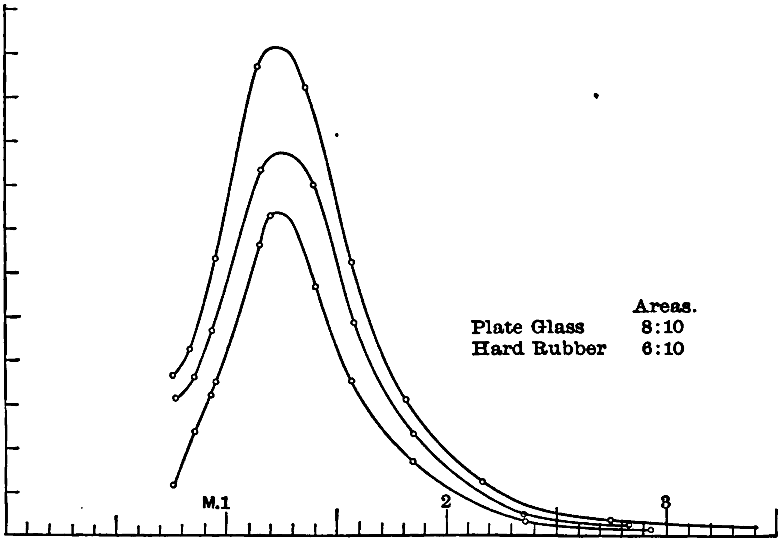

Fig. 3.

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[1] Annalen der Physik und Chemie, N. F. XLV., p. 260.

[2] Ofversigt af Kongl. Vetenskaps-Academiens Forhandlingar, 1892, No. 7, Stockholm, p. 345.

[3] Report on Mt. Whitney Expedition, p. 231; also Draper, American Journal of Science, 1872.

were so nearly constant that the observed points on the diagram were connected by a curve similar to the energy curve of the source.

Table V. contains the transmission percentages for quartz and lamp-black.

TABLE V.

λ	.778	.971	1.14	1.33	1.55	1.78	2.14
Quartz	81.2	88.7	90.0	91.9	90.9	92.8	94.5
Lamp-black . .	Too small to measure.	Too small to measure.	Too small to measure.	01.5	02.5	01.5	01.0

The quartz was an unusually clear plate of rock crystal 6.8 mm. thick, cut perpendicular to the optical axis of the crystal. The lamp-black was a deposit formed by smoking a plate of the glass described in Table IV. In this way a known medium was introduced as a carrier for the smoke film. The film was thickened until a square centimeter of area was obtained. It was uniform and just heavy enough so that the outline of the filament of a glow-lamp at normal candle power was barely visible through it, when the lamp was held 30 cm. away from the eye in a dark room.

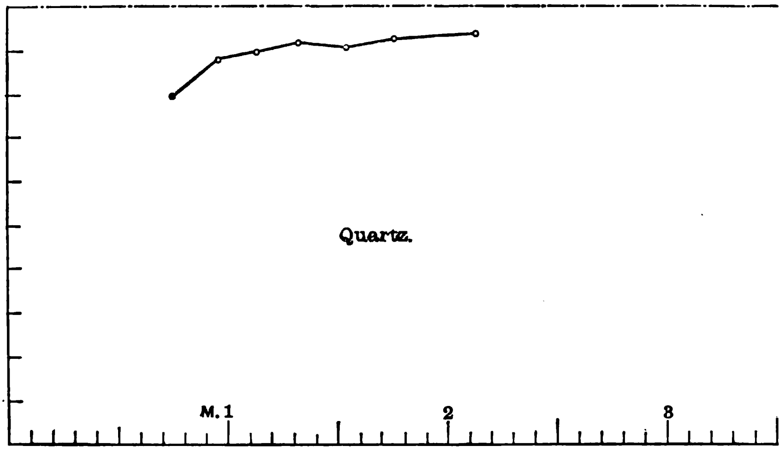

Fig. 4.

Figures 4 and 5 show these results plotted according to the conventions explained for Figs. 2 and 3. It will be noticed that the diathermancy of quartz increases with the wave-length as far, at least, as $\lambda = 2^{\mu}.14$. The results for lamp-black are not shown graphically.

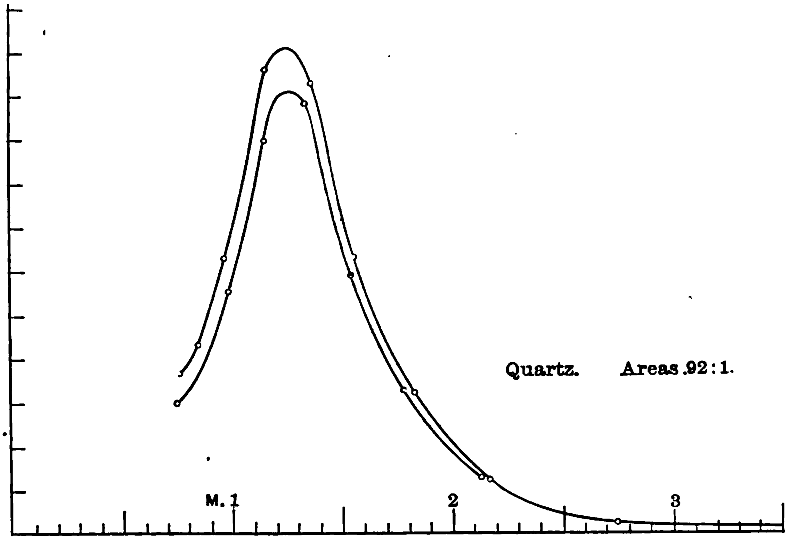

Fig. 5.

Table VI. gives similar results for cobalt glass.

TABLE VI.

λ	.775	.862	.940	1.19	1.41	1.58	1.86	2.37	2.86
Cobalt glass. . .	79.6	80.1	66.7	20.0	20.7	22.6	45.7	55.4	52.9

The plate was 2.8 mm. thick. In the visible spectrum cobalt glass shows a number of absorption bands, and Table VI. shows the presence of a deep band in the infra-red beginning at $\lambda = 0^{\mu}.862$. At $\lambda = 1^{\mu}.86$ the diathermancy has increased somewhat from the lowest values, but it does not, within the region studied, reach its former value again. Figures 6 and 7 show the results for cobalt glass graphically.

Table VII. shows the transmission percentages for absolute alcohol and a solution of chlorophyll in alcohol. These percentages are for layers of alcohol and chlorophyll solution 1 cm. thick, contained in a flat cell with thin glass walls. The percentages are computed on the basis of energy transmitted through glass sides and all. The same cell was used for both solutions, so the results are comparable.

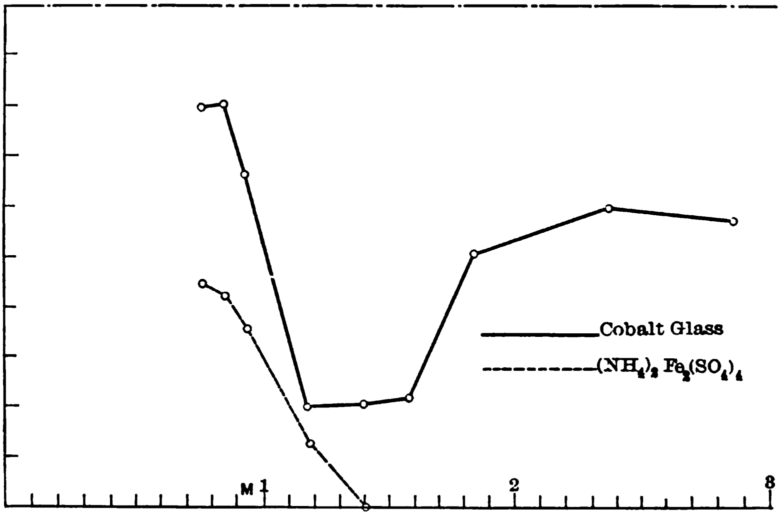

Fig. 6.

TABLE VII.

λ	.776	.863	.941	1.19	1.41	1.58	1.85	2.32	2.83
Alcohol (C_2H_6O)	75.9	77.3	75.8	40.2	01.8	01.5	too small to measure	too small to measure	too small to measure
Chlorophyll . . .	66.1	77.1	70.3	32.1	01.9	too small to measure	"	"	"

At $\lambda=0^{\mu}.863$ and $\lambda=1^{\mu}.41$ the solutions were equal in absorbing power. The transmission spectra for both liquids came to an end near $\lambda=1^{\mu}.4$. These results are shown graphically in Figs. 8 and 9.

Table VIII. contains the transmission percentage for the empty cell; for water; for a solution of oxyhæmoglobin, made up of 5 cc. cat's blood and 2cc. chloral in a liter of water; for a solution of commercial potassium aluminium sulphate in water, saturated at 0°; and for a solution of commercial ammonium aluminium sulphate in water, saturated at 0°. Centimeter layers of all these solutions were examined in the same cell, which was similar in every respect to the one used for alcohol and chlorophyll. These results are shown in Figs. 10 and 11.

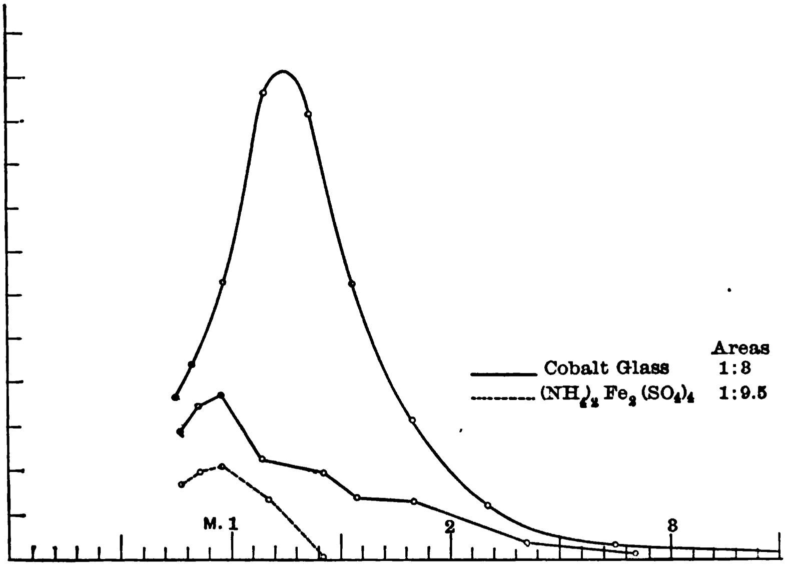

Fig. 7.

TABLE VIII.

λ	.779	.865	.945	1.19	1.41	1.59	1.89	2.29	2.82
Empty cell . . .		72.	73.	73.					
Water	76.2	74.4	58.4	14.4	too small to measure	too small to measure	too small to measure	too small to measure	too small to measure
Blood	68.9	74.7	55.6	16.0	"	"	"	"	"
$K_2Al_2(SO_4)_4$ . .	73.6	72.2	51.3	16.2	"	"	"	"	"
$(NH_4)_2Al_2(SO_4)_4$	73.8	76.2	54.8	14.7	"	"	"	"	"

For $\lambda = 0^{\mu}.865$ the transmission of the empty cell is seen to be a trifle less than that of the cell filled with the several solutions. This is not an impossible relation, because the reflection from the inner surfaces of the cell is cut down considerably when the cell is filled with the solutions, and the solutions were found highly diathermous at this point. The transmission spectra of these three solutions look very much like the spectrum of pure water. This seems to show that in alum solutions, commonly used to cut out dark heat, the alum is inactive. Whether alum in solution absorbs more radiant heat than distilled water has been questioned before, and H. N. Draper asks in "Nature,"[1] "Why use alum dissolved in water instead of water alone, to cut off obscure radiation?" In the next issue of that periodical he is answered on *a priori* grounds by Ch. Ed. Guillaume, that solid alum in its absorption occupies nearly the same place in the scale of solids that water does among liquids; consequently, if there be any difference in the selective absorption of the two substances, the alum solution will naturally be expected to cut out more radiation than the water alone.

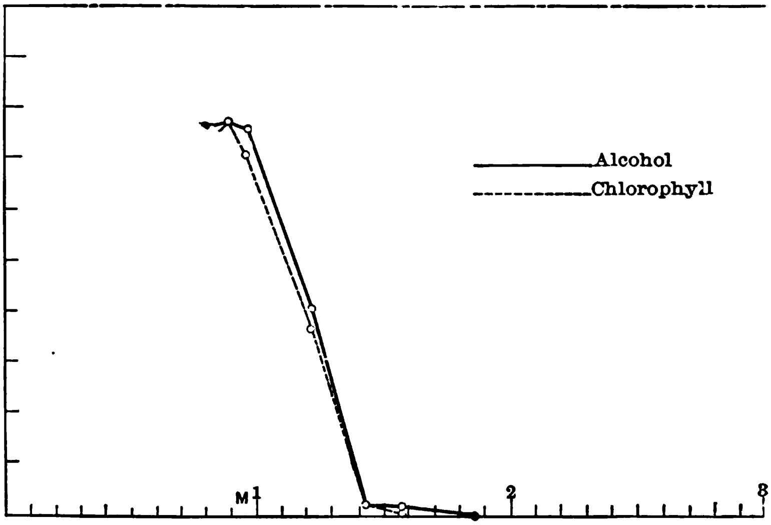

Fig. 8.

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[1] Vol. 44, p. 446.

Melloni,[1] using an Argand lamp with a glass chimney as a source, found that a layer of alum solution 9.21 mm. thick transmitted 12% of the total incident radiation, and that distilled water under like circumstances transmitted only 11%.

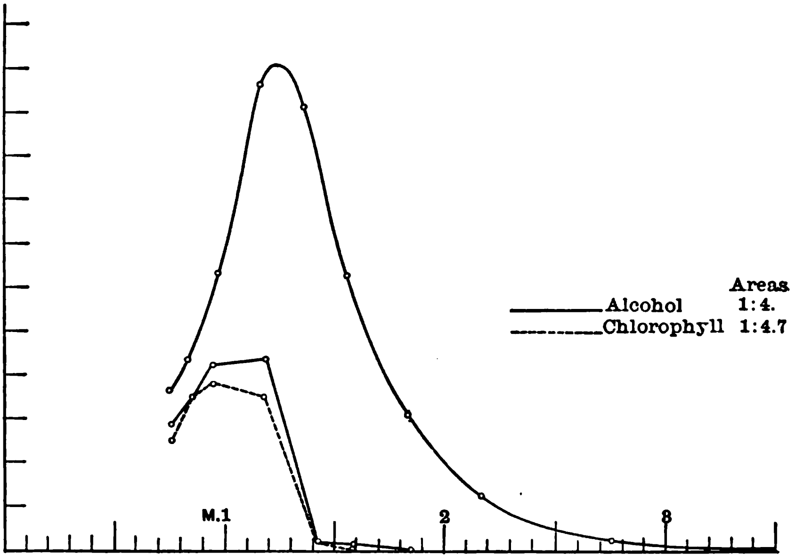

Fig. 9.

Shelford Bidwell,[2] using a paraffine lamp as a light source and the thermopile, obtained the following results: —

Solutions.	Diathermancy.
Empty cell	1000
Water, distilled	197
Tap water	200
Alum, saturated solution	204

F. C. Porter,[3] using the electric arc at 40 volts and 6.4 ampères, as a source and the Crooks radiometer, reached the following results: —

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[1] Thermochrose, p. 165.
[2] Nature, Vol. 44, p. 565.
[3] Nature, Vol. 45, p. 29.

	No. of revolutions of radiometer arms.	Time in half-seconds.
1. Unimpeded radiation	25	160
2. Through empty cell	25	254
3. Unimpeded radiation	25	152
4. Cell + water	25	820
5. Unimpeded radiation	25	164
6. Cell + water (second experiment) . .	25	832
7. Unimpeded radiation	25	160
8. Cell + alum solution	25 (time of two revolutions actually taken)	43,200
9. Unimpeded radiation	25	160

The spectra of all these solutions stopped before wave-length $1^{\mu}.41$ was reached. It was thought that possibly the quantity of alum in a centimeter layer of the solutions was not enough to produce a noticeable effect; consequently layers two centimeters thick were examined under like circumstances. The results appear in Table IX.

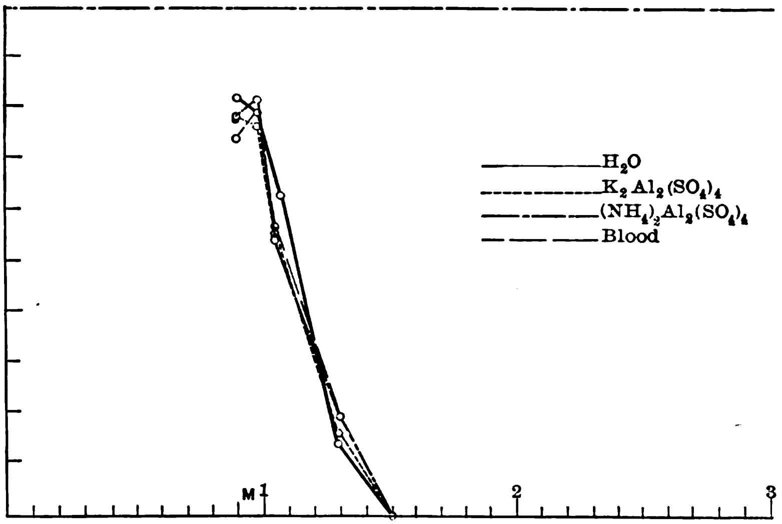
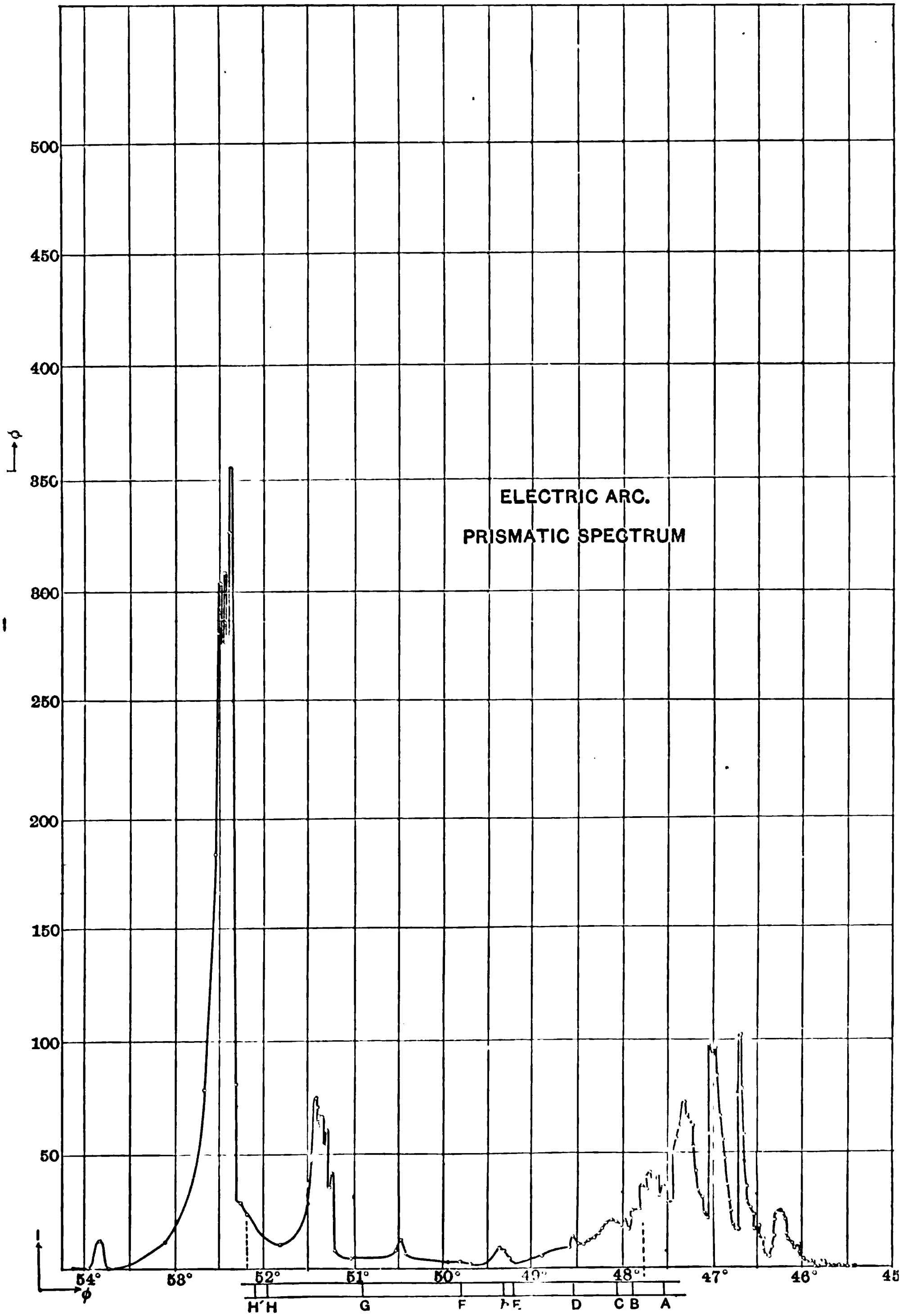


Fig. 10.

Fig. 3.


ELECTRIC ARC.
PRISMATIC SPECTRUM
500
450
400
350
300
250
200
150
100
50
ϕ
54°
53°
52°
51°
50°
49°
48°
47°
46°
45
H′ H
G
F
b E
D
C B
A

B. W. SNOW.

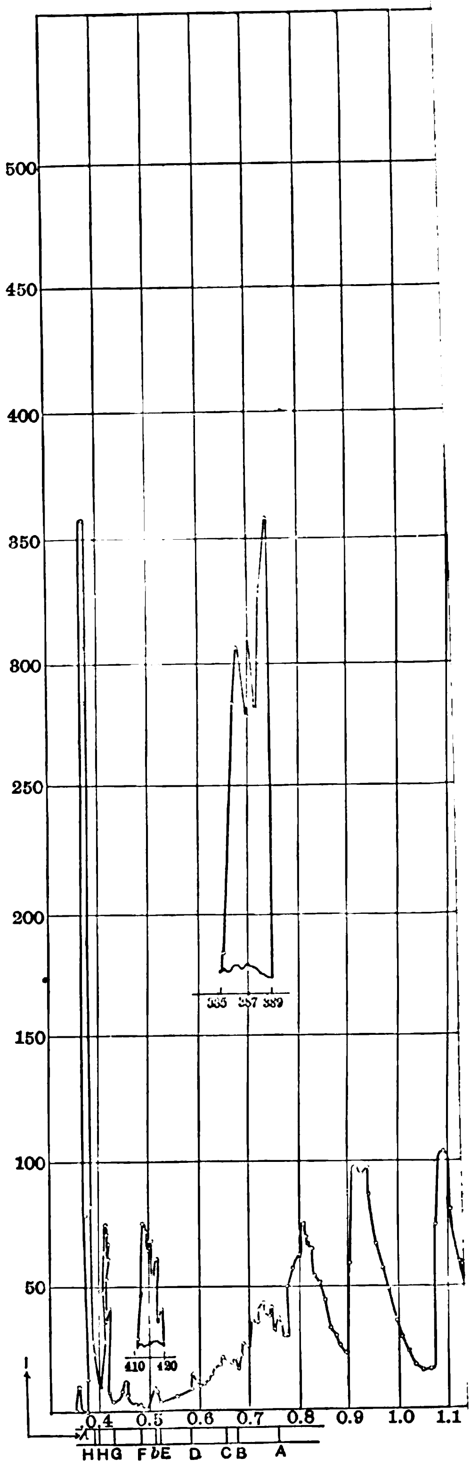

500
450
400
350
300
250
200
150
100
50
I
λ
0.4
0.5
0.6
0.7
0.8
0.9
1.0
1.1
H H G F b E D C B A
410
420
385
387
389

TABLE IX.

λ	.779	.865	.945	1.19	1.41	1.59	1.89	2.29	2.82
Water	77.1	71.7	44.2	03.7	too small to measure	too small to measure	too small to measure	too small to measure	too small to measure
Blood	72.0	68.4	42.3	05.1	"	"	"	"	"
$K_2Al_2(SO_4)_4$ . .	74.1	69.5	39.5	03.9	"	"	"	"	"
$(NH_4)_2Al_2(SO_4)_4$	75.8	69.6	42.6	05.4	"	"	"	"	"

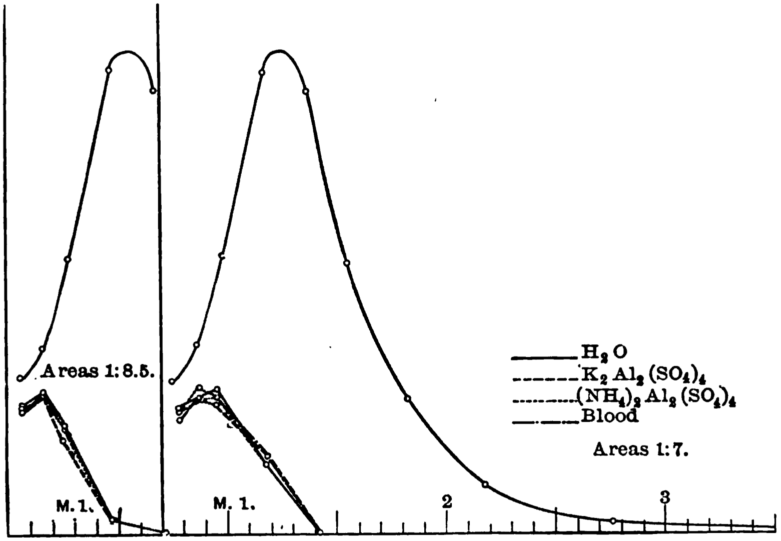


Fig. 11.

Table X. gives the transmission percentages for a centimeter layer of water solution of commercial ammonium-iron alum saturated at 0°. The cell was the one used for the other alum solutions.

TABLE X.

λ	.777	.863	.943	1.19	1.41	1.58	1.88	2.30	2.80
$(NH_4)_2Fe_2(SO_4)_4$	45.0	42.9	35.5	12.5	too small to measure	too small to measure	too small to measure	too small to measure	too small to measure

The contents of this table are shown graphically in Figs. 6 and 7.

Summary of Results.

1. The diathermancy of plate glass is fairly uniform through the infra-red down to $\lambda = 3$. The highest and lowest percentages of transmission are 82 and 72.

2. The diathermancy of hard rubber rises very rapidly just beyond the limits of the visible spectrum reaching the percentage 59 of transmission at $\lambda = 0^{\mu}.940$. It does not rise above 65% nor fall below 52% between $\lambda = 0^{\mu}.940$ and $\lambda = 3^{\mu}$.

3. The diathermancy of quartz increases with the wave-length from the limit of the visible spectrum to $\lambda = 3^{\mu}$.

4. Lamp-black absorbs almost non-selectively throughout the region studied.

5. Cobalt glass has a wide absorption band between $\lambda = 0^{\mu}.862$ and $\lambda = 1^{\mu}.85$.

6. The diathermancy of alcohol is nearly constant between $\lambda = 0^{\mu}.776$ and $\lambda = 0^{\mu}.941$. Beyond the latter wave-length it decreases rapidly and ceases at $\lambda = 1^{\mu}.4$. Chlorophyll in solution does not decrease the diathermancy of alcohol at $\lambda = .863^{\mu}$ and at $\lambda = 1^{\mu}.41$.

7. Neither potassium alum nor ammonium alum in solution alters the diathermancy of distilled water in the region studied.

8. Ammonium-iron sulphate in solution decreases the diathermancy of distilled water.

The work described in the foregoing paper was done in the Physical Laboratory of Cornell University during July and August, 1892, under the direction of Dr. Edw. L. Nichols, to whom I take pleasure in acknowledging my indebtedness for assistance and many important suggestions. I am indebted to Mr. E. S. Ferry also for help in making some of the observations preliminary to this study.

COLGATE UNIVERSITY, HAMILTON, N.Y.
January, 1893.

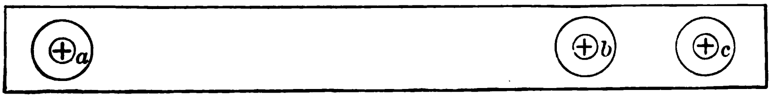
ON THE RELATION BETWEEN THE LENGTHS OF THE YARD AND THE METER.

BY WILLIAM A. ROGERS.

THE present determination of the length of the meter, expressed in terms of the yard, rests upon a combined yard and meter constructed by the writer in 1880. The description of the method of construction employed, and of the authorities upon which the total lengths depend, will be found in the Proceedings of the American Academy of Arts and Sciences for 1882–83, Vol. XVIII. It will be useful to give here a brief recapitulation of the results there found.

(*a*) Diagram showing the form of the bronze bar R_2, the ingredients of which are 16 parts of copper, 2½ parts of tin, and 1 part of zinc.

The inner circles, *a*, *b*, *c*, represent the polished surfaces of platinum-iridium plugs inserted in the metal. The lines traced upon these plugs are protected by circles of thin cover-glass cemented to the surface of the metal by means of Canada balsam. These cover-glasses are represented by the larger circles.



(*b*) Corrections to the yard (*ab*) and the meter (*ac*) adopted in 1882.

(*ab*) + .000008 in. = the Imperial Yard of Great Britain = Y;
(*ac*) − 1.3μ = the Metre des Archives = A_0.

Coefficient of expansion of the bronze bar $R_2 = 17.17\mu$ for each degree C.

(c) The meter depends on the following authorities: —

(1) A meter upon a copper bar having the X form, presented to the writer by the late Professor Tresca, who was at that time in charge of the Conservatoire des Arts et Metiers at Paris. This meter, designated T, was transferred by M. Tresca from a copy of the Metre des Archives, designated "No. 19," which was at that time provisionally adopted as the equivalent of the original standard. The transfer was made at two o'clock in the morning of February 6, 1880. From subsequent comparisons it appeared that —

$$T - 1.0\mu = \text{"No. 19" at } 13.70° \text{ C.},$$

and

$$T - 118.9\mu = \text{Metre des Archives at } 0° \text{ C.}$$

(2) An end-measure meter by Foment = F, of which the following relation to A_0 had been found: —

$$F - 8.4\mu = A_0 \text{ at } 0° \text{ C.}$$

(3) A combined yard and meter upon brass, made at the Bureau of Weights and Measures, Washington, and now in the possession of the Stevens Institute. The meter upon this bar, which is designated CS, was in 1883 compared with the provisional standard at that time adopted at the International Bureau of Weights and Measures with the resulting relation: —

$$CS + 310\mu = A_0 \text{ at } 0° \text{ C.}$$

(4) A meter belonging to the United States Coast Survey, known as the Berlin meter "No. 49," for which the following relation is given by Foester: —

$$\text{"No. 49"} + 21.4\mu = A_0 \text{ at } 0° \text{ C.}$$

The yard upon R_2 depends upon the following authorities: —

(1) A yard R_n upon nickel-plated steel made by the writer in 1880, and compared with Y by Mr. H. J. Chaney, the Warden of the Imperial Standards.

$$R_n + .000316 \text{ in.} = Y \text{ at } 62°.$$

(2) A yard upon the brass bar CS, also compared with Y by Mr. Chaney.

$$CS + .000081 \text{ in.} = Y \text{ at } 62°.$$

(3) The yard of the United States Bureau of Weights and Measures known as "Bronze 11."

$$\text{"Bronze 11"} + .000088 \text{ in.} = Y \text{ at } 62°.$$

Two values of the relation between the relative lengths of the yard and the meter have been published by the writer. The first one is given on page 398 of the Proceedings of the Academy, viz.: —

$$Y + 3.37027 \text{ in.} = A_0 \text{ at } 62°.$$

During the year 1887 the writer constructed for the Standards' Department of the London Board of Trade a combined yard and meter upon a bar of Baily's metal, which was transmitted by Mr. Chaney for this purpose. Before this standard, designated C_2, was returned to London it was compared with R_2, with the following results: —

For the Meter.			For the Yard.
No. of Comparisons.	τ	$R_2 - C_2$	$R_2 - C_2$
8	31.66°	-2.7μ	-0.1μ
9	33.83	-2.3	-1.2
9	40.42	-1.7	-0.2
8	47.52	-1.7	$+0.8$
8	72.75	-2.9	-0.8
	Means	-2.26	-0.30

Soon after the arrival of the standard at London a series of comparisons with the Imperial yard was made by Mr. Chaney. The results are given on page 7 of the Report to the Board of Trade for 1886–88, from which it appears that —

$$C_2 + .000128 \text{ in.} = Y.$$

Since $C_2 - .000012 = R_2$, we have $R_2 + .000140 \text{ in.} = Y$.

The original relation given must therefore be corrected by $+ .000132$ in. and the relation between the length of the yard and the meter is —

$$Y + 3.37014 \text{ in.} = A_0.$$

A remeasurement of the distance (*bc*) made at this time, the details of which are omitted, gave, using the new correction to R_2, —

$$Y + 3.37012 \text{ in.} = A_0.$$

This value was communicated to Professor Hazen of the Signal Office at Washington and was made the basis of the linear measure tables in his Handbook of Meteorological Tables, published in 1888. In the letter in which this result was communicated, the writer somewhat incautiously expressed the opinion that the true relation would be found to fall between the limits 39.37012 in. and 39.37015 in. In the preface to the volume, the first value is erroneously printed 39.3702 in. With the limitation that the corrections to the yard and the meter of R_2 are correctly given, this statement will probably be found to hold nearly true.

In view of the importance of a direct measurement of the space (*bc*) without the interference of the cover-glasses, it was decided to transfer to the edge of R_2 a new yard and meter, and in order that the conditions of comparison might vary as widely as possible, a provisional yard and meter were first laid off with the following subdivisions: —

Meter designated $R_{2e_1} = (ac)$. Yard designated R_{2e_1} was subdivided into 9 equal parts. Two separate 4-inch spaces were subdivided to inches, and the last inch to tenths of inches. The space from 3.3 in. to 3.4 in. was further subdivided into 10 equal parts, thus allowing a direct comparison of the space (*bc*) with the nearest hundredth of an inch. These auxiliary scales are designated $4\,i_1$ and $4\,i_2$. After the entire investigation had been completed, a second transfer was made to the other edge of the bar in which the 4-inch subdivisions of the yard were given rather large periodic errors. A new 4-inch space was also laid off. The former is designated R_{2e_2} and the latter $4\,i_3$.

The determination of the true value of (*bc*) consists of three operations, viz.: —

(*a*) The measure of the true value of the space $4\,i$ obtained by comparing it with each of the 4-inch spaces of the yard, corrected for the amount of the error in the total length. The correction for the line 3.37 in. will thus become known.

(*b*) The comparison of the space (*bc*) with the 4*i* space, the deviation from the distance 3.37 in. being measured with the micrometer of the microscope.

(*c*) The subtraction from the measured value of (*bc*) of the algebraic sum of the corrections to the distances (*ac*) and (*ab*). Thus, since the meter of R_2 is 1.3μ too long, and the yard is .00014 in. too short, the space (*bc*) will be .00019 in. too long.

The results, expressed in divisions of the micrometer, of the different measurements of the space (*bc*), are given below. One division of micrometer = .203μ = .000008 in.

I. Measurement of (*bc*) in terms of a 4-inch scale upon speculum metal. This scale has the last inch subdivided into 100 equal parts.

II. Measurement of (*bc*) in terms of a decimeter subdivided to millimeters, with the last millimeter subdivided to tenths of millimeters. For a description of these scales, see Astronomical Journal, No. 281.

Date.	τ	I. Excess over 3.37 in. at 62°.	II. Excess over 85.6 mm. at 62°.
1893 Jan. 14	40.3°	– 49.3 div.	– 37.7 div.
" 15	49.5	– 45.0	– 35.7
" 15	56.3	– 48.6	– 34.1
" 16	57.8	– 48.4	– 34.1
" 17	50.6	– 44.6	– 39.1
" 18	61.8	– 45.0	– 33.8
" 19	62.0	– 46.2	– 33.3
" 19	68.6	– 48.4	– 34.4
" 20	50.8	– 48.6	– 34.1
" 21	46.4	– 43.2	– 35.9
" 22	36.0	– 45.7	– 33.6
		means – 46.6	– 35.1

I.	Measured excess over 3.37 in.	– 46.6 div.
	Space (*bc*) too long	+ 24.0
	Correction to line 3.37 in.	+ 3.5
	Excess over 3.37 in.	– 19.1
	(*bc*)	= 3.370153 in.
II.	Measured excess over 85.600 mm. . . .	– 35.1 div.
	Space (*bc*) too long	+ 24.0
	Correction to line 85.6 mm.	+ 1.8
	Excess over 85.6 mm.	– 9.3
	(*bc*) = 85.60186 mm. =	3.370154 in.

III. Measurement of (bc) of R_2 and of (bc) of R_{2e_1} in terms of $4\,\tilde{i}_1$ and $4\,i_2$.

RELATIONS FOR TOTAL LENGTH.

For meter R_2,	*For yard R_2,*	*For meter R_{2e_1},*	*For yard R_{2e_1},*
$R_2 - 6.5$ div. $= A_0$.	$R_2 + 17.5$ div. $= Y$.	$R_{2e_1} + 1.2$ div. $= A_0$.	$R_{2e_1} + 13.6$ div. $= Y$.

Cumulative corrections to the 4-inch subdivisions of the yard R_{2e_1}: —

Subdivisions in inches	4	8	12	16	20	24	28	32	36
Corrections	$+1.0^d$	-2.9^d	-1.5^d	$+4.8^d$	$+6.7^d$	$+4.1^d$	$+5.6^d$	$+2.8^d$	$+0.0^d$.

CORRECTIONS TO SPACES $4\,i_1$ AND $4\,i_2$.

In the following tables the first column represents the summed series of corrections to the separate inches which are marked [ ], and the distributed corrections for each tenth of the fourth inch. The second column contains the corrections to the 4-inch space due to the error in total length. The third column contains

TABULAR CORRECTIONS TO $4\,i_1$.									TABULAR CORRECTIONS TO $4\,i_2$.						
Lines.	**Σ.**			**Sum.**	**$\frac{1}{100}$ in.**				**Σ.**				**$\frac{1}{100}$ in.**		
in.	div.	div.	$\frac{1}{10}$ in.	div.					div.	div.	$\frac{1}{10}$ in.	div.			
0	+ 0	+0		+ 0					+0	+ 0		+ 0			
[1]	+23	+0		+23					+9	− 9		+ 0			
[2]	+37	+0		+37	in.	div.	div.	div.	+6	−18		−12	div.	div.	div.
[3]	+45	+0	+ 0	+45	3.30	+46	+0	+46	+9	−27	+ 0	−18	−24	+ 0	−24
3.1	+40	+1	+ 8	+49	3.31	+43	+0	+43	+8	−28	+10	−10	−24	− 1	−25
3.2	+36	+1	+20	+57	3.32	+39	−3	+36	+7	−29	+14	− 8	−24	− 9	−33
3.3	+31	+1	+14	+46	3.33	+36	−9	+27	+6	−30	+ 0	−24	−24	−15	−39
3.4	+27	+1	−16	+12	3.34	+32	−7	+25	+5	−31	+ 0	−26	−24	−15	−39
3.5	+22	+1	−10	+13	3.35	+29	−7	+22	+4	−32	+ 2	−26	−24	−11	−35
3.6	+18	+1	− 8	+11	3.36	+25	−2	+23	+3	−33	+ 5	−25	−25	− 3	−28
3.7	+13	+2	− 1	+14	3.37	+22	−3	+19	+2	−34	+ 6	−26	−25	− 4	−29
3.8	+ 9	+2	+10	+21	3.38	+18	−2	+16	+2	−35	+16	−17	−25	− 8	−33
3.9	+ 4	+2	+11	+17	3.39	+15	−2	+13	+1	−36	+10	−25	−26	+ 0	−26
[4.0]	+ 0	+2	+ 0	+ 2	3.40	+12	+0	+12	+0	−37	+ 0	−37	−26	+ 0	−29

SPACES (*bc*) IN TERMS OF THE CORRECTED VALUES OF $4 i_1$ AND $4 i_2$.

Date.	*From* R_2.	*From* R_{2e_1}.	*From* R_2.	*From* R_{2e_1}.
Jan. 30.	−58.8 div.	−12.2 div.	−49.5 div.	− 2.9 div.
" 31.	−66.8	−15.1	−60.3	− 6.4
Feb. 1.	−64.1	−11.7	−55.6	− 3.2
" 1.	−65.4	−15.2	−56.0	− 5.8
" 1.	−60.2	−12.5	−54.9	− 4.8
" 2.	−59.4	− 9.8	−49.0	+ 0.6
Means.	−62.4	−12.7	−54.2	− 3.7
Corrections to (*bc*). . . .	+24.0	+24.0	+12.4	+12.4
Corrections to line 3.37 in.	+19.0	−29.0	+19.0	−29.0
Excess over 3.37 in.. . .	−19.4	−17.7	−22.8	−20.3
Excess in parts of an inch.	−.000155	−.000142	−.000182	−.000162

the corrections to the $\frac{1}{10}$-inch subdivisions of the fourth inch. The fourth column is formed by taking the sum of the three previous columns. The subsidiary tables contain the corrections to the lines from 3.30 in. to 3.40 in. The correction to line 3.37 in. is required.

IV. Measurement of (*bc*) of R_2 and of R_{2e_2} in terms of $4 i_3$.

In order to introduce a wide variation in the conditions of comparison, the corrections are given for both edges of the terminal lines of the yard and the meter R_{2e_2}.

RELATIONS FOR TOTAL LENGTH.

For the right edge of the terminal line of —		For the left edge of the terminal line of —	
The meter.	*The yard.*	*The meter.*	*The yard.*
$R_{2e_2} - 6.2$ div. $= A_0$.	$R_{2e_2} + 0.1$ div. $= Y$.	$R_{2e_2} - 49.5$ div. $= A_0$.	$R_{2e_2} - 37.0$ div. $= Y$.

Cumulative corrections to the 4-inch divisions of the yard R_{2e_2}: —

Subdivisions in inches	4	8	12	16	20	24	28	32	36
Corrections	-9.3^d	-15.0^d	-16.6^d	-12.9^d	-22.0^d	-18.0^d	-5.3^d	-4.9^d	+0

CORRECTIONS TO THE 4-INCH SPACE $4 i_3$.

Lines.	Comparison with the Right Edge of the Terminal Line of the $4 i_3$ Space.								Comparison with the Left Edge of the Terminal Line of the $4 i_3$ Space.							
	Σ.			Sum.	$\frac{1}{100}$ inch.							Sum.	$\frac{1}{100}$ inch.			
in.	div.	div.	$\frac{1}{10}$ inch.		in.	div.	div.	div.	div.	div.	$\frac{1}{10}$ inch.	div.	in.	div.	div.	div.
0	+ 0.0	0.0		0.0					0.0	0.0						
[1]	− 0.6	+ 0.5		− 0.1					+ 9.6	− 10.1		− 0.5				
[2]	+ 5.9	+ 1.1		+ 7.0					+ 26.2	− 20.2		+ 6.0				
[3.0]	+ 2.7	+ 1.6	+ 0.0	+ 4.3	3.30	+ 4.1	+ 0.0	+ 4.1	+ 33.0	− 30.3	+ 0.0	+ 2.7	3.30	+ 2.7	+ 0.0	+ 2.7
3.1	+ 2.4	+ 1.7	− 2.9	+ 1.2	3.31	+ 3.8	+ 3.3	+ 7.1	+ 29.7	− 31.3	+ 1.2	− 0.4	3.31	+ 2.4	+ 3.3	+ 5.7
3.2	+ 2.1	+ 1.7	+ 4.6	+ 8.4	3.32	+ 3.5	− 4.1	− 0.6	+ 26.4	− 32.3	+ 12.8	+ 6.9	3.32	+ 2.1	− 4.1	− 2.0
3.3	+ 1.9	+ 1.8	+ 0.4	+ 4.1	3.33	+ 3.3	+ 3.0	+ 6.3	+ 23.1	− 33.3	+ 12.7	+ 2.5	3.33	+ 1.8	+ 3.0	+ 4.8
3.4	+ 1.6	+ 1.8	− 2.0	+ 1.4	3.34	+ 3.0	− 3.9	− 0.9	+ 19.8	− 34.4	+ 14.4	− 0.2	3.34	+ 1.5	− 3.9	− 2.4
3.5	+ 1.3	+ 1.9	− 5.2	− 2.0	3.35	+ 2.7	− 3.9	− 1.2	+ 16.5	− 35.4	+ 15.3	− 3.6	3.35	+ 1.2	− 3.9	− 2.7
3.6	+ 1.1	+ 1.9	− 8.9	− 5.9	3.36	+ 2.5	+ 1.8	+ 4.3	+ 13.2	− 36.4	+ 15.7	− 7.5	3.36	+ 0.9	+ 1.8	+ 2.7
3.7	+ 0.8	+ 2.0	− 4.8	− 2.0	3.37	+ 2.2	− 0.2	+ 2.0	+ 9.9	− 37.5	+ 23.9	− 3.7	3.37	+ 0.5	− 0.2	+ 0.3
3.8	+ 0.5	+ 2.0	− 2.4	+ 0.1	3.38	+ 1.9	− 4.1	− 2.2	+ 6.6	− 38.5	+ 30.4	− 1.5	3.38	+ 0.2	− 4.1	− 3.9
3.9	+ 0.3	+ 2.1	+ 0.2	+ 2.6	3.39	+ 1.6	+ 1.1	+ 2.7	+ 3.3	− 39.5	+ 37.1	+ 0.9	3.39	− 0.1	+ 1.1	+ 1.0
4.0	+ 0.0	+ 2.2	+ 0.0	+ 2.2	3.40	+ 1.4	+ 0.0	+ 1.4	+ 0.0	− 40.5	+ 0.0	− 40.5	3.40	− 0.4	+ 0.0	− 0.4

The mean of the two values + 2.0 and + 0.3 of the correction for line 3.37 in., viz. + 1.1 div., will be adopted.

SPACES (*bc*) IN TERMS OF THE CORRECTED VALUE OF $4 i_3$.

Date.	$R_2 - 4 i_3$.	$R_{2e_2} - 4 i_3$.	$R_2 - 4 i_3$. Left edge.	$R_{2e_2} - 4 i_3$. Right edge of line for meter. Left edge of line for yard.	$R_{2e_2} - 4 i_3$. Left edge of line for meter. Right edge of line for yard.
1893. Feb. 27	−49.9 div.	−27.0 div.	−33.9 div.	+10.7 div.	−70.1 div.
" 28	−48.0	−30.9	−32.8	+11.0	−68.8
" 28	−45.6	−29.4	−33.0	+14.0	−67.5
" 28	−44.0	−28.8	−37.2	+16.8	−72.2
" 29	−44.2	−24.9	−36.6	+16.2	−63.0
" 29	−48.2	−24.0	−34.9	+13.8	−67.9
" 29	−45.1	−24.9	−29.7	+11.5	−65.7
Mar. 1	−47.6	−26.7	−32.4	+11.8	−69.1
" 1	−43.4	−24.8	−31.9	+17.3	−70.0
Means.	−46.2	−26.8	−33.6	+13.7	−68.3
Corrections to (*bc*). . .	+24.0	+ 6.3	+12.5	−30.8	+49.6
Correction of line 3.37 in.	+ 1.1	+ 1.1	+ 1.1	+ 1.1	+ 1.1
Excess over 3.37 in. .	−21.1	−19.4	−20.0	−16.0	−17.6
Excess in parts of an inch.	−.000169	−.000155	−.000160	−.000128	−.000141

Adopting the mean of the 11 determinations, we have finally:—

$$Y + 3.370155 \text{ in.} = A_0.$$

Dr. Gould, in his annual address before the American Meteorological Society, gives for the result of the latest determination at Breteuil:—

$$Y + 3.37000 \text{ in.} = A_0.$$

It is to be noted, however, that this determination is the result of only secondary comparisons through the toise. If this relation should eventually be found to be true, meter R_2 would require a further correction of -3.9μ, on the supposition that the correction for the yard is correctly given.

SHANNON PHYSICAL LABORATORY, COLBY UNIVERSITY,
March 29, 1893.

ON THE INFRA-RED SPECTRA OF THE ALKALIES.[1]

By Benjamin W. Snow.

I. INTRODUCTION.

OUR knowledge of the infra-red portion of the solar spectrum is confined almost entirely to investigations which have been made during the last fifty years. As early as the year 1840, Sir John Herschel[2] succeeded in showing an uneven distribution of energy in that part of the solar spectrum lying beyond the red, by allowing the spectrum of the sun's rays to fall upon a paper moistened with alcohol, and observing that at certain places in the invisible spectrum the paper dried more rapidly than at others. From this he concluded that the continuous solar spectrum is crossed at intervals by cold bands.

Later investigators have employed a variety of methods to confirm these results. Draper,[3] the two Becquerels[4] and Lommel[5] by means of phosphorescent plates, Fizeau and Foucault[6] by using a thermometer, Lamansky,[7] Mouton,[8] and Dessains[9] with the thermopile, Abney[10] by photographic methods, and finally

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[1] From Wiedemann's Annalen Bd. 47 (communicated by the author).

[2] *Sir John Herschel*, Proc. Roy. Soc., p. 209, 1840. Phil. Mag., **16**, p. 331, 1840.

[3] *Draper*, Phil. Mag., **24**, p. 456, 1842; **11**, p. 157, 1881.

[4] *E. Becquerel*, Compt. rend., **69**, p. 999, 1869; **77**, p. 302, 1873. *H. Becquerel*, Compt. rend., **96**, p. 123, 1883; **99**, p. 417, 1884. Ann. de Chim. et de Phys., **30**, p. 33, 1883.

[5] *Lommel*, Wied. Ann., **40**, p. 681, 1890; **40**, p. 687, 1890.

[6] *Fizeau* and *Foucault*, Compt. rend., **25**, p. 449, 1847.

[7] *Lamansky*, Monatsber. der k. Acad. d. Wiss. zu Berlin, p. 638, 1871. Pogg. Ann., **146**, p. 207, 1872. Phil. Mag., **43**, p. 282, 1872.

[8] *Mouton*, Compt. rend., **89**, p. 298, 1879.

[9] *Dessains*, Compt. rend., **95**, p. 435, 1882.

[10] *Abney*, Phil. Trans., II., p. 653, 1880; p. 457, 1886.

Langley[1] with the aid of the bolometer, have not only verified the three bands first suspected by Herschel, but have found many new ones, and have given to each its respective wave-length.

Captain Abney appears to have been the first person who succeeded in studying the infra-red spectra of the metals. In a paper[2] published in 1879, he stated that the emissive spectrum of lithium contains an infra-red line which coincides in position with a line previously discovered by him in the absorption spectrum of this metal. Later experiments[3] showed that sodium also, when burned in the electric arc, has two infra-red lines having wave-lengths $\lambda = .8187\,\mu$ and $\lambda = .8199\,\mu$ respectively. The intensity of these lines was estimated at 3, the intensity of the D lines being taken as 10.

Apparently without a knowledge of Abney's work, H. Becquerel found, during the next four years, that many other metals besides sodium and lithium have interesting spectra in the infra-red. The two sodium lines discovered by Abney were again determined by Becquerel, and a third line of still greater wave-length observed.

His first method[4] for determining the wave-lengths of these lines consisted in projecting upon the slit of the spectrometer a real image of the arc in which the metals were burned, and receiving upon a phosphorescent plate the spectrum produced by a calibrated prism. These experiments were repeated a year later,[5] when the prism producing the spectra was replaced by a Rutherford grating. Mention will again be made of these researches of Becquerel, and a comparison drawn between his results and those now under consideration, when the latter are discussed at the close of the present paper.

From conclusions reached in a recent series of investigations, Kayser and Runge[6] have shown, in an article "Ueber die Spectren der Alkalien," that the wave-lengths of the spectral lines of the metals belonging to this group bear to each other a mathematical relation, and may be represented by a formula. The constants of these formulæ, being determined by measurements made in the visible and ultra-violet portions of the spectrum, can rightfully be used, strictly speaking, only in those regions for which they were found. Since, however, these authors ascribe to their formulæ a general validity, and inasmuch as the wave-lengths of the infra-red lines obtained from them by extrapolation do not in all cases coincide with the results of previous observers, it appears to the present writer not altogether without interest to submit anew to a careful study the heat spectrum of the alkaline metals.

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[1] *Langley*, Report of the Mt. Whitney Expedition, p. 131, 1884. Phil. Mag. (5), 26, p. 511, 1888.

[2] *Abney*, Phil. Mag. (5), 7, p. 316, 1879.

[3] *Abney*, Proc. Roy. Soc., 32, p. 443, 1881.

[4] *H. Becquerel*, Compt. rend., 97, p. 71, 1883. Ann. de Chim. et de Phys., 30, p. 45, 1883.

[5] *H. Becquerel*, Compt. rend., 99, p. 374, 1884.

[6] *Kayser* und *Runge*, Wied. Ann., 41, p. 306, 1890.

II. EXPERIMENTAL METHODS AND APPARATUS.

1. *The Bolometer.*

For a considerable period the bolometer has proven itself to be the best means of studying spectra in the infra-red. This instrument in the perfected state, to which it has been developed by such men as Langley, Ångström, Robert von Helmholtz, and Rubens, fulfilled the conditions necessary to the solution of the present problem. It has been shown that resistances sensitive to temperature can be made so narrow that even in a very pure spectrum they do not exceed in width the breadth of the spectral lines, and that the delicacy of this method far exceeds that of photographic processes.

After a long series of experiments, platinum was chosen as the material for the bolometer resistances. This metal, it is true, is never to be obtained in such a degree of purity[1] that its temperature coefficient equals that of silver, copper, tin, or iron, but it possesses, on the other hand, in its durability and convenience of working, such superior advantages over all other materials, that the loss in sensitiveness, conditioned by its smaller temperature coefficient, is more than compensated.

Professor Edward L. Nichols, of Cornell University, very kindly supplied me with a quantity of platinum wire drawn in silver, from the workshops of W. and L. E. Gurley, of Troy, N.Y. While still in its covering of silver, this fine platinum wire, only .005 mm. in diameter, was hammered flat on a polished anvil, after which the silver was dissolved with nitric acid from the platinum ribbon within. The platinum strip so formed had a breadth of .05 mm. From this and its slightly increased length due to the hammering its thickness was calculated to be .00036 mm.

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[1] The smallest traces of iridium seriously affect the result.

Two platinum threads prepared in this way were fastened side by side with shellac upon a little frame of mica. The frame itself was then fastened in such a way in a circular wooden case that one of the strips, which had previously been blackened by smoking in a turpentine flame, came directly opposite a rectangular opening in the cover, and was thus exposed to all radiations, while the other was completely protected by the walls of the case. Corresponding to the size of the opening in the diaphragm of the casing (3 x 7 mm.), the platinum threads, when in position, had a length of 7 mm.

Figure 1 represents the arrangement of the interior of the little wooden box. Copper wires, soldered to the ends of the platinum threads, connected these strips with the three brass sleeves *A*, *B*, *C*. These sleeves were then slipped over three corresponding rods in the bolometer sheathing or holder, and thus served to support the little case in its place, and at the same time to afford electrical contact with the interior. The points *A* and *C* led to the battery, while *B* formed one point of contact with the galvanometer.

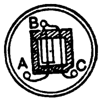

Fig. 1.

The two resistances were made as nearly alike as possible, and each measured about 75 ohms. The other two resistances in the Wheatstone's bridge were made of fine German silver wire, and wound non-inductively upon corks.

It was soon found that the task of balancing the Wheatstone's bridge would be made much lighter if small changes could be introduced at pleasure in the relation of the resistances forming the four arms. To attain this result, the two branches of the bridge last named, made of German silver, were connected by a German silver wire 50 cm. long and .5 mm. in diameter, upon which rested a sliding contact.

A short trial, however, soon proved that a simple displacement of this point of contact was not well adapted to a rapid and accurate balancing of the bridge. With the exceedingly high degree of sensitiveness of the galvanometer employed, a change in the position of the sliding contact of a fraction of a millimeter caused a throw of the galvanometer needle of several meters, a circumstance which rendered the manipulation of the apparatus exceedingly difficult. There was accordingly placed upon the bridge wire *MM'* (Fig. 2) a second movable contact, and both of these sliding contacts, *p* and *q*, were joined with the ends, *P* and *Q*, of a straight platinum wire, upon which slipped a small glass T-shaped tube filled with mercury. The latter formed the other terminal of the galvanometer circuit. *AB* and *BC* were the two platinum resistances in the circular case, and *AM* and *CM'* the other two resistances, joined by the bridge wire *MM'*. The battery connection was made at points *A* and *C*.

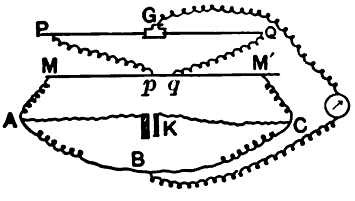

Fig. 2.

With this apparatus, the balancing of the bridge was effected in the following manner: The sliding contacts, *p* and *q*, were first placed in such a position that the point which had the same potential as the point *B* should fall between them. The distance between these movable contacts was then made as small as possible. Since the resistance of the connecting wires, *pP* and *qQ*, was negligible in comparison with that of the bridge wire, there existed the same fall of potential between the points *P* and *Q* as between the points *p* and *q*. It was possible, therefore, with this arrangement to vary at will the sensitiveness of the apparatus within wide limits. The approximate adjustment was made by displacing *p* and *q*, and the final balancing accomplished by slipping the tube *G* along the wire *PQ*.

To determine the sensitiveness of the instrument to thermal changes, a method was employed which has been described by Rubens and Ritter.[1]

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[1] *Rubens* and *Ritter*, Wied. Ann., 40, p. 62, 1890.

In parallel arc with one of the bolometer resistances of 75 ohms was placed a shunt of 1000 ohms, and around two ohms of this resistance was laid a second shunt of 100 ohms (see Fig. 3). In the latter circuit was placed a key. Opening or closing this key changed the resistance of the first shunt circuit by nearly $\frac{2}{1000} \cdot \frac{2}{100}$ of its former amount, whereby the entire resistance of the arm of the bolometer suffered a change amounting nearly to $\frac{75}{1000} \cdot \frac{2}{1000} \cdot \frac{2}{100}$. The temperature coefficient of the platinum employed being .0030, the sensitiveness to temperature, as calculated from the above figures and the throw α, of the galvanometer, was found to be

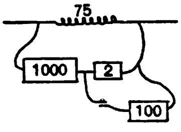

Fig. 3.

$$k = \frac{75}{1000} \cdot \frac{2}{1000} \cdot \frac{2}{100} \cdot \frac{1}{.0030\,\alpha}.$$

With the degree of astaticism used throughout the experiments, α was about 130 mm. The constant k was therefore found to be

$$k = \frac{1}{130000}\,°\,\mathrm{C}.$$

With this sensitiveness, a standard candle at a distance of 1 m. produced a deflection of 150 mm.

2. *The Galvanometer.*

It is well known that it is not difficult to make a bolometer sufficiently delicate to indicate minute differences of temperature, provided the surface exposed to the radiation may be made sufficiently large. If, however, it is desired, as in the present investigation, to discern fine lines in the spectrum by the heat which they emit, it is necessary to use for the purpose a linear bolometer in which the surface upon which the radiations fall comprises only a small fraction of a square millimeter. The illuminated filament of platinum in the bolometer already described, being 7 mm. in length and .05 mm. in breadth, exposed a surface of about $\frac{1}{3}$ sq. mm.

Since the sensitiveness of a bolometer is nearly proportional to the square root of its available surface, it was necessary to use a galvanometer of the highest degree of delicacy in order that the deflections might be sufficiently large to render the observations trustworthy. After many fruitless attempts to make use of the instruments already at hand, it was decided to construct one which would have the required degree of sensitiveness. Since a delicate galvanometer is often required in other lines of work, it may be not altogether without profit to give the details of its construction.

In order to obtain the highest efficiency from the galvanometer, it is necessary to use an astatic system which combines the smallest possible moment of inertia with the greatest strength and uniformity of magnetization. These two purposes were accomplished in the following arrangement.

After the model of Lord Kelvin's instruments, the magnets consisted of twelve little steel bars, 3 and 4 mm. long, fastened with shellac to the ends of an exceedingly fine, thin-walled capillary tube, three magnets being on the front side of the tube, and three on the back, at each end. A very small watch spring furnished the steel from which the magnets were made. This was first heated in potassium ferrocyanide and then hardened in mercury. Half-way between the two systems of magnets, and resting in its little stirrup of tissue paper, was placed a small mirror 5 mm. in diameter and .14 mm. thick.

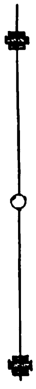

Fig. 4.

To make a mirror sufficiently accurate for the purpose, about fifty microscope cover-glasses were cut in pieces 5 mm. square, cleaned, and laid upon a piece of plane glass. These little plates were then viewed by the monochromatic light of the sodium flame, and the interference figures observed which were produced in the enclosed layer of air. It was found possible to tell by the form and position of these interference bands which pieces were adapted to the manufacture of mirrors.

Such interference figures are in general elliptically or hyperbolically curved, thus showing that the surface under examination has two noticeably different radii of curvature. It is clear that such surfaces can give no perfect images, inasmuch as an astigmatic eye would be necessary to see objects in their natural form.

If, on the other hand, the interference bands form systems of parallel straight lines or concentric rings, we have to deal with plane surfaces or surfaces of revolution, which are, in most cases, capable of giving good definition.

A number of these pieces were thus selected from the rest, silvered, and examined with a telescope and scale. Finally, that one which seemed the most nearly perfect was fastened, as already described, upon the needle of the galvanometer.

The weight of the entire needle, including the twelve magnets, the mirror, holder, and connecting rod, was about 80 milligrams.

As a suspension for the needle, an exceedingly fine quartz fiber, 40 cm. long, was employed, instead of one of the fibers 10 c.m. long which Professor C. V. Boys very kindly sent for the purpose. It seemed at that time that the torsion of the latter would be too great when compared with the very feeble directive force employed. This was, however, an erroneous belief, as was abundantly proven by later experiments.

Next to the construction of the needle, the greatest care was given to the design of the coils. Particularly did it seem of the greatest moment to make the best possible use of the space available for winding the wire. With this in view, several points were to be considered. The end to be attained demanded that the coils be given such a form that a current of a certain strength, and meeting within the coil the smallest possible resistance, should exert upon the needle the maximum magnetic effect. A simple consideration shows[1] that the space to be filled with wire must be given such a form that its surface will be bounded by the magnetic lines of force due to the current. The effectiveness will be still further increased if the wire is brought as close as possible to the needle.

In order to conform to these several conditions, a device was employed by which it was possible completely to dispense with the spool or thimble upon which the wire is usually wound, and at the same time to give the coils approximately the desired shape.

These ends were accomplished by providing a brass block,

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[1] *Maxwell*, Electricity and Magnetism, second ed., Art. 717; *Ayrton*, *Mather* and *Sumpner*, Phil. Mag. (5), 30, p. 63, 1890.

acefdb (shown in axial section in Fig. 5), the cylindrical part being 3 cm. in diameter, and the conical end at *ab* being 6 mm. in diameter. Upon this block was fastened, by the set screw *G*, the brass collar *H*. Against this ring as a shoulder rested the brass plate *CD*, having in its center a circular opening which allowed the plate to be supported upon the cylindrical part of the block. At the conical end, *ab*, a screw *S* held in place the plate *AB*. These two plates were then situated 15 mm. apart.

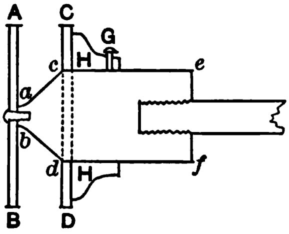

Fig. 5.

The space enclosed between this form and the two plates was then wound with wire. The part as far as *cd* contained 1100 turns of wire .25 mm. in diameter, and the remaining space was filled with 700 turns of wire .5 mm. in diameter.

The wire, while still in position between the plates upon the brass form, was then placed in a bath of melted paraffine, in which it remained until thoroughly heated. The vessel with all its contents was next placed under the receiver of an air-pump and the air withdrawn. As the vacuum became higher, the air which had been confined in the silk covering of the wire escaped, while at the same time the moisture was vaporized which had found its way, during the process of winding, to the insulating material around the wire. As soon as the atmosphere was again admitted to the receiver, the increased pressure caused the paraffine to fill the smallest interstices between the wire, so that, when cold, the whole formed a compact mass easy to handle.

The superfluous paraffine was then cut away, the screws *S* and *G* removed, and the plate *AB* quickly warmed in the flame of the Bunsen burner. Before the heat had been communicated to the remainder of the material, the plate slipped away from the wire, which still remained imbedded in the paraffine. The blast-lamp was then directed against the brass block, which in like manner became hot, and was removed without injury to the wire.

The coil in this form, held together as it was only by the paraffine, and supported solely by the rear plate *CD*, was now ready for use in the construction of the galvanometer.

Two coils made in this way were mounted in a rectangular case directly below two similar coils (see Fig. 6), and so placed that the surfaces which, in the process of construction, had been formed by the plate *AB*, were now situated opposite each other and 1 mm. apart. The front and the rear sides of the case, upon which the coils were mounted, were made of wood, the other two sides being formed simply of narrow panes of glass. It was easy, therefore, to examine from the outside the condition of the interior, so that any lack of freedom in the suspended system could readily be corrected. Binding-screws, placed upon the front wall of the galvanometer case, formed the means of electrical contact with the coils within. That part of the wall which supported the two front coils was movable, and could be taken out and replaced at pleasure. The interior was thus at all times accessible.

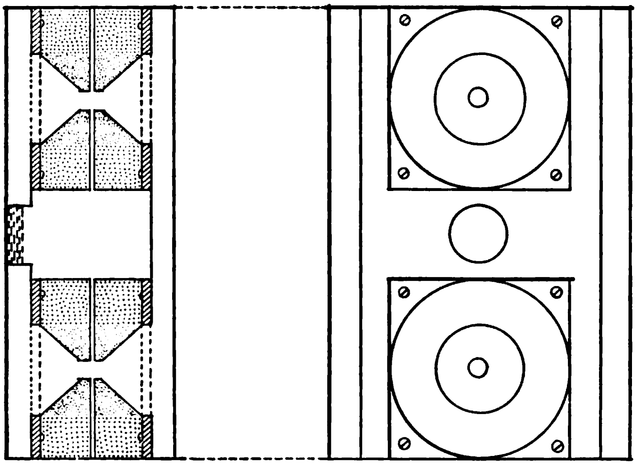

Fig. 6.

The description of the galvanometer will now be complete when mention is made of the fact that the instrument was placed upon a base, provided in the usual way with three levelling-screws, and that upon the upper part of the case was secured a glass tube 40 cm. long, at the upper end of which was fastened the quartz thread for the suspension.

The characteristic constants of this galvanometer were as follows: When the coils were connected in series, the current traversed 7200 turns of wire, of which the total resistance was 140 ohms. When the controlling magnet near the instrument was in such a position that a double vibration of the needle was performed in 20 seconds, 1 mm. deflection on the scale 3 m. away indicated a current of 1.5×10^{-11} ampères. In conducting the final experiments, however, it was found that a sensitiveness less than this was amply sufficient, and the time of vibration was accordingly reduced to 7 seconds. Even then its delicacy was notably higher than that of the commercial instruments.

3. *The Calibration of the Prism.*

A prism of medium silicate-flint glass of high dispersive power, mounted upon a spectrometer from the workshops of Schmidt and Haensch, formed the apparatus with which the spectra were studied. The prism was preferred for this purpose to a diffraction grating, because the superposition of the spectra produced by the latter rendered difficult the detection and identification of lines in the infra-red, and further because the energy in the prismatic spectrum is far more intense than is developed in any one of the numerous diffraction spectra.

After the method recently described[1] by Rubens, a number of calibrations of the prism were made, in all of which Dr. Rubens himself kindly assisted. The several experiments yielded results agreeing well among themselves, from which the dependence of the wave-length of the infra-red rays upon the angle of deviation could be determined within about five parts in a thousand.

The results of the first calibration are shown graphically in Fig. 1, Plate I. This figure represents the energy spectrum of the source of light, crossed by the heavy interference bands. The double thickness of the layer of air producing the interference, multiplied by the cosine of the angle of incidence i of the rays, gave $k = 6.677\mu$, as was determined from observations in the visible spectrum. The first infra-red interference band was of the 9th order, from which, accordingly, for this band, $n=9$. With the aid of these constants and the observed position of the maxima and minima, corrected before using by the method of envelopes given by Rubens, were calculated the values of the wave-lengths corresponding to the following angles of deviation ϕ.

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[1] *Rubens*, Wied. Ann., 45, p. 238, 1892.

TABLE I.

NAME.	ϕ	λ	NAME.	ϕ	λ
H_γ	50° 51½′	0.434 μ	b_3	46° 49′	1.026 μ
F	49° 46½′	0.486 "	a_4	39½′	1.111 "
D	48° 31½′	0.589 "	b_4	30′	1.214 "
C	2½′	0.656 "	a_5	20½′	1.336 "
a_1	47° 37′	0.753 "	b_5	9′	1.482 "
b_1	26½′	0.786 "	a_6	45° 55½′	1.669 "
a_2	16′	0.834 "	b_6	37½′	1.906 "
b_2	7′	0.889 "	a_7	14½′	2.225 "
a_3	46° 58′	0.952 "	b_7	44° 45½′	2.668 "

Two other series of observations were conducted in precisely the same way, except that the thickness of the layer of air which caused the interference was changed, so there were obtained in all three entirely independent calibrations of the prism. The wave-lengths corresponding to a series of equidistant deviations of the bolometer arm were determined by interpolation from each of these curves. In Table II. are contained the results which were thus obtained. The first column contains the angle ϕ, the following three the wave-lengths taken from the respective curves of dispersion, and the last the mean of these results. From the values contained in the last column, and the corresponding angles of dispersion, the curve of dispersion (Fig. 2, Plate I.) was constructed. This curve was used in all the determinations of wave-length which follow.

Since the dispersion in the visible spectrum was very great, this region was calibrated separately by reference to a large number of Fraunhofer lines, and a curve of dispersion for this portion of the spectrum was plotted on a larger scale than that used for the infra-red.

TABLE II.

ϕ	I.	II.	III.	MEAN λ.
50° 51′	0.434 μ	0.434 μ	0.434 μ	0.434 μ
49° 46′	0.486 "	0.486 "	0.486 "	0.486 "
48° 31½′	0.589 "	0.589 "	0.589 "	0.589 "
2′	0.656 "	0.656 "	0.656 "	0.656 "
47° 50′	0.695 "	0.690 "	0.686 "	0.690 "
40′	0.729 "	0.722 "	0.723 "	0.725 "
30′	0.767 "	0.761 "	0.766 "	0.765 "
20′	0.813 "	0.809 "	0.814 "	0.812 "
10′	0.872 "	0.864 "	0.872 "	0.869 "
0′	0.940 "	0.937 "	0.945 "	0.941 "
46° 50′	1.018 "	1.014 "	1.022 "	1.018 "
40′	1.109 "	1.104 "	1.112 "	1.108 "
30′	1.215 "	1.218 "	1.222 "	1.218 "
20′	1.339 "	1.347 "	1.348 "	1.335 "
10′	1.473 "	1.481 "	1.482 "	1.479 "
0′	1.611 "	1.610 "	1.617 "	1.613 "
45° 50′	1.746 "	1.743 "	1.751 "	1.747 "
40′	1.881 "	1.872 "	1.887 "	1.880 "
30′	2.017 "	2.004 "	2.022 "	2.014 "

4. *On the Manipulation of the Bolometer.*

Before mention is made of the details and results of the experiments themselves, a few words of suggestion should not be omitted which may be found useful in the manipulation of a bolometer.

As already stated in an earlier part of the paper, two arms of the bolometer were made of German silver, and the other two of platinum. Of the two latter branches, the one which was exposed to the radiations had been coated with lamp-black by smoking in a flame, and thus through this exposure to heat its temperature coefficient had to some extent been changed. As a result of this slight difference in the condition of the two arms, a perfect equilibrium could be maintained within the Wheatstone's bridge at a single temperature only. If, accordingly, the temperature of the room rose or fell, a drift of the needle in the one direction or the other became at once perceptible.

The disturbing effects of this drift were almost completely avoided if, some time before the observations were to begin, the precaution was taken to close the circuit through the bolometer, to light the burners illuminating the galvanometer scale,[1] to set in operation the electric lamp, the purpose of which will be described later, and at the same time to open the door of the room, in order that as regards all thermal changes an equilibrium might be established.

Even with the utmost care it was impossible to entirely eliminate the drift of the needle, but it was easily possible to make this disturbance so small that for an interval of many minutes the cross-hairs remained upon the same division of the scale. So sensitive was the apparatus to any changes in the temperature of the room, that it was invariably found possible to reverse the direction of the slow, ever-present drift simply by increasing or decreasing, often by not more than 1 cm., the amount by which the door stood open. This simple but effectual regulation frequently made it an easy matter to conduct the entire series of observations, lasting three hours, without altering the position of the sliding contact G (Fig. 2) in the Wheatstone's bridge.

The effect of the feeble drift which even this expedient failed to remove was still further reduced by observing the zero point of the needle after the throw as well as before, and taking the mean as the true zero. By this method of observation a uniform drift would be wholly without influence upon the result.

In the entire series of measurements it was not the final deflection, but the first throw of the needle, which was observed. There was secured by this means not only a saving of no inconsiderable amount of time, but also a greater freedom from the mechanical, thermal, and magnetic changes to which, notwithstanding all precautions to the contrary, a crowded laboratory, situated in the midst of a large city, is constantly exposed.

In a recent paper,[2] Professor Merritt has shown that the first throw of a galvanometer, when used with a thermopile or a bolometer, bears a constant ratio to the final deflection, and is therefore proportional to the heat radiated.

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[1] The scale was removed from the telescope and placed about 3 m. from the bolometer, from which it was separated by a cardboard screen.

[2] *Merritt*, Am. Journal of Science (3), 41, p. 422, 1891.

At each position throughout the spectrum the energy was measured by two such observations and the mean taken.

III. THE SPECTRUM OF THE ELECTRIC ARC.

Many vain attempts were made to detect traces of lines in the infra-red spectra of the alkalies when these metals were burned in a Bunsen burner or in the oxyhydrogen flame. Not until the electric arc was used for this purpose did efforts in this direction meet with success.

Inasmuch as it was suspected that a number of the observed infra-red spectral lines owed their origin to the arc itself and not to the metals therein consumed, it seemed all important to postpone the study of metallic spectra until a careful investigation of the distribution of energy in the infra-red spectrum of the electric arc had been made.

As a means of producing the arc an old Duboscq lamp was used, which, when supplied with current from the constant potential mains of the Berlin Central Station (110 volts), burned very quietly. By introducing resistance the current through the lamp was maintained constant at from 7.5 to 7.8 ampères.

After many experiments with carbons of various sizes, it was found that more perfect uniformity in the length of the arc (6½ mm.), together with greater quietness and regularity of burning, were secured by using carbons 8 mm. in diameter. The arc exhibited also less of a tendency to wander when a cored carbon was used as the anode.

By means of a lens, a real image of the arc, enlarged about twofold, was projected upon the slit of the spectrometer. Immediately before the slit was placed a large screen in such a way that the images of the incandescent carbons, as well as the lateral portions of the arc itself, were intercepted, and only a small strip, 7 mm. long and exceedingly narrow, belonging to the central region of the projected arc, was allowed to enter the collimating tube of the instrument.

In the entire series of measurements which follow, the opening of the slit did not exceed .1 mm. in width, and subtended an angle in the spectrum of 1.68 minutes. The width of the slit had been adjusted until its image in the telescope was just superposed upon the thread of the bolometer with its lamp-black coating.

Although the strength of the current in the Wheatstone's bridge was maintained as nearly constant as possible at $\frac{1}{40}$ ampère, yet before each series of observations the sensitiveness of the apparatus to temperature was determined according to the method already described, so that the results of the different determinations could be compared.

The energy was always assumed to be proportional to the deflection of the galvanometer needle. This assumption, however, is justified by the papers of Ångström,[1] Rubens,[2] and Merritt.[3]

In the manner already described, a large number of observations were conducted in order to determine the character of the spectrum of the arc light. The almost perfect agreement of the results obtained makes it a matter of indifference which particular series is selected from the material at hand to be given here in all its details.

The data are given with greater completeness in the following table than in the subsequent measurements, from the fact that the same order of procedure was followed throughout the entire investigation, and in the description of the distribution of energy in metallic spectra, to be given later, reference will be made to this series for many minor points.

The first column contains the angular deviations of the arm of the bolometer as read on the circle of the spectrometer, and the second contains the corresponding wave-lengths, determined with the aid of the curve of dispersion, Fig. 2, Plate I. In the third column are found the means of the galvanometer deflections, which were taken in the manner described above, and which were used without further correction as the relative values of the energy measured. (See Table III. on pages 45 and 46.)

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[1] *Ångström*, Wied. Ann., 26, p. 262, 1885.

[2] *Rubens*, Wied. Ann., 37, p. 256, 1889.

[3] *Merritt*, Am. Journal (3), 37, p. 167, 1889. *Merritt*, Am. Journal (3), 41, p. 422, 1891.

Sensitiveness of the Apparatus to Temperature. — *Deflections:* 128, 135, 128, 132, 132, 134. Mean: 132.

$$a = \frac{136 + 132}{2} = 134.$$

Wherefore $k = \frac{1}{134000}$° C.

The data contained in this table are graphically represented in the curve Fig. 3, Plate I. Angular deviations of the arm of the bolometer are chosen as abscissæ, and corresponding deflections in millimeters as ordinates.

In order that the positions of the maxima and minima of this curve may be more easily recognised, a number of Fraunhofer lines are drawn for comparison. The limits of the visible spectrum, as ordinarily accepted, are indicated by dotted lines. The assignment of these limits is naturally attended by much uncertainty, since the investigations of Helmholtz,[1] Esselbach,[2] Becquerel,[3] and Langley[4] have shown that under peculiarly favorable conditions the eye can perceive radiations, the wave-lengths of which lie far beyond these boundaries. The present paper also contains facts tending to confirm these statements.

In Fig. 4, Plate I., the same curve is represented, with the single exception that wave-lengths are chosen as abscissæ instead of divisions on the circle.

A glance at these curves shows at once the remarkable fact that *in the spectrum of the electric arc, the maximum of the energy lies farther in the direction of the short wave-lengths than the Fraunhofer lines H and K.*

The principal maximum of the energy curve extends from

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[1] *Helmholtz*, Pogg. Ann., 94, p. 12, 1855.

[2] *Esselbach*, Monatsber. d. k. Acad. d. Wiss. zu Berlin, p. 757, 1855; *Esselbach*, Pogg. Ann., 98, p. 514, 1856.

[3] *H. Becquerel*, Compt. Rend., 97, p. 73, 1883.

[4] *Langley*, Phil. Mag. (5), 21, p. 396, 1886.

TABLE III.

ELECTRIC ARC.

Sensitiveness of the Apparatus to Temperature. Deflection in Millimeters:
137, 138, 138, 130, 136. Mean: 136.

Observations of Energy.

φ	λ	I	φ	λ	I
54° 7½′		0	49° 56½′	0.4770 μ	3
			49′	0.4838 "	3
53° 57′		0	41′	0.4920 "	2
50½′	0.365(?)μ	11	21′	0.5140 "	9
44½′		0	19½′	0.5155 "	8
4½′	0.3770 "	13	15′	0.5213 "	4
52° 39½′	0.3820 "	80	48° 51½′	0.5540 "	6
29½′	0.3850 "	185	34½′	0.5835 "	9
27½′	0.3858 "	284	31½′	0.5892 "	14
26′	0.3861 "	306	28½′	0.5945 "	11
24½′	0.3868 "	279	26′	0.600 "	11
23′	0.3871 "	310	21′	0.611 "	12
22′	0.3874 "	282	17½′	0.619 "	13
20½′	0.3880 "	328	14′	0.627 "	15
19½′	0.3883 "	357	11′	0.635 "	18
18′	0.3889 "	83	8½′	0.642 "	19
16½′	0.3894 "	29	5½′	0.649 "	21
10½′	0.3918 "	24	2½′	0.657 "	20
51° 48½′	0.4015 "	10	47° 59½′	0.666 "	20
30½′	0.4110 "	29	57′	0.673 "	21
27′	0.4128 "	48	55′	0.679 "	17
24½′	0.4140 "	76	53′	0.685 "	22
23′	0.4150 "	72	50½′	0.693 "	26
22′	0.4157 "	64	48′	0.701 "	26
21′	0.4163 "	68	45′	0.711 "	36
19½′	0.4170 "	68	43′	0.718 "	35
18½′	0.4177 "	54	41′	0.726 "	40
17′	0.4183 "	62	39½′	0.731 "	43
15½′	0.4194 "	35	38′	0.737 "	39
14′	0.4203 "	42	36½′	0.743 "	39
11½′	0.4210 "	8	35′	0.749 "	41
½′	0.4250 "	4	33½′	0.755 "	32
			32′	0.761 "	37
50° 31′	0.4488 "	7	30½′	0.768 "	35
28′	0.4508 "	12	29′	0.775 "	30
25′	0.4532 "	6	27½′	0.781 "	30

Table III. — *Continued.*

φ	λ	I	φ	λ	I
47° 26′	0.788 μ	50	46° 25′	1.276 μ	12
24½′	0.794 "	57	23½′	1.295 "	8
23′	0.801 "	62	22′	1.312 "	5
21½′	0.808 "	74	20½′	1.330 "	5
20′	0.815 "	68	19′	1.352 "	8
18½′	0.823 "	66	17½′	1.372 "	13
17′	0.830 "	65	16′	1.392 "	21
15½′	0.838 "	54	14½′	1.413 "	24
14′	0.846 "	51	13′	1.435 "	24
12½′	0.854 "	44	11½′	1.453 "	24
10½′	0.865 "	33	9½′	1.482 "	21
9′	0.875 "	30	8′	1.502 "	14
7½′	0.886 "	26	6½′	1.523 "	11
6′	0.896 "	23	5′	1.543 "	10
4½′	0.906 "	60	3½′	1.563 "	8
3′	0.917 "	98	2′	1.583 "	9
1½′	0.928 "	95	½′	1.603 "	6
0′	0.940 "	97			
			45° 59′	1.626 "	4
46° 58½′	0.951 "	86	57½′	1.645 "	3
57′	0.962 "	67	56′	1.665 "	2
55½′	0.974 "	57	54½′	1.685 "	2
54′	0.987 "	48	53′	1.705 "	2
52½′	0.999 "	36	51½′	1.725 "	0
51′	1.012 "	29	50′	1.747 "	0
49½′	1.024 "	24	48½′	1.769 "	1
48′	1.038 "	18	47′	1.789 "	0
46½′	1.050 "	16	45½′	1.810 "	0
44½′	1.068 "	16	44′	1.830 "	0
43′	1.082 "	75	42½′	1.850 "	0
41½′	1.096 "	103	41′	1.870 "	0
40′	1.109 "	80	39′	1.897 "	0
38½′	1.128 "	59	37½′	1.918 "	0
37′	1.144 "	47	36′	1.938 "	0
35½′	1.158 "	35	34½′	1.958 "	0
34′	1.176 "	29	33′	1.978 "	0
32 ′	1.192 "	26	31½′	2.000 "	0
31′	1.208 "	23	30′	2.020 "	0
29½′	1.225 "	17	28½′	2.042 "	0
28′	1.243 "	18	27′	2.062 "	0
26½′	1.258 "	13			

$\lambda = .385\,\mu$ to $\lambda = .388\,\mu$; a secondary maximum is comprised within the region between $\lambda = .411\,\mu$ and $\lambda = .420\,\mu$.

In examining these exceedingly intense bands with the eye, it was found that the first consisted of five narrower bands, and the second of six, all of which were sharply defined on the side looking toward the red end of the spectrum, but which grew rapidly indistinct and hazy in the opposite direction.

A peculiarity of the arc spectrum was observed in this connection which, as was found later, had also been noticed by Kayser and Runge,[1] viz.: that when the slit of the spectroscope is made very narrow, the entire spectrum of the electric arc consists of many thousand fine lines which lie exceedingly close together at those places where the bright bands are visible, and are arranged at slightly greater intervals in the other portions of the spectrum.

With the bolometer, in the form in which it was used, it was not possible to detect these individual lines, which are often not more than a second of arc apart. Only a summation could be obtained which represents an integral of all the smaller portions of energy which fell upon the filament of the bolometer, 1.6 minutes of arc in width. The energy in the arc spectrum was accordingly measured with a breadth of slit approximately equal to the width of the sensitive bolometer thread. What was then observed was the characteristic banded spectrum, formed by a partial overlapping of this large number of fine lines.

Shortly before the close of the entire investigation, there was detected, first optically and then with the aid of the bolometer, a feeble band which lies still farther in the direction of the extreme violet than the group from $\lambda = .385\,\mu$ to $\lambda = .388\,\mu$. By an extensive extrapolation of the curve of dispersion an approximate wave-length, $\lambda = .365\,\mu$, was ascribed to this band, but this value, it must be remembered, lays no claim to being correct. Without doubt this band is identical with the one found by Kayser and Runge[2] to lie from $\lambda = .3584\,\mu$ to $\lambda = .3590\,\mu$.

In their spectrophotometric comparison of the various sources of illumination, Nichols and Franklin[1] have described a band in the spectrum of the electric light, which can be easily identified with the series of bands from $\lambda = .4110\mu$ to $\lambda = .4210\mu$; for, although representing far less energy than the group between $\lambda = .385\mu$ and $\lambda = .388\mu$, the former are optically much brighter than the latter. It is also to be noticed that the limits for the wave-length of this band, as given by Nichols and Franklin, lie nearer the group between $\lambda = .4110\mu$ and $\lambda = .4210\mu$.

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[1] *Kayser* und *Runge*, Wied. Ann., **38**, p. 81, 1889.

[2] *Kayser* und *Runge*, Wied. Ann., **38**, p. 90, 1889.

In a recent paper, P. Drude and W. Nernst[2] mention two bands in the spectrum of the arc light which excite a peculiarly active fluorescent effect upon a solution of fluorescene. They determined the wave-lengths of these bands with the aid of a diffraction grating, and obtained for the more refrangible and at the same time optically weaker band the value $\lambda = .386\mu$. This determination coincides so closely with the values found in the present paper for bands in this region as to leave no doubt as to the identity of those bands with the ones here investigated.

The energy throughout the remaining portions of the visible spectrum of the electric arc, as compared with the unusually large amount to be found in the violet, is quite unimportant. A small quantity of a sodium salt, always present as an impurity in the carbons, may be detected by the appearance of several of the characteristic sodium lines. The stronger of these, especially the D-lines, may be measured with the bolometer. In the visible spectrum are found, moreover, several indistinct bands which, on account of their feeble energy, are hardly to be distinguished from the bright background, and therefore are scarcely to be recognized in the figure. Not until the region of the infra-red is reached are well-developed maxima and minima again found, and here the bolometer detects five intensely warm bands, similar in many respects to those in the extreme violet, but showing, on the other hand, the noticeable difference that their sharply defined edges are turned toward the visible portion of the spectrum, while an indistinct border bounds each of these bands on the side of the longer wave-lengths. In all probability these bands consist of a warm background upon which is superposed a series of lines more or less sharply defined. The dispersion of the prism employed was, however, too feeble to permit of a complete separation of these lines by the bolometer. Their presence was nevertheless detected by the small irregularities which the curve shows in the neighborhood of each of the five maxima.

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[1] *Nichols* and *Franklin*, Am. Journal (3), **38**, p. 106, 1889.

[2] *Drude* und *Nernst*, Wied. Ann., **45**, p. 460, 1892.

The wave-lengths of these five infra-red bands are as follows:—

TABLE IV.

A.	from	$\lambda = 0.700\mu$	to	$\lambda = 0.770\mu$
B.	"	$\lambda = 0.785$ "	"	$\lambda = 0.860$ "
C.	"	$\lambda = 0.900$ "	"	$\lambda = 1.000$ "
D.	"	$\lambda = 1.075$ "	"	$\lambda = 1.160$ "
E.	"	$\lambda = 1.370$ "	"	$\lambda = 1.500$ "

IV. ON THE GRAPHICAL REPRESENTATION OF THE DISTRIBUTION OF ENERGY THROUGHOUT THE SPECTRUM.

In general it is not possible, in transforming to the normal spectrum the distribution of energy as observed by means of a prism, to retain in their original lengths the ordinates which in the diffraction spectrum represent the energy throughout the region measured. At each point of the curve, the lengths of the new ordinates must be so chosen that the curves (see Fig. 7), bounded by the wave-lengths λ and λ' and comprised between the interval $(\lambda - \lambda')$ on the axis of abscissæ and the portion of the curve immediately above, must, both in dispersion and diffraction spectrum, be equal.

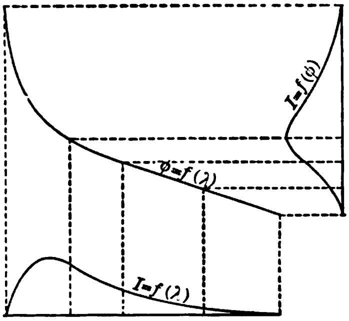

Fig. 7.

Professor Langley has suggested[1] a simple means of accomplishing this purpose, and illustrates his method by a drawing from which the idea of the accompanying figure has been taken.

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[1] *Langley*, A Report of the Mt. Whitney Expedition, p. 231, 1884. Phil. Mag. (5), 17. p. 211, 1884.

While this method serves to transform to the normal a continuous prismatic spectrum, it fails in its application to a bright line or discontinuous spectrum. In the latter case the width of the lines, so long as these do not overlap, is entirely independent of the amount of dispersion produced by the prism, and the reduction to the normal spectrum can be made without changing the lengths of the ordinates.

With the aid of a micrometer eye-piece it was determined that the width of the sharp spectral lines, for the amount of dispersion used in these determinations, was the same throughout all regions of the visible spectrum. Nevertheless, it was obviously impossible to consider the distribution of energy, shown in Fig. 4, Plate I, as the distribution in the normal spectrum, inasmuch as the lines, of which the entire series of bands consists, to a greater or less extent overlap. Moreover, the spectrum, strictly speaking, is not a continuous one, and therefore the reduction, as given by Langley, cannot be here applied. In the delineation of the energy contained in the radiations from the electric arc as well as in that of the metals to be later described, the author has been compelled to use as ordinates the values obtained from the dispersion spectrum, and to plot wave-lengths as abscissæ instead of angular deviations.

Mention must be made at this point, however, of the fact that if the assumption be made that the spectrum of the electric arc is a continuous one, in the sense in which the spectrum of an incandescent solid is so considered, the violet bands at $\lambda = .388\,\mu$ and $\lambda = .415\,\mu$ would be many times more intense, as compared with the energy in the infra-red bands, than appears in the drawing, Fig. 4, Plate I.

[*To be concluded.*]

THE CRITICAL CURRENT DENSITY FOR COPPER DEPOSITION AND THE ABSOLUTE VELOCITY OF MIGRATION OF COPPER IONS.

By Samuel Sheldon and G. M. Downing.

IT is a well-known fact that, in the electro-deposition of copper from solutions of its salts, if with a given exposed surface of cathode a certain current strength be exceeded, the deposit assumes a chocolate brown color when the solution is concentrated, or a dead black color when the solution is dilute. It is often stated that this modified form of deposit is but copper in a finely divided state. Magnus,[1] however, long ago stated that it was hydride of copper. He assumed that, for some reason or other, after a certain *critical current density* was reached, either the water of solution or H_2SO_4, formed by the union of liberated SO_4 with the water, took part in the conduction near the cathode. He endeavored to measure this current density by varying the current which was passing through an electrolytic cell containing $CuSO_4$ and then noting the strength at the moment when the hydrogen bubbles began to appear at the cathode. His results were not very concordant. C. L. Weber[2] explained the appearance of the copper hydride as follows: as the current which is passing through a certain copper solution with definite sized electrodes increases, the rate of deposit increases, and as a result, the rate of abstraction of copper from the portion of the solution around the cathode. Inasmuch as the copper ions have a definite absolute velocity, this rate of deposit is limited. If the current strength demands a greater deposit, the SO_4 ions, at the moment of electrolytic severance from the copper, join with the water, form H_2SO_4, and this takes part in the conduction, depositing hydrogen alongside of the copper. Weber endeavored to measure the absolute ionic velocity of the copper ions. He passed a current through a variable resistance, a galvanometer, and a solution contained in a glass tube. The electrodes were pieces of copper, ground to fit the ends of the tube. The potential difference between the electrodes was measured by means of a high resistance galvanometer. The current was increased by steps and closed for 15 seconds. The galvanometer deflections were read just before the one of high resistance indicated a sudden rise in the potential difference, owing to the polarization resulting from the newly deposited hydrogen. It is not necessary to give his method of calculating the ionic velocities, for it is analogous to the one to be explained later. His results were not wholly satisfactory, errors of 10 per cent being admitted.

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[1] Magnus, Pogg. Ann., 112, p. 23, 1857.

[2] Ztschr. f. phys. Chem., 4, pp. 182–188, 1889.

The present investigation considers the causes which affect the critical current density for copper sulphate solutions, and includes a determination of the absolute velocities of migration of the copper ions in solutions of varying concentration.

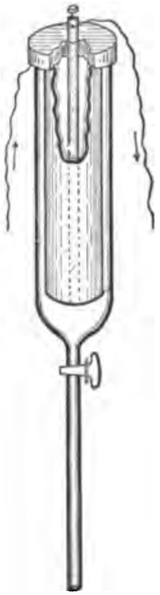

Fig. 1.

Apparatus. — The current was taken from the Edison street mains at 115 volts, and its strength was regulated by a bank of lamps. The current was measured by a Weston 0–5 ampère ammeter. The potential differences were measured by a Weston 0–150 volts voltmeter, whose series high resistance coil was short-circuited. A resistance of 314 ohms was inserted in the voltmeter circuit, causing the pointer to deflect the whole 150 scale divisions for 3 volts. The electrolyte was contained in a glass vessel of shape indicated in Fig. 1. The anode was a hollow copper cylinder of 4 cms. internal diameter. The cathode was a copper-plated brass rod of .648 cm. diameter. This was fastened into the center of a wooden cover, grooved to fit exactly over the top of the glass vessel.

Method. — The solution to be investigated was placed in the vessel and a current of somewhat less than the critical strength was sent through. This produced the normal copper deposit. The stopcock at the bottom of the vessel was then opened, and as the electrolyte ran out, the cathode surface diminished. Inasmuch as the resistance offered by the electrolyte was very small compared to that of the whole circuit, there was very little alteration in the current strength during the flow. Accordingly, after a certain amount of the electrolyte had flowed out, the surface of the cathode was reduced so as to produce the critical current density. The deposit immediately assumed the dark color, and upon removal of the cathode, the area could be readily measured by means of a micrometer gauge and a scale. The line separating the reguline deposit from the dark-colored one was very distinct, especially when produced from a moderately dilute solution. In all cases its position could be determined within .3 mm. The current was generally adjusted so that the height of the dark deposit was about 125 mm.

Influence of Rapidity of Flow. — Inasmuch as the appearance of the dark deposit is intimately associated with the alteration in density of the solution near the cathode, it is evident that the flow, upon turning the stopcock, must be so regulated as not to disturb the solution at this place. The form of apparatus was a happy one in this respect. This was proved by the following observations: A large quantity of $CuSO_4$ solution was prepared; portions of this were allowed to run, in succession, through the vessel at different rates. The current strengths (in ampères) divided by the areas (in square cms.) covered by the black deposit, *i.e.* the *critical current densities* D, are given in the following table, opposite to the times required to flow out of the vessel. The values of D are corrected for slight differences of temperature.

An inspection of this table shows that if the time which it takes the electrolyte to flow out of the vessel be greater than eight or nine minutes, the values of D will be practically the same as those which would be obtained were the electrolyte at rest.

TIME.		$D_{15°}$.
Min.	Sec.	
0	23	0.0886
0	34	0.0581
1	0	0.0455
1	30	0.0405
2	16	0.0375
3	3	0.0361
5	0	0.0342
8	49	0.0334
15	0	0.0330
20	0	0.0332

Influence of Temperature. — As might be expected, temperature has a great influence upon the value of the critical current density. Theory would indicate that the temperature coefficient should be the same as that for the specific conductivity of the solutions. Kohlrausch's[1] values referred to the conductivity at 18° are from .021 to .024; *i.e.* the specific conductivity at 18° increases for each degree rise of temperature from 2.1 to 2.4% according to concentration.

We have failed to obtain coefficients as small as these, ours ranging from .031 to .038. Our method of determination was to fill several glass-stoppered flasks with a solution taken from a stock bottle which had been previously shaken. These flasks were placed in water baths and brought to various temperatures, suffering no evaporation in the process. These solutions were successively placed in the density vessel, and as soon as the dark deposit appeared the temperature was taken. It seems hardly possible that the values given above are far from correct.

Influence of Concentration. — The critical current density increases with the concentration. We have investigated solutions containing 2, 1, .5, and .1 gram equivalents to the liter of solution. (A gram equivalent solution of $CuSO_4$ contains 31.55 grams of copper in a liter of the solution.) During preliminary observations the most concentrated solution became somewhat impoverished of copper. Not all of the anions set free by the current were able to unite with the anode. Accordingly, after the final determinations, the specific gravity of the solution was obtained. This was also done with the other solutions. By means of tables[1] and the value given by Kohlrausch[2] for the specific gravity of a normal solution (1.0775 at 18°.2) we were enabled to determine the concentration.

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[1] F. Kohlrausch, Wied. Ann., 6, 1 and 145, 1879.

Our results are given in the following table, where m represents the gram equivalents to the liter of solution, and D the current density in ampères per square centimeter. The values are reduced to 19°.

m	$D_{19°}$	D_{19}/m
1.89	0.1005	0.0532
1.	0.0528	0.0523
0.5	0.0257	0.0514
0.1	0.00542	0.0542

If the method of observation were susceptible of greater accuracy, it would undoubtedly be found that the values of D_{19}/m would increase with dilution in the same manner as the values of the specific molecular conductivity.[3]

For industrial or experimental purposes an approximation formula for determining the critical current density when the specific gravity of the solution is known may be of value. The excess of the specific gravity of solutions at 18°C above .9987 (the density of water at 18°) is very nearly proportional to the percentage of salt contained. Accordingly, if we represent the specific gravity of the solution by d, the critical current density in ampères per square centimeter of cathode surface

$$D_{18} = (d - .9987).65$$

within a few per cent. This is the limiting current allowable when the solution is at rest. By means of circulation at *the cathode surface* this value may be increased many times.

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[1] Landholdt und Börnstein, Tabellen, p. 147.

[2] F. Kohlrausch, Wiedemann's Ann., Bd. XXVI., p. 174, 1885.

[3] F. Kohlrausch, l.c., p. 196.

Velocity of the Copper Ions. — If the assumption be made that the cause of the abnormal hydride of copper deposit is the inability of the copper ions to migrate fast enough to keep up the supply for deposit, which is required by Faraday's law, the velocity of migration can be determined in the following manner. Suppose that a current of C ampères is the critical current for a deposit upon q square centimeters of cathode surface from a solution of m gram equivalents to the liter. Consider a layer of the solution around the cathode, of radial thickness dr_1. This layer contains $31.55\, mqdr_1$ milligrams of copper. The current requires that, in the short time dt, $.3281\, Cdt$ milligrams be deposited. If we suppose dr_1 to be of such a size that $31.55\, mqdr_1 = .3281\, Cdt$, and remember that C is of such a magnitude that the ions migrate with exactly the requisite velocity to maintain the reguline character of the deposit, it will be evident that all the copper ions in the elementary layer considered have migrated to the cathode in the time dt, and their velocity has been

$$V = \frac{dr_1}{dt} = \frac{.3281 \cdot C}{31.55\, mq}. \tag{1}$$

If the current were less than the critical amount C, the velocity would be smaller, as would also be the drop in potential along dr_1. In all cases when the current is less than C, the velocity is directly proportional to the drop in potential. The velocity in absolute measure must be expressed as so many centimeters per second under an impressed difference of potential of 1 volt per centimeter, measured along a line of current flow.[1] Let dE represent the volts of potential difference between the concentric surfaces enclosing the layer of thickness dr_1. Then the velocity of the copper ions in absolute measure,

$$V_0 = \frac{V}{\frac{dE}{dr_1}} = \frac{.3281\, Cdr_1}{31.55\, mqdE}. \tag{2}$$

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[1] It must be remembered that the expression for the velocity of certain ions does not imply that all the individual ions migrate with that velocity, but that it is the expression for the mean value of their velocities.

Consider a single observation with the arrangement of apparatus which we have employed. Let

E = volts between electrodes,
r_1 = radius of cathode,
r_2 = radius of anode,
R = resistance of solution,
s = the specific resistance of the solution,
h = height of solution.

We have $E = CR$

and

$$dE = CdR. \tag{3}$$

The resistance of an elementary hollow cylinder of the solution of radius r is

$$dR = \frac{sdr}{2\pi hr}. \tag{4}$$

Substituting this value in (3), we have

$$\frac{dE}{dr} = C\frac{s}{2\pi hr}. \tag{5}$$

If we consider $r = r_1$, we can substitute by merely adding the subscript to r in equation (2). But this expression is not in a convenient form for introduction into the expression for V_0, for it involves the value of the specific resistance of the solution. The formula can be altered so as to eliminate s, but it should be borne in mind that any lack of homogeneity in the solution during an observation, affecting s as it does, affects the validity of the application of the modified formula to the observations. By applying Ohm's law and integrating (4), we obtain

$$R = \frac{E}{C} = \int_{r_1}^{r_2} \frac{sdr}{2\pi hr} = \frac{s}{2\pi h}\log\left(\frac{r_2}{r_1}\right). \tag{6}$$

$$\therefore \frac{s}{2\pi h} = \frac{E}{C\log(r_2/r_1)}. \tag{7}$$

Substituting this value in (5) and again in (2), we have

$$V_0 = \frac{.3281\, Cr_1 \log(r_2/r_1)}{31.55\, mqE}. \tag{8}$$

In our experiments $r_1 = .324$ cm. and $r_2 = 2$ cm. Hence the formula reduces to

$$V_0 = .006136 \frac{C}{qmE}$$

The results which we have obtained are tabulated below, where m represents, as before, the gram equivalents to the liter of the $CuSO_4$ solutions.

m	C	q	E	V_0
1.89	2.59	25.16	1.55	0.000204
1.	1.35	25.56	1.26	0.000257
0.5	0.732	28.32	1.03	0.000308
0.1	0.128	22.78	0.775	0.000445

It is very probable that these values of V_0 are too large, owing to an increase in the specific resistance of the solution around the cathode, which follows the closing of the circuit. To eliminate this error as far as possible, in every observation the auxiliary resistances in circuit were so adjusted that the current, upon closing the circuit, was slightly less than the critical current for the exposed cathode surface. The surface was then decreased by allowing the fluid to flow out at the bottom of the vessel. The appearance of the abnormal deposit, indicated by a sudden rise in the voltage, followed the closing of the circuit in less than a minute.

The values given by Weber are less than the ones obtained by us. His method, too, involves the specific resistance and is open to the same criticisms as ours.

POLYTECHNIC INSTITUTE OF BROOKLYN,
March, 1893.

NOTE. GEOMETRICAL PROOF OF THE THREE-AMMETER METHOD OF MEASURING POWER.

By Frederick Bedell and Albert C. Crehore.

THE methods, now well-known, for measuring power by three voltmeters or three ammeters, first shown by Professor Ayrton and Dr. Sumpner,[1] are applicable to the measurement of power of any circuit irrespective of the nature of the impressed electromotive force, and the general proof of the methods is given in the paper referred to. In the case of an harmonic electromotive force the methods are capable of simple geometrical proof. The writers have shown this[2] for the voltmeter method, and in this note will give the corresponding proof for the three-ammeter method.

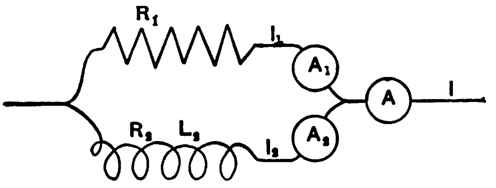

Fig. 1.

Let R_2L_2, Fig. 1, be an inductive circuit whose power is to be measured, and R_1 a non-inductive resistance in parallel with it. If the maximum values of the main and branch currents be denoted by I, I_1, and I_2, respectively, they may be represented as shown in Fig. 2. The current I_1 is in phase with the impressed electromotive force E; the main current, I, lags behind it by an angle θ; and the current I_2 lags behind it by an angle θ_2. The tangent of θ_2 is $\frac{L_2\omega}{R_2}$; and the tangent of θ is $\frac{L'\omega}{R'}$, where R' and L' denote the equivalent resistance and self-induction of the parallel circuit,[1] and ω is 2π times the frequency.

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[1] "The Measurement of Power given by Any Electric Current to Any Circuit." Proc. Roy. Soc., Vol. XLIX., p. 424.

[2] "Alternating Currents," p. 230.

The power expended in the inductive circuit is $W_2=\frac{1}{2}EI\cos\theta_2$. From the geometry of the figure

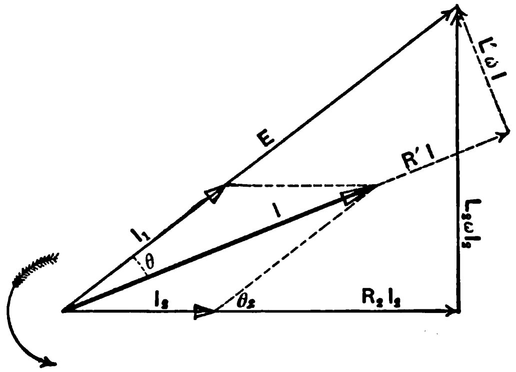

Fig. 2.

$$\cos\theta_2=\frac{I^2-I_1^2-I_2^2}{2I_1I_2};$$

whence

$$W_2=\frac{E}{4I_1}(I^2-I_1^2-I_2^2),$$

where E, I, I_1, and I_2 represent maximum values. Writing virtual or square root of mean square values as obtained from ordinary measuring instruments, the expression for the power becomes:

$$W_2=\frac{\overline{E}}{2\overline{I}_1}(\overline{I}^2-\overline{I}_1^2-\overline{I}_2^2).$$

The power in the non-inductive branch is

$$W_1=\overline{E}\overline{I}_1;$$

and the total power in the two branches is

$$W=\frac{\overline{E}}{2\overline{I}_1}(\overline{I}^2+\overline{I}_1^2-\overline{I}_2^2).$$

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[1] "Equivalent Resistance, Self-Induction and Capacity of Parallel Circuits with Harmonic Impressed Electromotive Force." Bedell and Crehore, Phil. Mag., September, 1892.

The method is thus geometrically established for harmonic currents, which may be represented by lines in a vector diagram. For an alternating current, not harmonic, the proof does not hold unless we assume the current to be equivalent to an harmonic current, and the question then arises as to what will be the equivalent harmonic current. The equivalent harmonic current must be such that its square root of the mean square value, and the expenditure of energy in the circuit, are the same as in the case of the given current which was not harmonic.

NOTES.

The International Congress of Electricians. — The committee appointed to arrange a programme for the World's Fair Congress of Electricians met in Washington on the 21st of April. There were in attendance T. C. Mendenhall, chairman, H. A. Rowland, H. S. Carhart, A. E. Kennelly, Elisha Gray, F. B. Crocker, and E. L. Nichols. It was decided that the Congress should be divided into three sections for the reading and discussion of papers: Section I., for the consideration of matters purely theoretical; Section II., of matters of mixed theory and practice; Section III., of matters purely practical. These sections may be further subdivided if they become unwieldly. Two sessions are to be held daily for the above purposes, in addition to which there will be three or four evening lectures and general sessions devoted to matters of wider interest to electricians. Papers on subjects covered by the three divisions already referred to are invited, the same to be submitted to the programme committee before August 1. The question of the acceptance of such papers for presentation will rest with sectional committees to be appointed later. The legislative work of the Congress, such as the establishment and legalization of electrical standards, will be performed by a special body consisting of five members from each country represented in the Congress. This body will be termed the Chamber of Delegates. It will report the result of its deliberations to Congress at a final sitting. The delegates appointed by the Secretary of State to represent the United States, are H. A. Rowland, T. C. Mendenhall, H. C. Carhart, Elihu Thomson, and E. L. Nichols.

Spring Meeting of the National Academy of Sciences. — The National Academy of Sciences held its regular spring meeting in the lecture hall of the National Museum, April 18, 19, and 20. Twenty-one papers were read either in full or by title, of which the following were of immediate interest to physicists: —

"On the Nature of Certain Solutions, and on a New Means of Investigating Them," by M. C. Lea.

"Telegraphic Gravity Determinations," by T. C. Mendenhall.

"Comparison of Latitude Determinations at Waikiki," by T. C. Mendenhall.

"A One Volt Standard Cell," by H. S. Carhart, introduced by T. C. Mendenhall.

"Fundamental Standards of Length and Mass," by T. C. Mendenhall.

"On the Potentiality of Internal Work in the Wind," by S. P. Langley.

"On the Bolograph in the Infra-Red Solar Spectrum," by S. P. Langley.

Professor Langley's papers were read by title.

Superintendent Mendenhall, in his paper upon gravity determinations, described an ingenious method of comparing the force of gravity at distant stations by the interchange of chronometer and pendulum signals. The method was given an excellent test in transferring the base for pendulum work from the Smithsonian Institute to the basement of the Coast Survey Building near the Capitol. The method is one which may be expected to give close comparisons between stations where telegraphic communication can be established.

Dr. Mendenhall's paper on the latitude of Waikiki was a summary of the careful investigations made at that station by Dr. Marchuse, representing the International Geodetic Association, and Mr. E. D. Preston, of the U. S. Coast Survey, in order to test the question of the periodic variation of latitude. The observed latitudes at two stations about forty feet apart were plotted for a period of more than a year. These show a marked and well-defined periodicity, the two curves running parallel to each other with a difference of ordinates corresponding closely to the difference in latitude of the two instruments. Both curves show a principal period of about 378 days, the oscillations being such as would be accounted for by a rotation of the earth's pole through a circle of forty feet in diameter.

In the discussion of this paper, Professor Chandler, who has paid much attention to the subject of fluctuations of latitude, pointed out that a reduction of observations obtained at Berlin and Waikiki gives curves which agree closely in amplitude, so that the question of the phenomenon's being local must be considered as settled. He pointed out the important effects of this movement upon various astronomical constants. It will be necessary to take cognizance of it in the zero point of right ascensions. The constant of nutation will be affected, and also that of aberration. There will result also an error in the accepted value of the precession, as well as in that of the obliquity of the ecliptic. In the opinion of Professor Chandler there is decided evidence of a third term in the equation of the curve which is perhaps periodic, with a period of from seven to fourteen years. It is of the highest importance in the further study of this problem that the old observations of the European and American observatories be worked over with a view to obtaining data covering long periods of time. What is most needed in the way of new work is the establishment of a competent observer in the southern hemisphere.

Professor Carhart described a standard cell of a type devised by Helmholtz, and since then employed by Ostwald. Carhart finds that the electromotive force of such cells can be controlled by changing the density of the solution, and that the cell can be adjusted so as to give precisely one volt. The form adopted is similar to Carhart's modification of the Clark cell. The elements are mercury, mercurous chloride, zinc chloride, and zinc. The resistance is 1500 ohms. The temperature coefficient is +.000094 instead of the negative coefficient of −.00077 of the Clark cell. After heating to 50° C. the return was nearly but not quite complete. The electromotive force is expressed by the equation $E_t = 1 + .000094T$, where the temperature T is reckoned from 15° C.

The only other paper relating to physics read in full at this meeting was that upon "Standards of Length and Mass," by Dr. Mendenhall. This was an announcement of the official acceptance of the international prototypes of the meter and the kilogram recently received by the government. These copies are the result of modern refinement of construction, and are better adapted for use than the earlier standards of length. The result of this action is to make the meter the official standard of the United States, and to lead to the definition of the legal yard in terms of the former.

On the afternoon of April 18 the Academy was received by President Cleveland at the White House. On the following evening it attended a reception at the residence of Alexander Graham Bell. Messrs. Brush, Gould, Langley, Mendenhall, Newcomb, and Remsen were added to the council. No new members were elected at this meeting.

The Annual Meeting of the Institute of Electrical Engineers. — The American Institute of Electrical Engineers held its annual meeting in New York, May 16th and 17th. At the meeting on Tuesday afternoon the election of Professor Houston of Philadelphia as President for the coming year was announced, and various matters of routine business were transacted. In the evening sixty-five members attended a dinner, at which speeches were made by the retiring President, Mr. Sprague, President Houston, Mr. T. D. Lockwood, Mr. T. C. Martin, and others. Wednesday was devoted to the reading of papers.

Following is the programme: —

1. "On the Behavior of Fuse Metals in Direct and Alternate Current Circuits," by Mr. Charles P. Matthews, of Cornell University, Ithaca, N.Y.

2. "A Modified Deprez D'Arsonval Galvanometer," by Lieutenant Charles D. Parkhurst, of Watervliet Arsenal, West Troy, N.Y.

3. "The Variation in Economy of the Steam Engine Due to Variation in Load," by Professor R. C. Carpenter, of Cornell University, Ithaca, N.Y.

4. "An Automatic Printing Speed Counter for Dynamo Shafting," by Professor George S. Moler, of Cornell University, Ithaca, N.Y.

5. "Practical Aspects of Electrical Resonance," by Dr. M. I. Pupin, of Columbia College, New York City.

6. "The World's Electrical Congress of 1893," Discussion of the Provisional Programme prepared by the Institute Committee.

7. "A New Method and Apparatus for Measuring Conductivity," by E. S. Willyoung, of Philadelphia.

8. "The Heating of Armatures," by Messrs. A. H. and C. E. Timmerman, of Cornell University.

The paper which excited the greatest amount of interest was that of Dr. Pupin, of Columbia College. For the purpose of witnessing the demonstrations which accompanied its presentation the Institute adjourned to the Chemical Lecture Room of the School of Mines of Columbia College. Dr. Pupin's experiments upon the tuning of electrical resonators of low period were highly successful, and it was made evident to all who witnessed them that as a means of studying electrical oscillations, the period of which lies within musical limits, the telephone is likely to be of great service. Whereas in the case of the shorter wave-lengths hitherto studied by Hertz and his many followers, the period had to be determined by indirect and at best rather unsatisfactory means, it becomes possible in the case of the range of wave-lengths to which the telephone is sensitive to determine the frequency by ordinary acoustical methods. Those who witnessed Dr. Pupin's demonstrations were struck with the extreme delicacy of the methods which it was the purpose of his paper to exhibit.

It was reported at the close of the meeting that within seven years the membership of the Institute had increased from seventy to nearly seven hundred. The question of extending the usefulness of the Institute by authorizing the local organization of its members in various cities distant from headquarters for purposes of holding meetings was discussed, but no action was taken. Specimens of the new badge adopted by the Institute were exhibited by Mr. Hammer.

NEW BOOKS.

The Theory and Practice of Absolute Measurements in Electricity and Magnetism. Vol. II., Parts I. and II. By ANDREW GRAY, M.A., Professor of Physics in the University College of North Wales. London, Macmillan and Co., 1893. — 8vo. pp. xxiii + 868.

The first volume of this work appeared in 1888. It includes chapters on electrostatic theory, elementary electrodynamics, units and dimensions, general physical measurements, electrometers, the comparison of resistances and of electrostatic capacities, and the measurement of specific inductive capacity. The volume now before us contains chapters on magnetic theory, the determination of H, electric current actions, Lagrange's dynamical method, the electro-magnetic theory of light, the calculation of the constants of coils, galvanometry, the measurement of inductances, units and dimensions, the absolute measurement of resistances, the determination of v, alternating current measurements, the measurement of intense magnetic fields and permeability, and electric oscillations. An appendix giving a brief sketch of spherical harmonic functions is well placed.

The book suffers somewhat from the extensive use of small type, for which, however, the author makes due apology. The printing is good, and the book is provided with an exhaustive table of contents, a good index, and marginal notes, which will add greatly to its usefulness. A greater number of references would be an improvement. In notation Professor Gray has taken Maxwell's treatise as a basis and has "followed a suggestion of Heaviside in the use of clarendon type instead of German capitals for directed quantities." It will, no doubt, be a matter of regret to many that the condensed notation of the vector analysis has not been made use of in the purely theoretical parts of the subject, for it is almost as much of a task to read carefully a book containing numerous Cartesian formulæ as it is to write it. This is especially true if singularities of notation occur, which, to a certain extent, are inevitable to a work of this kind on account of "the large number of sets of quantities to be symbolized." Professor Gray assumes considerable knowledge on the part of the reader concerning the fundamental phenomena of electricity and magnetism. Still, the absence of terse statements of fundamental phenomena, laws, and conceptions to accompany the formulation of them is to be deplored. No mathematical treatment can be wholly satisfactory unless so sustained. The cure for a difficulty in mathematical physics cannot be *more mathematics*, except in such developments as partake of the nature of problems as distinct from the development of formal theory. If the difficulties of the student, like those to which Professor Gray refers in the preface, are mathematical difficulties, this work will go a great way towards clearing them up; but if he labor under the more common and serious difficulties which come from weakness in fundamental conceptions, then, to use a term often applied by English critics to German works of this class, Professor Gray's book is surely not sufficiently "metaphysical" to be of much assistance.

It seems that the book could have been made slightly smaller without detracting in the least from its value, by limiting the discussion of such topics as the force action in cavities in magnetized iron to the two important cases; for, in fact, with the exception of just these two cases the whole matter is meaningless. In large cavities the condition of undisturbed magnetization cannot be fulfilled, and in small cavities the conception of uniform magnetization loses its meaning. It is misleading to treat the subject as if our knowledge extended to the inside of a magnet: all magnetic measurements without exception depend upon phenomena which occur outside of iron. Chapter V., on the electromagnetic theory of light, would have been briefer and in some respects more presentable if the symmetrical form of the equations of the electro-magnetic medium due to Heaviside and Hertz had been used. Chapter VI., on the calculation of coil constants, deserves special mention. This is a subject to which Professor Gray has made some valuable contributions. The chapter gives a concise sketch of the subject and gives a number of formulæ which will be of great value to the practical electrician. On the whole, the subject-matter of the book is chosen in accordance with the title; Chapters V. and XIV. (on electric oscillations) being apparently digressions from the plan as announced in the preface to Vol. I. Of no little value and interest are the accounts of experimental investigations, in which the author places "before the reader not merely a skeleton of the method followed and the result arrived at, but such a statement in each case as may serve to show the procedure adopted, the difficulties met with, the mode in which they were overcome, the corrections made, and the reduction of the observations to the final result." Professor Gray has been eminently successful in his "attempt to avoid too sharp a distinction between what is theoretical and what is practical." The main object of his book is the development of absolute measurements, and this object is fully attained.

W. S. Franklin.

Alternating Currents: An Analytical and Graphical Treatment for Students and Engineers. By F. BEDELL, Ph.D., and A. C. CREHORE, Ph.D. New York, The W. J. Johnston Co.— 8vo. pp. 325.

The object of the above work is to give a clear and succinct treatment of the fundamental principles governing the flow of variable or alternating currents; and the authors aim on the one hand to give exact and rigorous demonstrations, and on the other to express the results in such a way as to be perfectly intelligible to those who do not desire to follow the methods of proof, but are only interested in the conclusions reached.

For the task proposed logical order and clear writing were essential; we find them on every page. Thorough knowledge of the subject was essential; the authors are contributors to its original literature. In short, it may be said that the work is what the authors claim, and that they have succeeded in realizing their object in an admirable manner.

If the authors had been writing for professional mathematicians, the natural order would perhaps be to begin with the most general theorem, and proceed from it deductively to the special cases; but as they are writing for students and engineers, they adopt the more suitable method of taking first a very special theorem, then a more general, and last of all attacking the most general theorem considered. Thus they treat first of a simple circuit, afterwards of the complex circuit, and introduce the complexity gradually. In each of these main divisions they first consider a circuit having resistance and self-induction, then a circuit having resistance and capacity, and lastly, the general case of a circuit having all three. As the proofs hinge on the solution of differential equations, this mode of presentation gives the reader an education, probably needed, in the meaning and solution of such equations.

The simple circuit is fully treated by the analytical method, and accurately drawn diagrams are provided to illustrate the results obtained. The complex circuit is not treated analytically, but by means of a graphical method applied to generalize the results for a simple circuit. The work is very free from typographical errors, but the manner of printing the equations leaves something to be desired. On account of the wide spacing many of the equations are too long for one line, and not infrequently the division is made at an awkward place; for factors are disassociated from co-factors, and the function from the variable.

There are a few statements which are not sufficiently exact. Thus the square of a polynomial is said to be "equal to the sum of the squares of each term separately plus twice the product of each term by every other term." Here either the "twice" ought to be deleted, or else "following" added at the end. In the earlier part of the book (p. 41, and p. 80), Fourier's theorem is not stated with sufficient exactness. Thus at p. 41: "Fourier has shown that any single-valued periodic curve may be built up by combining a number of simple sine-curves. Analytically this means that any single-valued periodic function may be expressed as the sum of a series of sine-terms; thus

$$y = f(x) = A \sin \alpha x + B \sin \beta x + C \sin \gamma x + \cdots \text{etc.},$$

where f is a single-valued function. This is true for any single-valued periodic function, even one represented by an irregular series of straight lines." Now the exact statement of the theorem is: If $f(x)$ is any single-valued periodic function of x of period p, then

$$f(x) = A_0 + A_1 \cos \frac{2\pi}{p} x + A_2 \cos 2\frac{2\pi}{p} x + \cdots + A_i \cos i\frac{2\pi}{p} x + \cdots$$
$$+ B_1 \sin \frac{2\pi}{p} x + B_2 \sin 2\frac{2\pi}{p} x + \cdots + B_i \sin i\frac{2\pi}{p} x + \cdots$$

Each A and B coefficient is independent of x, and has one and only one value; and Fourier shows how to find the value. The several arcs are not arbitrary; they are the integer multiples of the first; in other words, the successive periods have the relations

$$1, \ \tfrac{1}{2}, \ \tfrac{1}{3}, \ \tfrac{1}{4}, \ \tfrac{1}{5}, \ \text{etc.}$$

At p. 89 and p. 157 this relation is mentioned, but it is not stated with sufficient prominence.

When a circuit has a resistance R, self-induction L, and capacity C, the discharging current i at any time t is given by the equation

$$i = c_1 \epsilon^{\frac{-RC + \sqrt{R^2C^2 - 4LC}}{2LC} \cdot t} + c_2 \epsilon^{\frac{-RC - \sqrt{R^2C^2 - 4LC}}{2LC} t},$$

where c_1 and c_2 are the arbitrary constants of integration. Now if R^2C is greater than L, the expression $\sqrt{R^2C^2 - 4LC}$ is real; but if R^2C is less than L, it is imaginary. Nevertheless there is physical continuity in the change of the character of the current; in the former case the current is non-oscillatory, in the latter it is oscillatory, and in the bounding case of R^2C being equal to L it has an intermediate character. Now this being so, it follows that $\sqrt{-1}\sqrt{4LC - R^2C^2}$ is physically real, being the complement of $\sqrt{R^2C^2 - 4LC}$. The geometrical meaning of this is that the tangent to a circle is the complementary continuation of the ordinate, the latter being $\sqrt{a^2 - x^2}$, the former $\sqrt{-1}\sqrt{x^2 - a^2}$. The authors transform the imaginary expression to a real form in the usual manner — by what

Fleming, in "The Alternate Current Transformer," calls 'a well-known algebraic device'—namely,

$$\sin\theta = \frac{e^{\sqrt{-1}\theta} - e^{-\sqrt{-1}\theta}}{2\sqrt{-1}} \text{ and } \cos\theta = \frac{e^{\sqrt{-1}\theta} + e^{-\sqrt{-1}\theta}}{2}.$$

No real student of physics feels particularly satisfied when an algebraic device is presented to him for a reason; he wishes to reason not darkly, but by evident principles. Any one who wishes to see an explanation founded entirely on geometrical principles will find such explanation in a paper which I have recently published on "The Imaginary of Algebra."

At the beginning of "Part II., Graphical Treatment," the authors explain why analytical treatment is not applied to complex circuits. "The analytical solutions derived in Part I. apply merely to a single circuit having resistance, self-induction, and capacity in series. The problems which arise in case there is not simply a single circuit but a complicated network of conductors might be treated analytically, though the process would be exceedingly laborious and the results too cumbersome to handle. Fortunately, however, by making use of the analytical solutions already given in Part I. and extending them by graphical methods, we are enabled to solve problems with compound circuits which offer considerable difficulty to analytical investigation. These graphical methods are most easily and advantageously adapted to problems in which we deal with an harmonic impressed E.M.F."

The principle of the graphical method is to represent a simple harmonic motion by means of the vector to the point describing the auxiliary circle; for a simple harmonic motion is uniform circular motion resolved along a particular diameter. Hence one may reason on the relations of these vectors instead of the algebraic displacements along a diameter. The reason why the ordinary analysis is here found too complex, laborious, and unwieldy, is because it is adapted to quantities which may be represented on a straight line, not to quantities in a plane, still less to quantities in space. But there is a generalized algebra which deals with quantities in a plane, and more generally in space; it is the analytical expression of this same graphical method, is equally direct and clear, but, being a generalization of ordinary analysis, is much more powerful than any graphical method.

ALEXANDER MACFARLANE.

William Gilbert of Colchester, on the Loadstone and Magnetic Bodies, etc. A translation by P. FLEURY MOTTELAY. New York, John Wiley & Sons, 1893.—8vo, pp. liv + 368.

The need of an English translation of Gilbert's "De Magnete" has long been felt. In this country especially, where copies of the original are rare and widely scattered, the difficulties that have stood in the way of consulting this most interesting book have been a matter of great regret to physicists and electricians. The appearance, in so attractive a form, of Mr. Mottelay's translation will therefore be hailed with satisfaction by all who are interested in the early history of physical science.

In addition to the text of the London edition of the year 1600, which is translated in full, the volume contains the laudatory address of Edward Wright "to the most learned Mr. William Gilbert, . . . the father of magnetic philosophy," together with a biographical memoir by Mr. Mottelay. The latter is necessarily brief on account of the slight knowledge of the great philosopher that has come down to us.

The annotations throughout the text are numerous, and show the result of Mr. Mottelay's wide acquaintance with early scientific literature. For those who wish to make a critical study of the book they cannot fail to prove of great assistance, while for the general reader they serve in many cases to explain obscure references which would otherwise be unintelligible. It is unfortunate, however, that in a few cases the notes themselves are capable of being misunderstood. For example, on page 117 the reference to Porta is explained in a footnote by a quotation from Porta's "Natural Magick," published in 1658. Since the last authentic edition of "De Magnete" appeared in 1633 the quotation serves rather to confuse the reader than to help him. In this case, as in several others of a similar kind, a few words of explanation would probably make the matter clear.

From an artistic standpoint the book is deserving of high praise. Print, paper, and binding are alike excellent, and far superior to what is ordinarily seen in scientific books. Among the features which add to the attractiveness of the volume may be mentioned the reproduction of the title-page of the edition of 1600, and the use of the Gilbert coat of arms on the cover. At the end of the book the title-pages of the editions of 1628 and 1633, and that of Gilbert's "Philosophia Nova" (1651) are also shown. The numerous illustrations that appear throughout the text are reproduced by photographic processes from the original edition.

The translation appears to be accurate and as nearly literal as is consistent with good English and a correct interpretation of the author's meaning. In a careful comparison of some five or six chapters, taken at random through the book, with the original, I have found but three errors that are worthy of notice. On page 106, line 10, the word "magnetic" should be replaced by "electric." In the first sentence on page 105 I doubt if the full significance of "commouentem" has been brought out in the translation. On page 231 the word "only" should be inserted after "not" in the second line from the bottom, so that the sentence shall read: "In southern latitudes also, and at sea, far away from the equator and toward the Antarctic, and not *only* in northern latitudes near those magnetic mountains, is variation frequent and great." Although not free from faults, the translation will, I think, prove satisfactory to the great majority of readers. Mr. Mottelay is especially to be congratulated on his success in retaining in his translation some of the quaintness of style which is so attractive in the original and so appropriate to the time at which the book was written.

ERNEST MERRITT.

The Voltaic Cell. By PARK BENJAMIN, Ph.D. New York, John Wiley & Sons. — 8vo. pp. iv + 562.

According to the preface the object of this large volume is "to assist the student and investigator . . . by placing before him the record of the principal discoveries and researches relating to chemical generators of electricity." The author further remarks that "the number of sources laid under contribution is large. Chief among them is Dr. D. Tommasi's recent treatise 'Les Piles Électriques,' from which copious extracts have been translated."

Dr. Tommasi's book was published in 1889. It is a large encyclopedic volume of 680 pages, relating to a great variety of combinations which have been proposed as voltaic cells, and containing numerous tables of the numerical results of many investigations relating to electromotive force and the like. The striking resemblance, even at a glance, between Mr. Benjamin's book and that of Tommasi has led me to compare them somewhat at length. The result is to ascertain that of the 535 pages of "The Voltaic Cell," up to the Bibliography, over 300 have been directly translated from Tommasi. These "copious extracts" are taken from ten to fifty pages *en bloc*, with only slight additions interlarded. They include the larger part of Tommasi's tables, which he doubtless collected largely from original sources, and to which Mr. Benjamin has made only a very few additions. So truthful is the translation and so exact the copy that footnotes are frequently included exactly as in the original. The description of cells also very generally follows Tommasi page after page, with an occasional addition drawn from American examples. It is therefore evident that a review of the present volume would be substantially a review of "Les Piles Électriques." According to the literary ethics with which I am familiar, it would have been nearer the facts in the case if Mr. Benjamin had described his book as a translation of Tommasi with additions and an introduction.

The early history of the voltaic cell is well told in the first chapter, and many facts not commonly known are brought out.

The next chapter on definitions is less successful. Volta's invention is so defined as to comprise a modern secondary battery or accumulator. But it appears somewhat like forcing language to suit one's purpose to speak of "a secondary voltaic cell or voltaic accumulator." This definition scarcely justifies the title of the book, which is made to cover both primary and secondary cells.

Polarization is said to produce "a current in opposite direction to the normal current of the cell." The polarization of an electrolytic cell may produce a current opposite in direction to the one effecting electrolysis, but this is not a correct description of the effect of polarization in a voltaic cell. It is in either case a counter or back electromotive force.

Local action is described as "chemical action occurring within the cell." Of course it is that chemical action which contributes nothing to the energy represented by the current through the cell.

The ohm is incorrectly defined as "the resistance of a column of mercury at 0° C. of 1 sq. mm. section and nearly 105 cm. in length." The length is now well known to be very close to 106.3 cm. So also the electrochemical equivalent of silver is given on page 24 as 1.1340 milligrams per coulomb. It should be 1.118 milligrams per coulomb, or 4.025 grams per hour for a current of one ampère.

The summary of the advantages of sodium bichromate on page 186, taken from my "Primary Batteries," includes one in which I am in error; and this gives me an opportunity to correct it. The sodium salt does not contain a larger proportion of available oxygen than the potassium salt, for the reason that the former has two molecules of water of crystallization.

Among standard cells my own is dismissed with the statement that "the employment of pure chemicals and careful handiwork is claimed to result in a reduction of the temperature coefficient to about .03 of one per cent per degree centigrade." This reduction of the temperature coefficient to .039 per cent is effected in a very different way, and the evidence of it is given numerically in tabular form in my "Primary Batteries," to which Mr. Benjamin makes frequent reference.

The arrangement of materials from Tommasi has been improved by Benjamin, and his bibliography is much superior to that of the French writer. Attention may be called to the fact that the third edition of Hauck's excellent little book on "Die Galvanischen Batterien" appeared in 1890. The 1883 edition catalogued by Benjamin was the second. Also the *Journal de Physique* does not appear among French periodicals bearing on the subject. The whole of Bouty's investigations, quoted from Tommasi, are given in the volume of this journal for 1880.

The book belongs to a class too common in the literature of electricity — books which show that the author is not in touch with the subject by actual contact with the laboratory or with practical work. Science learned from books and science learned in the laboratory present certain pronounced differences readily apprehended but not easily described.

H. S. CARHART.

Report of Studies of Atmospheric Electricity. By Professor T. C. MENDENHALL. National Academy of Sciences, Vol. V., pp. 113–318.

This monograph, which is a report of attempts made by the United States Signal Service to determine the value of observations upon atmospheric electricity as a means of forecasting the weather, is of great interest to the physicist as well as to the meteorologist. The comprehensive historical sketch of early work upon atmospheric electricity will be welcome to all students in this domain. The discussion of the electrometer, and of the best methods of using that instrument for the measurement of atmospheric potentials, is likewise of great interest.

One cannot but deplore the limitations put upon work of the kind described in this report by the policy of the United States government in such matters. The problem of atmospheric electricity is one of great complexity, and it is one that cannot be solved save by the united action of a large number of observers distributed over wide territory. What precise bearing the results may have upon our knowledge of the weather it is impossible to say, but enough work has been done in Europe and in this country to show that the electrical conditions of the atmosphere play an important part in climatology. It is sad that a government like ours, with unlimited resources at its command, should hesitate to expend the time and money necessary to an exhaustive study of such problems, simply because it cannot be demonstrated in advance that the outcome will be of direct and immediate monetary value.

The instructions under which Professor Mendenhall undertook the direction of this work for the Signal Service were, however, most explicit, and he was confined strictly to the question whether simultaneous observations on atmospheric electricity were of service in forecasting the weather. That no answer to so complex a problem should have been obtained from the comparison of observations extending over two or three years at four or five stations is not surprising. The records which were made and which have been incorporated in this memoir are full of suggestion, and they make it evident that in order to determine ultimately and finally the relation of electrical conditions of the atmosphere to the weather a much greater number of stations should be established, and that the work should be carried on by a great variety of methods. Whether or not such systematic and exhaustive study would result in adding to the certainty of our weather forecasts, the data obtained would be a notable addition to our knowledge of electricity. The study of electricity for science's sake in the past has found important and far-reaching application. So, doubtless, it would be in this case; but were no results attained, the value of which could be measured in dollars and cents, the United States government, by pursuing work of this description, which, as has already been pointed out, lies beyond the reach of any individual laboratory or institution, would be performing one of those functions which certainly in the future we must look to government to perform. It would thus enable a people to reach knowledge unattainable by individual and unorganized effort. It is to be deplored that these observations were abandoned after so short a trial; and it is to be hoped that one of the results of the divorce of the Signal Service and the Weather Bureau will be to hasten the day when subjects like this may be attacked in a manner in some sense adequate to their solution.

E. L. NICHOLS.

Handbuch der Physik. Herausgegeben von DR. A. WINKELMANN. Breslau, 1893 (Edward Trewendt). — Band 3, I., pp. 546.

Winkelmann's Handbook is one of the two great general treatises on physics now in process of production. Being the work of more than a dozen colaborers, it lacks that unity and conciseness of treatment which characterizes Violle's great treatise; but it exhibits in a striking manner the indefatigable industry characteristic of German men of science. It is especially rich in the two features which render such a work of value to the practical physicist; viz. unusual fulness in the matter of references to original authorities, and a wealth of tabulated material. Theoretical development, on the one hand, and elementary description and explanation, on the other, have wisely been given a secondary place, and the work is chiefly to be regarded as a compendious summary of well-established results.

Volume III., the first part of which has just appeared, is principally from the hands of Dr. Graetz of Munich, and of Professor Auerbach of Vienna, with chapters on the electric conductivity of gases by Professor Staenger of Dresden, and upon thermo-electricty by Professor Braun.

Graetz, after giving a brief development of the potential theory, treats of electro-statics at considerable length, devoting special attention to the properties of the dielectric. The data gathered under this head from a variety of sources will be found to be of great value to workers in the new field of electro-optics.

Current electricity, beginning with contact phenomena, and including a brief treatise upon voltaic cells and methods of measuring current and electromotive force, is treated by Professor Auerbach.

The subject of electrical conductivity has been divided into three parts: the resistance of metallic bodies, by Graetz; of electrolytes, solid and liquid, by the same author; and the conductivity of gases, by Dr. Staenger. Electrolytic conductivity is entirely separated from electrolysis proper, to which an important chapter is given in another portion of the volume.

The third volume of this work is to be devoted entirely to electricity, and Part I., which is now under consideration, forms in itself a book of more than five hundred large pages.

Considering the increasing difficulty of following the results of investigation in the original sources, the appearance of such a compilation as Winkelmann's Handbook will be welcome to physicists, if for no other reason, because it will greatly facilitate acquaintance with the contents of the numerous papers whence the mass of material contained within its covers has been derived, and through the accompanying bibliographical work, with the memoirs themselves.

In a work of this character, especially when it is the joint product of several writers, one does not look for uniformity of treatment. In the present treatise much that has gone to make up the bulk of previous text-books is passed over briefly or is altogether omitted. It is in topics heretofore very inadequately treated that Winkelmann's work is especially strong.

E. L. NICHOLS.

Some Recent Text-Books in Physics.[1] — The first book in the list given below is intended as a text-book for classes in high schools, or schools of a similar grade, where the mathematical preparation of the student is of the most elementary character. For this reason the subject has been presented in such form that only the simplest ideas of geometry and algebra are required, while the only mention of a trigonometric function occurs in the discussion of the tangent galvanometer. The difficulties that are met with in preparing a satisfactory text-book under these restrictions are obvious. In some respects it would be easier to write an elaborate treatise. The authors have been unusually successful, however, in avoiding the faults which are too often met with in books of this class. The statements and explanations are, in general, brief but clear. In no case has accuracy been sacrificed for apparent simplicity. Although not free from faults, the book cannot fail to be of assistance in raising the standard of elementary teaching.

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[1] *The Elements of Physics.* By HENRY S. CARHART and HORATIO N. CHUTE. Boston, Allyn and Bacon, 1892. — 8vo. pp. 382.

Discussion of the Precision of Measurements. By SILAS W. HOLMAN. New York, John Wiley & Sons, 1892. — 8vo. pp. 176.

A Student's Manual of a Laboratory Course in Physical Measurements. By WALLACE CLEMENT SABINE. Boston, Ginn & Co., 1893. — 8vo. pp. 120.

A Manual of Physics. By WILLIAM PEDDIE. New York, G. P. Putnam's Sons, 1892. — 8vo. pp. 501.

Of the different branches of physics that are discussed, the treatment will strike the reader as least satisfactory in the case of electricity and magnetism, although it is difficult to tell just where the fault lies. It is to be remembered, however, that the subject of electricity is probably the most difficult in physics to present in an elementary manner. The omission of all mention of double refraction and polarization in the section on light seems unfortunate. It is doubtless intentional, and probably results from the experience of the authors in the difficulty of teaching these subjects to an elementary class. I cannot help thinking, however, that a few well-written pages devoted to the simpler phenomena of polarization would add to the value of the book.

The numerous illustrative experiments that are described throughout the book will doubtless prove of considerable value to teachers. These experiments are, in most cases, so simple that they could be performed in the class-room, or by the student alone. The illustration of physical laws by reference to the familiar facts of everyday life is also deserving of mention, the sections on heat and sound being in this respect especially satisfactory. There are several cases, however, in the section on mechanics where the explanations and illustrations seem inadequate. The subject of the composition of velocities, for example, might be more clearly presented. I doubt also if the discussion of fluid pressure on pages 90–91 will make the matter entirely clear to a beginner. The most striking omission, however, is in the case of the third law of motion, which is barely stated, and left with practically no explanation. Such an omission is especially to be regretted in a book of this kind, which is unfortunately too often placed in the hands of a teacher whose training is not sufficient to enable him to supply its deficiencies.

Professor Holman's "Discussion of the Precision of Measurements" is intended for the use of students in the laboratory, and presents in a clear and concrete form the usual methods of treating the accidental errors of observation. The book will be found to differ widely, however, from the ordinary text-book of least squares. The results of the theory of probabilities are stated without proof, and serve as a basis for the discussion which follows. It appears to me doubtful whether the omission of proof in the more fundamental cases is advisable. References should at least be given to books where the mathematical treatment can be found. But the discussion of the principles involved is so excellent that criticism on this point is in large part disarmed.

The scope of the book will perhaps be best indicated by the titles of its four principal divisions: I. Direct Measurements; II. Indirect Measurements; III. Best Magnitude of Components; IV. Solutions of Illustrative Examples. In the first section will be found a general discussion of the constant and variable errors of observation, illustrated by carefully chosen examples. The use of the "average deviation" (the average value of the deviation from the mean, neglecting signs) instead of the "probable error" will meet with the approval of all who have observed the difficulty that is experienced by students in obtaining a proper understanding of the latter term. The second division is devoted to the consideration of cases where the final result is some function, more or less complex, of the quantities directly observed. This chapter would be improved if the treatment were made more brief. The detailed discussion of the special forms that may be assumed by the function which expresses the result seems entirely out of place. In fact the twelve pages that are devoted to this purpose rather detract from the usefulness of the book by giving an appearance of complexity to a subject which is really simple. The examples at the end of the chapter would gain in clearness if they were explained by a direct use of the principles of the method. The reference to special formulæ, which requires the student to retain in his memory an arbitrary system of notation, does not appear to me to be justified in the present case.

The fourth section, which is devoted to the solution of illustrative examples, is in some respects the most valuable in the book. Of the seven problems that are discussed, the following may be mentioned as of especial interest: the calibration of a voltmeter; the measurement of power by the cradle dynamometer; and the discussion of the sources of error in the use of a tangent galvanometer. Apart from their illustrative value the examples discussed in this section will prove useful for reference in the laboratory.

It may not be out of place to call attention to the misprint on page 132, where we find the statement that "1 lb. = 13825 *g* ergs." So much misunderstanding already exists in regard to the two systems of units that it is unfortunate that any avoidable cause of confusion should be added.

The "Manual of a Laboratory Course in Physical Measurements," by Mr. Sabine, represents a grade of work that is intermediate in character between the course which would be expected in a preparatory school and that which is usually required in colleges. The elementary character of the book is doubtless explained by the fact that the course, as given by Mr. Sabine, is intended to replace the experimental lectures which in most colleges are made to precede work in the laboratory. Although the desirability of this arrangement of work appears to me very questionable, it would be out of place to discuss the subject here. For the purposes in view the book is an excellent one.

It is the intention of the author that the experiments should be performed in the order in which they occur in the book, the class being broken into sections if necessary, and working under the direction of an instructor. To render this plan practicable without an excessive outlay for equipment, the experiments have been so planned as to require only the simplest apparatus. The reader cannot fail to be impressed with Mr. Sabine's success in this part of his task. A danger that arises, however, from the use of very simple apparatus lies in the fact that the student is apt to look upon his experiments as merely illustrative. Careless habits of observing result, and although at the end of his course the student may have obtained a fair understanding of physical phenomena, he has failed entirely to acquire a grasp of the quantitative methods of the laboratory. It is in the lack of sufficient precautions against this tendency that I consider the book chiefly in fault. The general directions in the introduction are excellent, while the explanations of the principles involved in the various experiments are unusually clear. But in the discussion of the sources of error, and of the methods to be used in determining the corresponding corrections, the book is uniformly unsatisfactory. The value of the course would be greatly increased if the treatment were made more rigorous, even if this should require a considerable reduction in the number of experiments.

Although the book is in general free from errors, two instances may be mentioned where the statements are not above criticism. On page 45 the author states that "theoretically mercury increases in volume as much when the temperature is changed from 0° to 50° as when it is changed from 50° to 100°." The word "theoretically" seems to be inserted in order to indicate that the statement is not strictly true. Although the use of the term in this sense is not uncommon, it is certainly not to be commended. The use of the gram as the unit of force on page 23 is unfortunate in view of the statement made in the introduction that the C. G. S. system would be adhered to. This error will probably correct itself, however, since the dyne is defined and correctly used on the following page.

The "Manual of Physics" by Dr. Peddie can scarcely be called a text-book in the sense in which the term is ordinarily used. As stated in the preface, the book is intended to give a review of the whole subject of physics for the use of students in the universities. For this purpose it has many advantages. The treatment throughout the book is extremely suggestive, while the amount of valuable matter that is contained in its 500 octavo pages can scarcely be realized until the book is read. The evil effects of crowding are noticeable, however, throughout. There is an indescribable something that suggests an approaching examination, and which cannot fail to give to the reader a feeling of hurry and unrest.

The mathematical treatment, especially in the early part of the book, presents some novel features. The discussion of the theory of contours in Chapter III., and the use of the simpler methods of vector analysis in Chapter V., will meet with approval, as indicating the advances that are being made in the application of graphical and vector methods to physics. In both cases, however, the discussion will prove satisfactory only to those who have studied these subjects elsewhere. For a student who is approaching the matter for the first time the treatment is quite inadequate. This remark will apply with equal force to most of the mathematical work in the early part of the book. Chapter IV., which is devoted to the explanation of the elementary processes of differentiation and integration, seems out of place in a manual of physics, and is insufficient to prepare the student for the applications of the calculus which follow. The elementary explanation of the use of Cartesian co-ordinates in Chapter V. seems also unnecessary.

In what might be called the philosophy of physics the book is strong. The discussion of the uses of theories and hypotheses in physical science is good, although the author weakens his position by the bold statements that "as knowledge advances the truth of some theories will be proven," and that "an elementary molecule or atom has absolutely unalterable properties." The parallel discussion of the undulatory and corpuscular theories in the section on light is instructive, and affords an opportunity for illustrating the methods of discriminating between theories which at the outset appear equally plausible. On the whole the book seems to be well suited for the purpose set forth in the preface. I doubt, however, if it will prove of value as a text-book.

ERNEST MERRITT.

Volume I. *September–October, 1893.* *Number 2.*

THE

PHYSICAL REVIEW.

ALTERNATE-CURRENT CONDENSERS AND DIELECTRIC HYSTERESIS.[1]

BY FREDERICK BEDELL, N. F. BALLANTYNE, AND R. B. WILLIAMSON.

WITHIN recent years the electromagnetic effects of alternating currents have been quite thoroughly studied, but the electrostatic phenomena have not received such careful attention, and many points of interest still remain to be investigated, among which may be mentioned that of hysteresis loss in the dielectric of a condenser. Not only from a purely physical point of view is it desirable to know as much as possible about the behavior of condensers, but also for practical reasons, inasmuch as condensers are likely to be used more extensively in the future in connection with alternating currents. For these reasons it was thought that the results of some experiments recently performed might be of interest. A description of what little was done may furnish some suggestions to those working along the same lines.

The condensers, six in number, upon which the experiments were performed are intended to be used commercially on 500-volt circuits. The plates are of tinfoil, the useful part of which is of the following dimensions: length, 10½ inches; width, 8 inches; thickness, .0007 inch. The dielectric is of waxed paper .0043 inch thick. There are sixty-five sheets of tinfoil (total) in each slab, and two of these slabs are placed together in one tin case.

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[1] A paper read at the Madison Meeting of the American Association for the Advancement of Science, August, 1893.

As one of the first things to be determined about a condenser is its capacity, this was measured for each of them by Thomson's method of mixtures, which is described in various laboratory manuals.

Another method used for finding the capacity was that of measuring the current in the condenser for any particular value of an harmonic electromotive force impressed at the terminals. Since the current in a circuit containing capacity alone when $e=E \sin \omega t$ is $I=CE\omega$, we have at once $C=\frac{I}{E\omega}$, in which I and E are either maximum or mean square values of current and electromotive force respectively, and ω is 2π times the frequency. The current was measured by means of a Thomson milli-ampere balance, and difference of potential by a Thomson multicellular static voltmeter. The frequency may be easily found when the number of poles on the dynamo and the speed of the armature are known.

The data and results obtained by this method for one of the six condensers are given in Table I., and, for comparison, the measurements made by Thomson's method are given in Table II. In almost all cases the values of the capacity obtained by the method of mixtures were slightly higher than those found by the alternating current method. The equation $C=\frac{I}{E\omega}$ is true only when the impressed electromotive force is a sine-function of the time, and R is zero. Since neither of these conditions was strictly fulfilled, the results obtained were considered quite concordant, inasmuch as, taking all the measurements into consideration, the greatest difference between a single observation by the alternating current method and the method of mixtures was 7 per cent, while the greatest difference in the mean values obtained from a series of observations was 3.24 per cent.

TABLE I.

CAPACITY OF CONDENSER.

Volts. E.	Amperes. I.	ω	$C=\frac{I}{E\omega}$.
208.	.340	1127.	1.45
304.	.522	1127.	1.52
324.	.542	1110.	1.51
332.	.548	1110.	1.49
354.	.584	1097.	1.50
374.	.622	1102.	1.51
396.	.660	1108.	1.50
446.	.742	1108.	1.50
460.	.762	1104.	1.50
		Average . .	. . 1.50 (nearly)

TABLE II.

CAPACITY BY THOMSON'S METHOD.

Standard Capacity.	R_1.	R_2.	Capacity of Condenser.
.6 microfarads.	7000. ohms.	2752. ohms.	1.526 microfarads.
.4 "	7000. "	1835. "	1.526 "
.2 "	8000. "	1050. "	1.524 "
.1 "	8000. "	524. "	1.526 "
		Average . .	. 1.525 "

These results indicate that, for practical purposes, this method may be used where apparatus for more refined measurements is not available.

Since it was not known to what extent the capacity was dependent upon the temperature, a few determinations were made for the purpose of investigating this point. One of the condensers was placed in a tank of water, heated by means of a Bunsen's burner beneath, and at intervals the source of heat was removed and the capacity was measured by Thomson's method of mixtures, the temperatures of the water being noted each time. The variation of the capacity for rising and falling temperatures is graphically shown in Fig. 1. The capacity changes very little between 0° and 30° C. Above 30° the capacity decreases until a temperature of 40° is reached, while beyond this the capacity rises quite rapidly with increase of temperature. The reading for capacity was not taken until it became constant, at which time the temperatures of the condenser and of the water were assumed to be the same. It is not easy to explain the causes which give this curve its form.

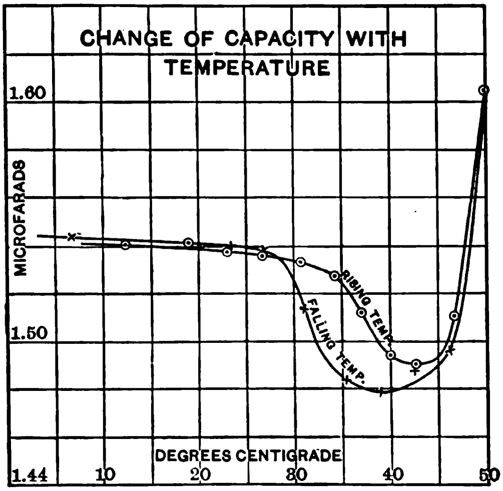

Fig. 1.

In order to determine the leakage through the dielectric, one of the condensers was given a charge, and then allowed to stand several hours, with its terminals connected to the multicellular voltmeter. The falling off in the potential is shown by curve I., Fig. 2. For comparison, the theoretical exponential curve, showing the leakage through a dielectric resistance of 25,000 megohms, is plotted, curve II., in the same figure. Several values of the dielectric resistance of the condenser were calculated from curve

I., on the assumption that, when t is small, the expression $R=\dfrac{t}{C\log_e\dfrac{V_0}{V_t}}$ is approximately correct. These values are plotted

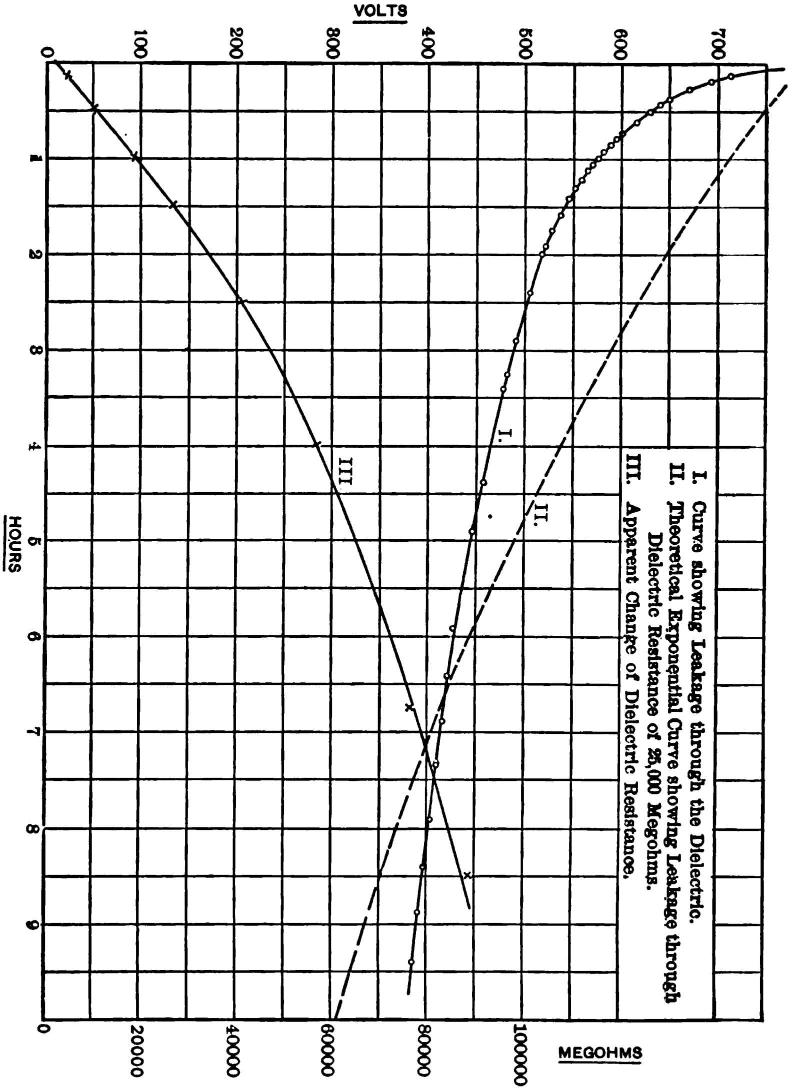

Fig. 2.

in curve III., Fig. 2, from which it appears that the dielectric resistance increases very greatly with increase of time and decrease of potential. This apparent increase is probably due to the soaking in of the charge, which would cause a more rapid falling off in the potential at first than the falling off due to loss of charge through leakage.

Since some idea of the loss that occurs in a condenser when rapidly charged and discharged may be had from the amount of heat liberated, one of the condensers was subjected for five hours to an alternating electromotive force of 500 volts, at which potential the current is about three-fourths of an ampere, when the frequency is 160 alternations per second. The temperature of the condenser, which was well covered with cotton batting to prevent radiation, was read at intervals of ten minutes. Curves were plotted showing the change of temperature of the room as well as that of the condenser. It was ascertained that under the above conditions, if there were no radiation, the condenser would heat at the rate of very little over 1° C. an hour. This amount of heating shows the loss to be small, and therefore, if there is hysteresis in the dielectric, it must be small.

The power expended in a condenser may be ascertained by the three-voltmeter method. In a perfect condenser, the current is 90° ahead of the impressed electromotive force, this indicating that no power is lost, since the current has no component in the direction of the electromotive force. Any change in the angle of phase from 90° would cause the current to have a component in the direction of the electromotive force, which would show that there was some expenditure of energy.

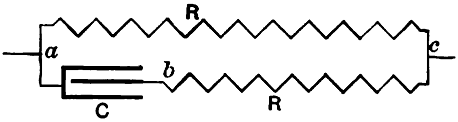

Fig. 3.

A circuit, as shown in Fig. 3, was subjected to an alternating electromotive force and the electromotive force was measured between *a* and *b*, *a* and *c*, and *b* and *c*. The values obtained are given in Table III.

TABLE III.

E. M. F. between $a-b$.	E. M. F. between $a-c$.	E. M. F. between $b-c$.
1165.	568.	1009.
1151.	564.	1003.
1178.	590.	1029.
Average, 1165.	Average, 574.	Average, 1014.

Using the averages of the three values of the electromotive force obtained for each, the electromotive force triangle shown in Fig. 4 was drawn. In this triangle the electromotive force *ab* impressed upon the condenser is so nearly at right angles to the current, which is in the direction of *cb*, that the loss is again shown to be very small.

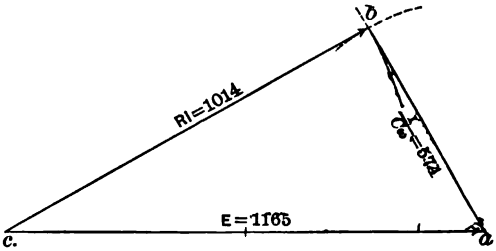

Fig. 4.

An attempt was made to obtain an hysteresis loop for a condenser by plotting a curve showing the relation between the charge and increasing and decreasing potentials. The condenser was charged up to any desired potential, measured by a multicellular voltmeter, and was then discharged through a ballistic galvanometer to find the quantity of charge. By successively recharging to different potentials, and discharging through the galvanometer, the corresponding quantities of charge were found. To show the relation between the potential and the quantity of charge, a curve was plotted in which abscissæ represented potentials, and ordinates represented the quantities of charge for increasing values of potential. To obtain the corresponding curve for decreasing potentials, the condenser was charged to a certain high potential, and then allowed to discharge through a high resistance to any desired potential, the quantity of charge at this potential being determined as before by discharge through the galvanometer. This operation was repeated, the condenser being charged to the same high potential in each case, and allowed to discharge to some lower potential. If the curves drawn for increasing and decreasing potentials enclosed an area, it would indicate an hysteresis loss; but the two curves so nearly coincided that no loop was discernible. As the results of later experiments show the presence of hysteresis, it is probable that the time element had some effect. The necessary delay in measuring the potential doubtless permitted the condenser to acquire the same charge irrespective of whether the potential was raised or lowered to the particular final value.

Since considerable may be learned about the behavior of a condenser subjected to an alternating electromotive force from the instantaneous curves for electromotive force and current, it was thought that the subject of hysteresis in the dielectric might well be investigated in this way. From such curves the power expended at any instant may be found by multiplying the instantaneous value of the electromotive force by the corresponding value of the current. By obtaining in this way a series of values of the power, a watt curve may be plotted showing the amount of energy supplied to the condenser and the amount taken from it. From this curve the power wasted and the efficiency of the condenser may be determined. When the current and time of flow are known, the quantity of charge in the condenser at any time may be found according to the relation $q=\int idt$. The quantity of charge at any time during a cycle, and also the potential being known, a curve may be drawn showing graphically the relation between the two.

The arrangement of the apparatus for finding instantaneous values of the impressed electromotive force, of the electromotive force at the terminals of the condenser, and the electromotive force at the terminals of a non-inductive resistance in series with it, is shown in Fig. 5. The alternator is represented at A; a row of twenty-two incandescent lamps in series, by means of which the impressed electromotive force could be measured in two steps, is shown by R_1; the condenser to be tested is C_1, while R_2 is a row of twenty-five incandescent lamps in series with the condenser. The several electromotive forces were measured with the multicellular voltmeter represented by *V*, to which the condenser C_2 was connected in multiple so as to prevent errors due to leakage.

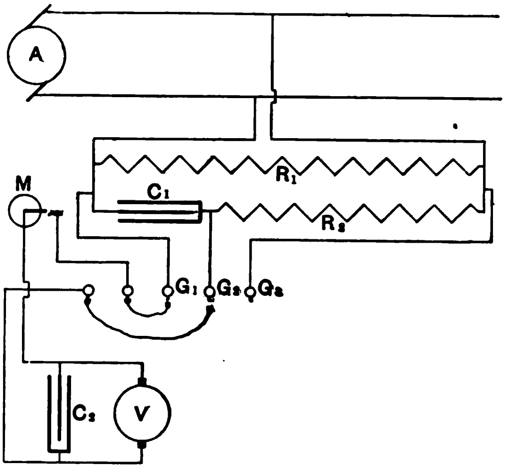

Fig. 5.

The electromotive force at any particular part of the cycle was obtained by means of the contact-maker *M*, through which the voltmeter circuit was completed for an instant once during every revolution of the armature. The contact can be so adjusted as to complete the circuit at any part of the revolution as desired. For convenience in making connections, the points between which it was desired to measure the difference of potential were joined to suitable terminals G_1, G_2, G_3 on a switch-board. Readings were taken at intervals of 2½° on the contact-maker. The alternator having ten poles, there are five complete periods for every revolution of the armature, hence 72° on the contact-maker corresponds to a cycle. From the data thus obtained, the instantaneous electromotive force curves shown in Fig. 6 are plotted.

The current in the condenser circuit was found by calculating the current in the non-inductive lamp-resistance, R_2, according to Ohm's law. The resistance R_2 was previously determined by the fall of potential method. Since the condenser current is in phase with the electromotive force of curve II., Fig. 6, and has its ordinates proportional to those of curve II., this may also be taken to represent the instantaneous values of the current when a suitable scale is applied. Multiplying corresponding ordinates of curves II. and III., the watt curve given in Fig. 7 was obtained. From the way in which this curve was plotted, the areas enclosed above the

X-axis represent energy imparted to the condenser, while areas below represent the energy given out by it. Taking the difference between the upper and lower areas, the power lost was

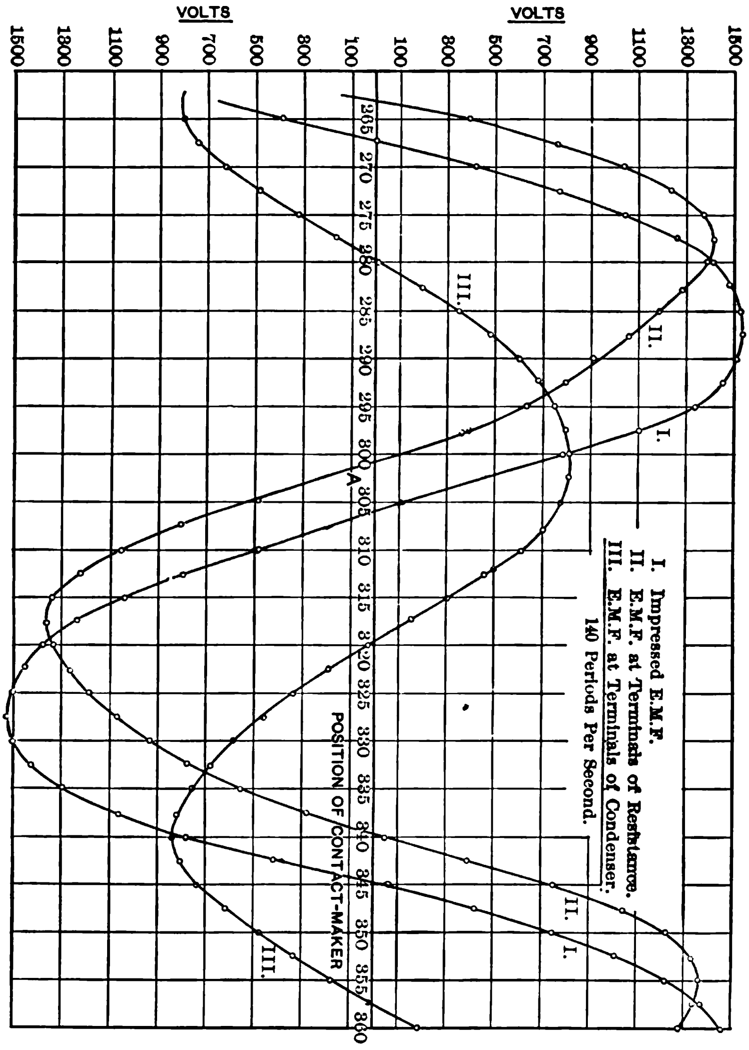

Fig. 6.

calculated to be 4.4 watts, which makes the efficiency of the condenser 96.9 per cent. In a similar preliminary run, the loss was found to be 2.47 watts, corresponding to an efficiency of 98.05 per cent.

The method devised for obtaining an hysteresis loop from the instantaneous curves for current and electromotive force is as follows. Since the quantity of electricity that flows through a circuit in the time *dt* is *idt*, if we start with no charge in a condenser and measure the value of the current from instant to instant, the total charge in the condenser at any time may be found by summing up

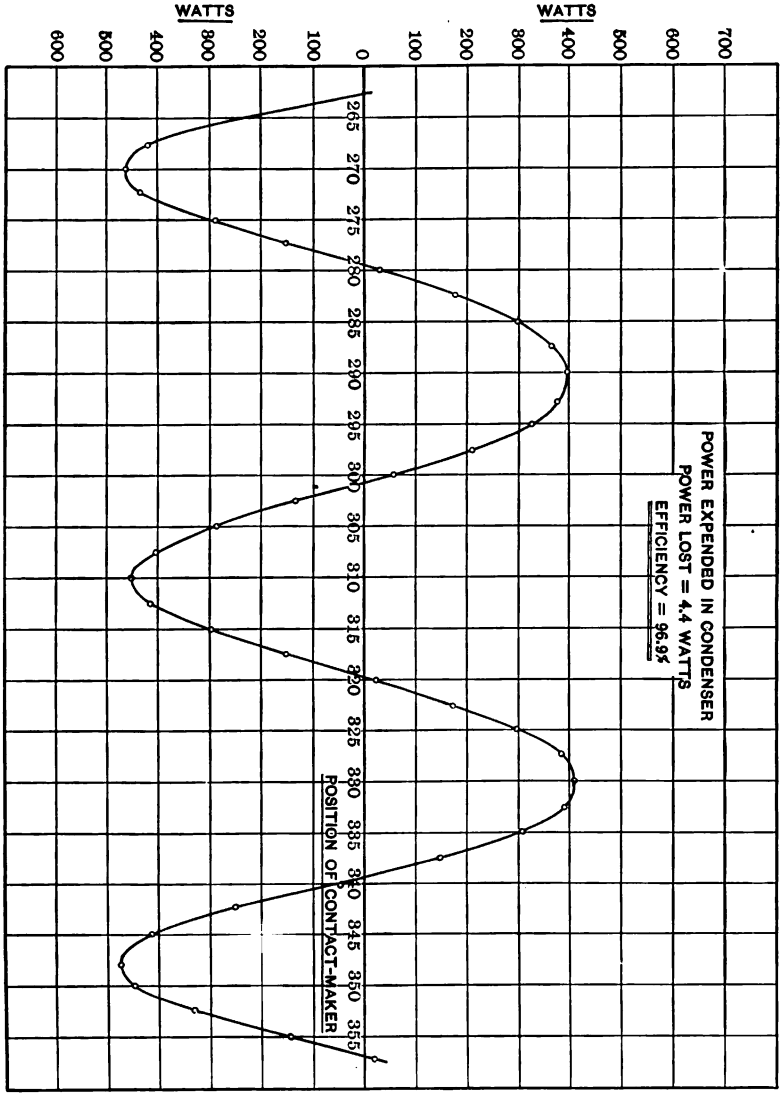

Fig. 7.

the instantaneous increments of charge. The total quantity of charge at the time t is $q=\int_0^t idt$, and is represented by the area enclosed between the X-axis, the current curve, and ordinates drawn at the points where $t=0$ and $t=t$. It being impossible to tell from the current curve at what point of the cycle the condenser contained no charge, the point A, Fig. 6, where the current is zero and the charge a maximum, was taken as a convenient place

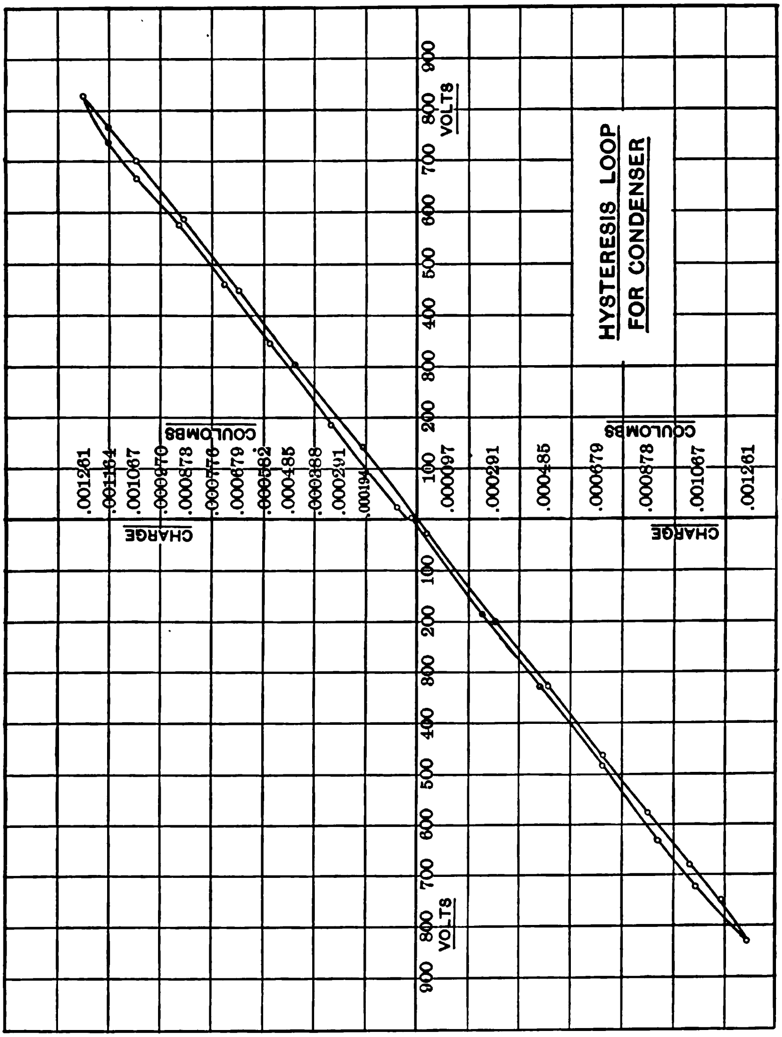

Fig. 8.

to start in obtaining the quantities of charge at different parts of the cycle. Between two successive points where the current is zero, the condenser becomes discharged, and then charged in the opposite sense, so that the area of one of the loops, enclosed between the current curve and the X-axis, represents double the maximum charge. Starting from A and working toward the left, the quantity of charge in the condenser was found, for different potentials, by subtracting successive areas. Similarly the quantities of charge at different parts of the cycle to the right of A were found by adding, algebraically, successive areas, remembering that areas below the X-axis are negative. The loop shown in Fig. 8 was obtained by plotting potentials as abscissæ and quantities of charge, obtained in the way described, as ordinates. A similar loop was obtained from the data of the preliminary run. Now the area of this loop is a measure of the amount of energy dissipated in the condenser per cycle, being equal to $\int vdq$. The area, measured by an Amsler planimeter, was found to be equivalent to $.0512 \times 10^7$ ergs. As there were 140 complete alternations per second, the rate of dissipation of energy in the condenser is $.0512 \times 10^7 \times 140 = 7.17 \times 10^7$ ergs per second $= 7.17$ watts. This result may be considered a rough check on the previous work, but cannot be depended upon by itself, inasmuch as the accurate determination of such a long and thin area is not readily made. The nature of the result is interesting, although of little quantitative value.

In alternating-current circuits the neutralization of the effects of self-induction by capacity opens a large field for work. How far this neutralization can be effected depends upon the nature of the impressed electromotive force and the condenser itself. With a perfect condenser and an harmonic electromotive force it would be possible to completely annul the effect of self-induction. With an alternating current which is not harmonic and a condenser which dissipates a certain amount of energy, *i.e.* is not perfect, complete neutralization of self-induction can only be approximated to. Experiments were made with the condensers in question, and curves obtained showing the variation in the current for changes in capacity for certain circuits. These curves showed a maximum point for the critical value of the capacity, but this was not as high or as well marked as the theoretical curves[1] plotted on the supposition that the condenser was perfect and the electromotive force harmonic.

In this paper we have given the results of an investigation of the loss in the dielectric of a condenser for one particular frequency. A complete investigation of the subject of dielectric hysteresis would include the determination of this loss for different frequencies and potentials, and also for different dielectrics. The law for dielectric hysteresis could thus be ascertained. The purpose of this paper has been merely to call attention to the existence of hysteresis and a method for its determination.

Physical Laboratory of Cornell University,
June 20, 1893.

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[1] Alternating Currents, by Bedell and Crehore, p. 139.

ON THE INFRA-RED SPECTRA OF THE ALKALIES.[1]

II.

By Benjamin W. Snow.

V. THE ENERGY SPECTRA OF THE ALKALIES.

THE problem which had to be solved when the various alkalies were burned in the electric arc consisted in finding a means whereby the metals or their salts could be continuously and uniformly introduced. The following methods, among others, appeared the most feasible: — 1. To surround the lower (positive) carbon with a crucible in which could be placed the salt to be investigated. 2. To throw, with the aid of a suitable apparatus, a jet of the salt solution in the form of a fine spray directly into the arc. 3. To bore out the carbon and to fill with the salt to be burned the cavity thus formed, as the wick is imbedded in a candle. 4. To mix with the powdered carbon, during the process of manufacture, a sufficient quantity of the metallic salt, and to make from this mixture the special carbons to be used. The first three of these methods were tried, the third giving the best results, so that this was the means which was uniformly used in the actual experiments.

In the axis of the positive (lower) carbon, 8 mm. in diameter, was bored a hole 3 mm. in diameter, in which the dry and finely powdered salt was pounded with a hammer to a solid core. The negative carbon was also prepared in a similar manner, with the exception that the diameter of the cavity was only 1.5 mm.

As soon as the electric lamp was set in operation, the glowing of the carbon points, in which the salt was imbedded, caused the salt to boil, and to send a perfectly uniform stream of metallic vapor directly into the arc.

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[1] Continued from page 50.

The appearance of the arc was completely changed by this introduction of the salt. Instead of the peculiar bluish violet color, was seen at once the characteristic color of the metallic vapor. Even the form changed immediately, from the little spherical ball playing uniformly about the two carbon ends, into a slender ellipsoidal figure whose vertices were never at rest. These extremities in general rotated slowly around the two rings formed by the ends of the cored carbons (see Fig. 8). Since it was found impossible to obviate this wandering of the arc, an assistant was secured who, throughout all the following investigations, changed the position of the projecting lens, as the arc moved, so that the slit of the spectrometer, upon which a real image of the arc was cast, was always symmetrically and uniformly illuminated. The length of the arc was, moreover, examined before each observation and regulated when necessary.

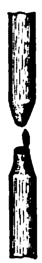

Fig. 8.

The consumption of electrical energy in the lamp was also quite different when carbons were used which were prepared in the manner already described. It was found by trial that to maintain the current constant at 7.7 ampères, a resistance of six ohms had to be included in the lamp circuit when the solid carbons were used, but that this resistance must be increased to ten ohms when carbons were used containing the core of salt. In both cases the length of the arc remained the same. Whether this change was due to a decreased resistance of the arc when filled with metallic vapor, or to a smaller counter-electromotive force of the arc, could not be decided without further experiments. Since this phenomenon had no vital connection with the more immediate objects of the investigation, no further experiments were made bearing upon this point.

Inasmuch as the employment of the different salts of a metal gave similar results, as, for instance, metallic sodium, sodium carbonate, and sodium chloride, only the chlorides of the metals were used in the investigation of the other alkalies.

With one exception, the salts employed were obtained in the purest possible condition from the Trommsdorff Chemical Works in Erfurt. Only in the investigation of the chloride of sodium was the ordinary commercial article used. Below is found the number of definitive experiments carried through with each salt: —

With NaCl, 6;
" KCl, 4;
" LiCl, 3;
" CsCl, 3;
" RbCl, 2.

So complete was the uniformity of the results obtained with each metal that no hesitation is felt in selecting at random, from the material at hand, a series of observations illustrative of each group.

The contents of the following tables may be found in graphical form in Figs. 5–9, Plates II. and III. Wave-lengths are again chosen as abscissæ, and galvanometer deflections as ordinates. Even these curves can in no way lay claim to representing the true distribution of energy in the normal spectrum, since here also the lines partially, at least, overlap, as may be clearly observed in the curve for sodium from the fact that the yellow D-line does not appear double.

Table V.

SODIUM. $k = \frac{1}{135000}$° C.

φ	λ	I	φ	λ	I
52° 37½′	0.3836 μ	0	48° 21½′	0.610 μ	29
28½′	0.3852 "	1	19′	0.616 "	91
23′	0.3870 "	0	16′	0.622 "	11
20½′	0.3880 "	0	10½′	0.637 "	13
13′	0.3908 "	0	7½′	0.644 "	22
7′	0.3932 "	31	6′	0.648 "	13
3½′	0.3950 "	2	3½′	0.654 "	8
			½′	0.663 "	6
51° 59½′	0.3967 "	31			
55½′	0.3983 "	0	47° 57½′	0.671 "	26
11½′	0.4218 "	1	56′	0.676 "	10
8½′	0.4236 "	42	54½′	0.680 "	10
5½′	0.4255 "	4	53′	0.685 "	4
			51½′	0.690 "	7
50° 10½′	0.4650 "	0	50′	0.695 "	5
7′	0.4677 "	11	48½′	0.699 "	7
4′	0.4703 "	1	47′	0.704 "	6
			45½′	0.710 "	14
49° 37′	0.4963 "	6	44′	0.714 "	13
34′	0.4996 "	62	42½′	0.720 "	12
31′	0.5028 "	4	41′	0.726 "	9
22′	0.5130 "	7	39½′	0.731 "	9
19′	0.5164 "	12	38′	0.737 "	10
16′	0.5200 "	3	36½′	0.743 "	5
13′	0.5240 "	4	35′	0.749 "	6
10½′	0.5271 "	16	33½′	0.755 "	9
7½′	0.5310 "	1	32′	0.761 "	12
			30½′	0.768 "	24
48° 50½′	0.5560 "	10	29′	0.775 "	12
48′	0.5600 "	19	27½′	0.781 "	6
46½′	0.5625 "	15	25½′	0.789 "	8
43′	0.5685 "	186	24′	0.796 "	8
39½′	0.5742 "	21	22½′	0.803 "	22
38′	0.5772 "	20	21′	0.811 "	166
36½′	0.5800 "	27	20¼′	0.814 "	603
31½′	0.5892 "	877	19½′	0.818 "	659
28½′	0.5950 "	96	18′	0.825 "	170
24½′	0.6040 "	28	16½′	0.833 "	27

TABLE V. — *Continued.*

SODIUM. $k = \frac{1}{135000}$° C.

ϕ	λ	I	ϕ	λ	1
47° 15′	0.841 μ	22	46° 21½′	1.317 μ	5
13½′	0.849 "	18	20′	1.337 "	4
12′	0.857 "	18	18½′	1.358 "	3
10½′	0.865 "	11	17′	1.380 "	5
9′	0.874 "	8	15½′	1.399 "	4
7½′	0.884 "	6	14′	1.421 "	3
6′	0.895 "	6	12½′	1.442 "	3
4½′	0.905 "	6	11′	1.461 "	4
3′	0.916 "	4	9½′	1.482 "	3
1½′	0.926 "	7	8′	1.503 "	2
0′	0.938 "	7	6½′	1.523 "	3
			5′	1.543 "	2
46° 58½′	0.949 "	5	3½′	1.563 "	0
57′	0.962 "	6	2′	1.583 "	6
55′	0.977 "	5	½′	1.603 "	2
53½′	0.988 "	8			
52′	1.003 "	8	45° 59′	1.624 "	3
50½′	1.014 "	4	57½′	1.645 "	2
49′	1.028 "	6	56′	1.667 "	0
47½′	1.041 "	5	54′	1.694 "	4
46′	1.055 "	7	52½′	1.713 "	6
44½′	1.067 "	12	51′	1.733 "	5
43′	1.082 "	12	49½′	1.755 "	17
41½′	1.096 "	11	48′	1.776 "	12
40′	1.111 "	45	46½′	1.795 "	17
38½′	1.127 "	415	45′	1.816 "	11
37′	1.143 "	112	43½′	1.836 "	33
35½′	1.158 "	24	42′	1.857 "	25
34′	1.176 "	11	40½′	1.878 "	11
32½′	1.191 "	7	39′	1.898 "	5
31′	1.208 "	8	37½′	1.918 "	4
29½′	1.223 "	10	36′	1.938 "	2
28′	1.242 "	30	34½′	1.958 "	0
26½′	1.257 "	16	33′	1.978 "	0
25′	1.276 "	8	31½′	2.000 "	0
23′	1.300 "	6			

Table VI.

POTASSIUM. $k = \frac{1}{127000}$° C.

ϕ	λ	I	ϕ	λ	I
52° 35½′	0.3830 μ	0	47° 54½′	0.680 μ	8
29½′	0.3850 "	2	51′	0.691 "	74
23½′	0.3870 "	0	49′	0.698 "	13
21′	0.3877 "	0	44½′	0.713 "	7
14′	0.3903 "	0	39′	0.733 "	12
			36′	0.745 "	27
51° 46′	0.4030 "	0	31′	0.767 "	1443
43′	0.4045 "	30	30′	0.770 "	622
40′	0.4060 "	0	28½′	0.777 "	82
12′	0.4214 "	2	27′	0.783 "	31
9′	0.4233 "	16	25½′	0.790 "	31
6′	0.4251 "	0	24′	0.796 "	27
			22½′	0.803 "	36
50° 17½′	0.4590 "	2	21′	0.811 "	34
			19½′	0.818 "	22
49° 46′	0.4870 "	2	18′	0.825 "	16
41½′	0.4915 "	0	16½′	0.832 "	16
39½′	0.4942 "	3	15′	0.841 "	17
32′	0.5023 "	0	13½′	0.850 "	14
25′	0.5095 "	4	12′	0.857 "	12
23½′	0.5113 "	5	10½′	0.866 "	12
20½′	0.5148 "	0	9′	0.876 "	13
8′	0.5307 "	0	7½′	0.886 "	12
5½′	0.5340 "	9	6′	0.896 "	9
4′	0.5362 "	6	4½′	0.906 "	8
2′	0.5388 "	0	3′	0.917 "	10
			1′	0.931 "	7
48° 40′	0.5737 "	1			
36½′	0.5800 "	14	46° 59½′	0.943 "	20
34′	0.5848 "	7	58′	0.954 "	20
31½′	0.5892 "	55	56½′	0.966 "	9
28½′	0.5950 "	3	55′	0.977 "	8
21½′	0.6093 "	2	53½′	0.990 "	7
19′	0.616 "	6	52′	1.002 "	9
10′	0.638 "	8	50½′	1.015 "	8
8′	0.643 "	11	49′	1.029 "	14
5½′	0.649 "	8	47½′	1.042 "	16

TABLE VI. — *Continued.*

POTASSIUM. $k = \frac{1}{127000}$° C.

ϕ	λ	I	ϕ	λ	I
46° 46′	1.055 μ	21	46° 3½′	1.563 μ	6
44½′	1.068 "	17	2′	1.583 "	3
43′	1.082 "	102	½′	1.603 "	2
42¼′	1.090 "	57			
41½′	1.096 "	25	45° 59′	1.625 "	4
40′	1.110 "	13	57′	1.652 "	2
38½′	1.127 "	35	55½′	1.672 "	2
37′	1.144 "	207	54′	1.693 "	4
35½′	1.158 "	389	52½′	1.713 "	3
34′	1.175 "	34	51′	1.733 "	3
32½′	1.193 "	14	49½′	1.755 "	2
31′	1.209 "	126	48′	1.776 "	5
29½′	1.225 "	190	46½′	1.795 "	0
28′	1.243 "	43	45′	1.816 "	0
26′	1.264 "	10	43½′	1.836 "	2
24½′	1.283 "	11	42′	1.857 "	5
23′	1.301 "	10	40½′	1.876 "	3
21½′	1.317 "	8	39′	1.898 "	2
20′	1.336 "	10	37½′	1.918 "	4
18½′	1.359 "	10	36′	1.938 "	0
17′	1.379 "	5	34½′	1.958 "	0
15½′	1.400 "	5	33′	1.978 "	0
14′	1.420 "	8	31½′	2.000 "	0
12½′	1.440 "	19	30′	2.021 "	0
11′	1.461 "	60	28½′	2.042 "	0
9½′	1.482 "	56	27′	2.062 "	0
8′	1.503 "	49	25′	2.087 "	0
6½′	1.523 "	12	23½′	2.108 "	0
5′	1.543 "	5	22′	2.128 "	0

TABLE VII.

LITHIUM. $k = \frac{1}{134000}$° C.

ϕ	λ	I	ϕ	λ	I
52° 33½′	0.3838 μ	2	47° 58′	0.670 μ	1191
26′	0.3860 "	2	56′	0.676 "	57
22′	0.3875 "	2	54½′	0.680 "	29
16′	0.3900 "	0	53′	0.684 "	24
11½′	0.3913 "	8	51½′	0.689 "	18
4′	0.3947 "	0	50′	0.694 "	15
			48½′	0.699 "	11
51° 29½′	0.4116 "	1	47′	0.704 "	10
25½	0.4140 "	58	45½′	0.709 "	10
22½′	0.4155 "	5	44′	0.715 "	11
11′	0.4220 "	0	42½′	0.720 "	8
8½′	0.4238 "	11	41′	0.725 "	7
4½′	0.4260 "	0	39½′	0.732 "	6
½′	0.4288 "	10	38′	0.737 "	5
			36½′	0.743 "	10
50° 57½′	0.4305 "	2	35′	0.749 "	6
18′	0.4590 "	8	33½′	0.754 "	6
14½′	0.4615 "	331	32′	0.760 "	11
10½′	0.4650 "	7	30½′	0.768 "	12
			29′	0.775 "	8
49° 37½′	0.4958 "	2	27½′	0.781 "	9
34½′	0.4990 "	34	26′	0.787 "	17
31½′	0.5023 "	2	24½′	0.794 "	18
			22½′	0.803 "	151
48° 59½′	0.5420 "	8	21¾′	0.807 "	292
34½′	0.5840 "	8	21′	0.811 "	238
31½′	0.5892 "	51	19½′	0.819 "	22
28½′	0.5950 "	8	18′	0.825 "	15
26½′	0.5993 "	5	16½′	0.833 "	12
25′	0.6025 "	20	15′	0.841 "	11
21½′	0.6102 "	570	13½′	0.849 "	15
18′	0.6180 "	17	12′	0.857 "	12
12½′	0.6308 "	7	10½′	0.865 "	10
5½′	0.6490 "	18	9′	0.876 "	9
½′	0.663 "	46	7½′	0.885 "	8

TABLE VII. — *Continued.*

LITHIUM. $k = \frac{1}{134000}$° C.

ϕ	λ	I	ϕ	λ	I
47° 6′	0.895 μ	5	46° 15½′	1.397 μ	2
4½′	0.904 "	7	14′	1.420 "	3
3′	0.916 "	9	12½′	1.440 "	3
1½′	0.926 "	11	11′	1.462 "	2
0′	0.937 "	11	9½′	1.482 "	4
			8′	1.502 "	3
46° 58½′	0.950 "	9	6½′	1.522 "	2
57′	0.962 "	9	5′	1.543 "	3
55½′	0.973 "	4	3½′	1.563 "	2
54′	0.987 "	8	2′	1.583 "	1
52′	1.002 "	5	½′	1.603 "	4
50½′	1.014 "	5			
49′	1.028 "	6	45° 59′	1.625 "	2
47½′	1.041 "	6	57½′	1.645 "	2
46′	1.054 "	5	56′	1.665 "	0
44½′	1.067 "	7	54½′	1.685 "	0
43′	1.082 "	14	53′	1.706 "	9
41½′	1.096 "	8	51′	1.732 "	10
40′	1.111 "	9	49½′	1.755 "	12
38½′	1.127 "	9	48′	1.776 "	16
37′	1.144 "	6	46½′	1.796 "	13
35½′	1.157 "	7	45′	1.816 "	14
34′	1.176 "	7	43½′	1.835 "	11
32½′	1.192 "	9	42′	1.857 "	10
31′	1.208 "	7	40½′	1.876 "	8
29½′	1.225 "	8	39′	1.898 "	12
28′	1.242 "	5	37½′	1.918 "	8
26½′	1.257 "	7	36′	1.937 "	3
25′	1.276 "	8	34½′	1.958 "	0
23½′	1.294 "	4	33′	1.978 "	0
21½′	1.317 "	4	31½′	2.000 "	0
20′	1.336 "	3	30′	2.020 "	0
18½′	1.357 "	2	28½′	2.042 "	0
17′	1.378 "	3			

TABLE VIII.

RUBIDIUM. $k = \frac{1}{132000}$° C.

φ	λ	I	φ	λ	I
52° 30′	0.3832 μ	0	48° 14′	0.627 μ	23
24′	0.3868 "	0	11′	0.633 "	12
20½′	0.3880 "	0	6′	0.648 "	10
16½′	0.4190 "	0	1′	0.662 "	8
51° 14′	0.4200 "	6	47° 58½′	0.669 "	11
12′	0.4215 "	4	55½′	0.677 "	8
9½′	0.4230 "	8	49½′	0.696 "	5
6½′	0.4250 "	0	45′	0.711 "	8
			41′	0.726 "	21
50° 18′	0.4588 "	0	39½′	0.731 "	17
			38′	0.737 "	19
49° 19′	0.5163 "	3	35½′	0.747 "	15
15′	0.5215 "	5	33½′	0.755 "	31
12¾′	0.5242 "	2	31½′	0.763 "	128
10½′	0.5270 "	6	30½′	0.768 "	188
6′	0.5332 "	2	29′	0.775 "	414
3½′	0.5367 "	4	27½′	0.781 "	262
½′	0.5406 "	2	25¼′	0.791 "	443
			24½′	0.794 "	279
48° 58½′	0.5435 "	4	23′	0.801 "	56
56′	0.5473 "	2	21½′	0.808 "	38
48½′	0.5592 "	9	20′	0.815 "	32
45½′	0.5642 "	3	18½′	0.823 "	42
43½′	0.5676 "	1	17′	0.831 "	24
41½′	0.5710 "	7	15½′	0.839 "	25
38½′	0.5762 "	2	14′	0.846 "	50
35½′	0.5820 "	3	12½′	0.854 "	14
31½′	0.5892 "	26	11′	0.863 "	11
28¾′	0.5942 "	5	9½′	0.872 "	45
25′	0.6020 "	4	8′	0.882 "	59
23′	0.607 "	8	6½′	0.892 "	17
21′	0.611 "	6	5′	0.902 "	10
19′	0.616 "	12	3½′	0.912 "	11
17′	0.620 "	11	2′	0.923 "	9
15½′	0.624 "	10	½′	0.934 "	9

TABLE VIII. — *Continued.*

RUBIDIUM. $k = \frac{1}{132000}$° C.

ϕ	λ	I	ϕ	λ	I
46° 59′	0.946 μ	10	46° 8½′	1.498 μ	64
57½′	0.957 “	7	7′	1.517 “	71
55½′	0.972 “	7	5½′	1.535 “	16
54′	0.986 “	21	4′	1.556 “	6
52½′	0.998 “	160	2½′	1.576 “	5
51′	1.011 “	32	1′	1.597 “	5
49½′	1.023 “	9			
48′	1.038 “	7	45° 59½′	1.618 “	5
46½′	1.050 “	8	57½′	1.645 “	4
45′	1.064 “	11	56′	1.666 “	4
43½′	1.077 “	10	54½′	1.685 “	2
42′	1.092 “	13	53′	1.705 “	3
40½′	1.105 “	11	51½′	1.725 “	3
39′	1.124 “	7	50′	1.747 “	3
37½′	1.138 “	13	48½′	1.770 “	0
36′	1.154 “	26	47′	1.790 “	0
34½′	1.170 “	10	45½′	1.812 “	0
33′	1.187 “	5	44′	1.831 “	3
31½′	1.203 “	6	42½′	1.850 “	1
30′	1.218 “	13	41′	1.870 “	0
28′	1.242 “	10	39½′	1.890 “	0
26½′	1.258 “	9	38′	1.913 “	0
25′	1.277 “	35	36½′	1.932 “	0
23½′	1.293 “	100	35′	1.952 “	0
22′	1.312 “	197	33½′	1.972 “	0
20½′	1.330 “	151	32′	1.993 “	0
19′	1.352 “	83	30½′	2.014 “	0
17½′	1.372 “	16	29′	2.036 “	0
16′	1.393 “	15	27′	2.062 “	0
14½′	1.413 “	40	25½′	2.082 “	0
13′	1.434 “	82	24′	2.102 “	0
11½′	1.454 “	94	22½′	2.120 “	0
10′	1.476 “	102			

TABLE IX.

CÆSIUM. $k = \frac{1}{132000}$° C.

ϕ	λ	I	ϕ	λ	I
52° 41½′	0.3812 μ	0	48° 7½′	0.644 μ	10
32′	0.3842 "	0	5′	0.650 "	9
25′	0.3868 "	0	2½′	0.658 "	15
22½′	0.3875 "	0	½′	0.663 "	19
51° 17′	0.4186 "	0	47° 58′	0.670 "	46
14½′	0.4200 "	4	56′	0.676 "	11
12½′	0.4212 "	2	54′	0.681 "	17
10½′	0.4225 "	7	52½′	0.687 "	26
7½′	0.4243 "	0	50′	0.694 "	63
			47½′	0.702 "	20
50° 23½′	0.4545 "	0	45′	0.711 "	11
21′	0.4565 "	15	42′	0.721 "	47
18½′	0.4584 "	2	39′	0.733 "	13
16½′	0.4600 "	6	34½′	0.751 "	12
13½′	0.4625 "	0	32½′	0.759 "	43
			31′	0.766 "	65
49° 41′	0.4920 "	4	29′	0.775 "	175
38½′	0.4950 "	4	27½′	0.781 "	95
0′	0.5412 "	4	26′	0.788 "	104
			24½′	0.794 "	105
48° 52½′	0.5528 "	7	23′	0.801 "	77
50′	0.5570 "	4	21½′	0.808 "	45
46′	0.5635 "	7	20′	0.815 "	29
43′	0.5686 "	2	18½′	0.823 "	26
38′	0.5772 "	4	17′	0.831 "	54
35′	0.5828 "	28	15½′	0.838 "	297
33½′	0.5856 "	23	14′	0.847 "	226
31½′	0.5892 "	54	12½′	0.854 "	59
28½′	0.5950 "	9	11′	0.863 "	151
25¾′	0.6010 "	14	9½′	0.872 "	140
23′	0.607 "	9	8′	0.882 "	345
20′	0.614 "	8	6½′	0.892 "	113
17½′	0.619 "	23	5′	0.902 "	154
15½′	0.624 "	8	3½′	0.913 "	83
13½′	0.629 "	12	2′	0.923 "	16
10½′	0.636 "	7	½′	0.934 "	9

TABLE IX. — *Continued.*

CÆLIUM. $k = \frac{1}{132000}$° C.

ϕ	λ	I	ϕ	λ	I
46° 59′	0.945 μ	10	46° 8½′	1.495 μ	19
57½′	0.956 "	9	7′	1.516 "	20
55½′	0.973 "	7	5½′	1.535 "	5
54′	0.986 "	129	4′	1.555 "	6
52½′	0.998 "	181	2½′	1.575 "	7
51′	1.011 "	17	1′	1.596 "	4
49½′	1.023 "	5			
48′	1.038 "	6	45° 59½′	1.617 "	3
46½′	1.050 "	4	58′	1.638 "	4
45′	1.064 "	6	56½′	1.658 "	3
43½′	1.077 "	6	55′	1.679 "	3
42′	1.092 "	4	53½′	1.700 "	2
40½′	1.105 "	6	51½′	1.725 "	2
39′	1.124 "	8	50′	1.748 "	2
37½′	1.138 "	8	48½′	1.768 "	1
36′	1.154 "	8	47′	1.789 "	2
34½′	1.170 "	4	45½′	1.807 "	1
33′	1.187 "	4	44′	1.830 "	0
31½′	1.202 "	7	42½′	1.850 "	0
30′	1.219 "	6	41′	1.871 "	0
28½′	1.236 "	4	39½′	1.890 "	0
27′	1.253 "	12	38′	1.912 "	0
25½′	1.269 "	13	36½′	1.930 "	0
23½′	1.293 "	27	35′	1.952 "	0
22′	1.312 "	64	33½′	1.972 "	0
20½′	1.327 "	80	32′	1.992 "	0
19′	1.352 "	31	30½′	2.012 "	0
17½′	1.372 "	7	29′	2.036 "	0
16′	1.392 "	11	27½′	2.055 "	0
14½′	1.412 "	35	26′	2.075 "	0
13′	1.433 "	38	24½′	2.095 "	0
11½′	1.453 "	51	22½′	2.120 "	0
10′	1.475 "	25	21′	2.142 "	0

VI. DISCUSSION OF THE OBSERVATIONS.

A hasty consideration of the curves of Plates II. and III. shows that the energy spectra of the alkalies have but little in common with that of the electric arc. While the latter produces, as has been shown above, the impression of a banded spectrum, the former is at once recognized as a spectrum composed of bright lines.

Kayser and Runge mention,[1] in their investigation, the fact that measurements upon the spectral lines of metals vaporized in the electric arc are often rendered difficult by the superposition of the metallic lines upon the banded carbon spectrum.

The present writer, however, has found, by following the method here described of filling the carbons with a core of the salt to be examined, that *the carbon spectrum is completely destroyed, and in its stead appears the pure metallic spectrum.* To attain this result, however, it is necessary to set the lamp in operation about five minutes before beginning the series of observations, since by the ignition of fresh carbons, the bands of the arc spectrum are often to be seen between the lines in the spectrum of the metal in question. After the lapse of this short interval of time, the bolometer, even when placed at the position of maximum heat in the arc spectrum (from $\lambda = .385\,\mu$ to $\lambda = .388\,\mu$), gives at most a deflection of but 2 or 3 mm., while large deflections at this same place are obtained when pure carbons are used.

It must be mentioned in this connection that to attain this result it is necessary that *both* carbons be filled with a core of salt, since when a solid cathode, for instance, is used, there always clings to the ends of the negative carbon a point of the characteristic violet light of the ordinary arc.

By increasing the dimensions of the cavity containing the salt in the carbons, the energy of the metallic lines may be many times increased. It was observed that a core 5 mm. in diameter increased the intensity of the lines nearly tenfold. On the other hand, the use of such large quantities of salt was attended with other disadvantages which made it desirable to return to the original dimensions of the holes bored; viz. 3 mm. and $1\frac{1}{2}$ mm. respectively.

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[1] *Kayser* und *Runge*, Abh. d. K. Acad. d. Wiss. zu Berlin, 2. Abs. p. 3, 1889. Wied. Ann., 38, p. 80, 1889.

It is to be noticed in all these drawings that the lines become broader as the more distant portions of the infra-red are reached. This effect does not, however, correspond to a physical cause, but is due in part to the method of representation here chosen, in part also to the chromatic aberration of the lenses of the spectrometer. As the bolometer filament wanders through the entire spectrum, there is concentrated upon its surface a far greater number of rays of different wave-lengths in the infra-red than in the region visible to the eye. If the thread of the bolometer were to be drawn at different places in Figs. 4–9, Plates II. and III., its breadth at $\lambda = 2\,\mu$ would appear nearly forty times greater than at $\lambda = .4\,\mu$. The same is true of the image of the slit itself. Metallic lines which in the energy curve of the dispersion spectrum appear equally broad, must, accordingly, appear much broader at $\lambda = 2\,\mu$ than at $\lambda = .4\,\mu$, when wave-lengths are chosen as abscissæ instead of the angular deviation of the bolometer arm.

To this effect is added, as mentioned above, that due to the lack of achromatism of the lenses used. This failure is far more noticeable in the infra-red than in the visible portions of the spectrum, and as a result the lines become more indistinct the farther they lie in the direction of the long wave-lengths. Abney[1] has also remarked that the focus of a lens for infra-red rays is quite different from its focus for radiations of greater refrangibility.

The cause of this failing achromatism in a system of lenses is to be found in the different characters which the curves of dispersion for flint and crown glass show in the infra-red. Rubens, in his investigation "Ueber Dispersion Ultrarother Strahlen," has shown[2] that the curves of dispersion for specimens of flint glass become nearly straight in the infra-red, but yet are always curved in the same direction, while in the curves of dispersion of crown glass a point of inflection occurs in the neighborhood of $\lambda = 1.4\,\mu$, which causes the direction of the curvature at this point to change.

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[1] *Abney*, Phil. Trans., Pt. II., p. 658, 1880.

[2] *Rubens*, Wied. Ann., 45, p. 255, 1892.

The extent to which this effect was noticeable in the measurements of the energy spectra may be illustrated by an example. When the telescope was focused on a green potassium line, the bolometer detected between the limits $\lambda = 1.440\,\mu$ and $\lambda = 1.540\,\mu$ a broad, poorly defined band, which passed into an easily recognized, although not perfectly sharp line, as soon as the focus was adjusted upon the line potassium α.

In conducting the definitive measurements, the observing telescope was so adjusted that the line potassium α appeared perfectly sharp. It is believed by thus focusing upon this most distant visible line toward the infra-red end of the spectrum, that a portion at least of the defective achromatism was avoided.[1]

Before a short description of the several metallic spectra is given in detail, a word may be said regarding the influence upon the intensities of the lines of the absorption within the measuring apparatus. In the less refrangible regions of the visible spectrum and in those portions of the infra-red here investigated, the absorption is inconsiderable, as is proven in the paper by Rubens so frequently cited. As is there shown, strong absorption is not perceptible in the specimens of crown glass examined until a wave-length $\lambda = 2.3\,\mu$ is reached, while flint glass exhibits a far greater transparency for infra-red radiations. The absorption produced in the violet end of the spectrum by the lenses of the spectrometer and by the prism is, without doubt, of much greater magnitude, though the amount could hardly be estimated. In a recent paper, Professor Nichols and the present writer have shown[2] that even a single crown glass lens can produce heavy absorption in this portion of the spectrum. It may then be supposed that the violet bands in the spectrum of the electric arc are far more intense than they appear from the curves shown in Figs. 3 and 4, Plate I.

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[1] It need hardly be mentioned here that these errors would not affect the position or wave-lengths of the lines, but would simply tend to diminish the intensity.

[2] *Nichols* and *Snow*, Phil. Mag. (5), 33, p. 380, 1892.

VII. ON THE METALLIC LINES.

The visible region of the sodium spectrum contains a series of lines which may be readily observed with the eye, but which are so lacking in energy that they exert no appreciable effect upon the bolometer. On the other hand, the two violet lines at $\lambda = .3932\,\mu$ and $\lambda = .3967\,\mu$, coinciding, as they do, very closely with the Fraunhofer lines H and K of the solar spectrum, appear sharply defined in the drawing.[1] The green line at $\lambda = .5685\,\mu$, the yellow at $\lambda = .5892\,\mu$, D, and the red at $\lambda = .616\,\mu$ are the strongest in the visible region. In the infra-red are found, besides a series of weaker lines at $\lambda = .770\,\mu$, $\lambda = .855\,\mu$, $\lambda = .930\,\mu$, $\lambda = 995\,\mu$, $\lambda = 1.075\,\mu$, $\lambda = 1.245\,\mu$, two lines of unusual intensity at $\lambda = .818\,\mu$ and $\lambda = 1.132\,\mu$. The energy of the first of these two is approximately equal to that of the D-line, and, indeed, exceeds the latter in several series of observations. The bolometer shows, further, in the region between $\lambda = 1.70\,\mu$ and $\lambda = 1.90\,\mu$, the presence of a small amount of energy, which, if the proper focus could be adjusted, might be resolved into a line or a group of lines. It has, however, not been possible thus to analyse this diffuse radiation, which also appears in a similar position in the lithium spectrum.

Much simpler than the spectrum of sodium is that of lithium, which, besides this feeble amount of energy between $\lambda = 1.70\,\mu$ and $\lambda = 1.90\,\mu$, shows only a single line at $\lambda = .811\,\mu$ in the infra-red. There is here noticeable, in addition to the eight visible lines measurable with the bolometer, also the yellow sodium line. This metal occurred in small quantities as an impurity in all the salts investigated, and in all cases made its presence known by a more or less prominent D-line.

In the potassium spectrum, the largest portion of the energy is concentrated in the one line potassium α. While this line exerts only a feeble effect upon the eye, it is, as regards its radiant energy, by far the strongest line found in the spectra of the five alkalies investigated. This is then an illustration of the fact that the human eye is far from sensitive to radiations of this wave-length.

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[1] Professor Kayser has kindly pointed out that these two lines are undoubtedly due to the presence of a trace of calcium in the salt used.

A noticeable difference between the spectrum of this metal and that of sodium and lithium is the relatively luminous background upon which the lines are superposed. Infra-red lines were found at $\lambda = .840\,\mu$, $\lambda = .885\,\mu$, $\lambda = .950\,\mu$, $\lambda = 1.086\,\mu$, $\lambda = 1.155\,\mu$, and $\lambda = 1.220\,\mu$, besides the diffuse line mentioned above at $\lambda = 1.470\,\mu$. The other lines occurring at $\lambda = .589\,\mu$ and $\lambda = .815\,\mu$ in Fig. 6, Plate II., belong to sodium.

The spectra of rubidium and cæsium are richer in infra-red lines than those thus far considered. In the visible region the background upon which the lines of these metals seem to be projected appears more luminous than that of potassium — a fact which is undoubtedly closely connected with the whitish appearance which the arc assumes as soon as these metals are introduced. The two blue lines at $\lambda = .4565\,\mu$ and $\lambda = .4600\,\mu$, to which the metal cæsium owes its name, are easily discernible in Fig. 9, Plate III., although of feeble intensity. On the other hand, the two red rubidium lines, the deep red color of which has given the name rubidium to this metal, have, as regards energy, a maximum intensity.

Inasmuch as in the cæsium spectrum indications are found of the presence of lines which obviously belong to the metal rubidium, for instance the group at $\lambda = .4200\,\mu$ and $\lambda = .4230\,\mu$, it is probable that in the cæsium chloride used a quantity of the corresponding rubidium salt was present as an impurity. The infra-red cæsium spectrum, as given here, may therefore be considerably affected by lines due to rubidium.

In their investigations,[1] Kayser and Runge have undertaken an approximate estimation of the intensity of many of the lines, and have introduced for this purpose a scale of luminosity consisting of six numbers. Since it may be assumed in the present case that with the possible exception of the violet end of the spectrum, the selective absorption of the apparatus plays no important part, it has seemed allowable in this investigation to regard the intensities of the lines as proportional to the galvanometer deflections. It is to be observed, however, that the more distant infra-red lines appear by this method too weak in intensity, due to the defective achromatism mentioned above.

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[1] *Kayser* und *Runge*, Wied. Ann., 41, p. 306, 1890.

For all this, the writer does not consider himself justified in introducing a correction in these figures which would thus apply to the intensity of but a few lines at most, and even here the correction is probably not large. In the following tables, in addition to the wave-lengths of the several lines, are added their respective intensities, as read directly from the ordinates of the curves, Figs. 5–9, Plates II. and III. While, indeed, the unit of the energy is an arbitrary one, yet the lines of all the metals examined were measured in terms of the same standard, since the current in the lamp and the sensitiveness of the measuring apparatus were subjected to but feeble fluctuations.

TABLES OF WAVE-LENGTHS AND INTENSITIES.

Table X.

LITHIUM.

λ	I	λ	I	λ	I
0.3913 μ	10	0.4615 μ	331	0.670 μ	1191
0.4140 "	58	0.4990 "	34	0.811 "	296
0.4238 "	11	0.6102 "	570	1.800(?)"	(?)
0.4288 "	10				

Table XI.

POTASSIUM.

λ	I	λ	I	λ	I
0.4045 μ	30	0.643 μ	11	1.086 μ	108
0.4233 "	16	0.691 "	74	1.155 "	395
0.5113 "	5	0.768 "	1443	1.220 "	205
0.5340 "	9	0.840 "	18	1.470 "	70
0.5362 "	6	0.885 "	13	1.500(?)"	50
0.5800 "	14	0.950 "	23		

TABLE XII.

SODIUM.

λ	I	λ	I	λ	I
0.3932 μ	31	0.5892 μ	877	0.770 μ	22
0.3967 "	31	0.616 "	91	0.818 "	660
0.4236 "	42	0.644 "	22	0.855 "	18
0.4677 "	11	0.671 "	26	0.930 "	8
0.4996 "	62	0.699 "	7	0.995 "	10
0.5164 "	12	0.710 "	14	1.075 "	13
0.5271 "	16	0.714 "	13	1.132 "	419
0.5600 "	19	0.720 "	12	1.245 "	30
0.5685 "	186	0.736 "	10	1.800(?)"	(?)

TABLE XIII.

RUBIDIUM.

λ	I	λ	I	λ	I
0.4200 μ	6	0.627 μ	23	0.945 μ	10
0.4230 "	8	0.669 "	11	0.997 "	151
0.5215 "	5	0.726 "	21	1.063 "	11
0.5270 "	6	0.737 "	19	1.090 "	13
0.5367 "	4	0.775 "	414	1.153 "	26
0.5435 "	4	0.791 "	443	1.224 "	13
0.5592 "	9	0.821 "	42	1.318 "	198
0.5710 "	7	0.845 "	50	1.475 "	102
0.607 "	8	0.878 "	60	1.520 "	71
0.616 "	12	0.913 "	11		

TABLE XIV.

CÆSIUM.

λ	I	λ	I	λ	I
0.4200(?)μ	4	0.646 μ	10	0.900 μ	155
0.4230(?)"	7	0.674 "	46	0.995 "	182
0.4565 "	15	0.694 "	63	1.150 "	9
0.4600 "	6	0.721 "	47	1.205 "	7
0.5528 "	7	0.775 "	175	1.323 "	81
0.5635 "	7	0.790 "	107	1.420(?)"	38
0.5828 "	28	0.833 "	297	1.450 "	52
0.6010 "	14	0.865 "	151	1.520 "	20
0.619 "	23	0.882 "	345	1.575 "	8
0.629 "	12				

The previously mentioned investigations of Becquerel on the wave-lengths of the infra-red sodium and potassium lines gave values which are compared in the following table with the results of the present observations.

TABLE XV.

BECQUEREL.		SNOW.
With the Prism.	**With the Grating.**	**With the Prism.**
Na λ = 0.819 μ	λ = 0.819 μ	λ = 0.818 μ
λ = 1.098 "	λ = 1.142 "	λ = 1.132 "
K λ = 0.770 "	λ = 0.770 "	λ = 0.768 "
λ = 1.003 "		
λ = 1.073 "	λ = 1.098 "	λ = 1.086 "
λ = 1.125 "	λ = 1.162 "	λ = 1.155 "
λ = 1.182 "	λ = 1.233 "	λ = 1.220 "

Becquerel's measurements with the aid of the prism are observed to be in general smaller than the results here presented, while the latter coincide more nearly with his determinations with the grating. More confidence was placed by Becquerel himself in the grating measurements, so that the better agreement between his values and those here given may be regarded as a partial confirmation of the determinations as given in this paper.

VIII. CONCLUSION.

This investigation was originally begun to test experimentally in the infra-red the empirical formulæ established by Kayser and Runge for the wave-length of lines in the visible and ultra-violet spectrum.

At the writer's request, Professor Runge very kindly furnished a table of the wave-lengths as calculated by himself and Professor Kayser. These values as given in the following table may be in error by an amount from 10 to 20 $\mu\mu$ on account of the extensive extrapolation demanded.

TABLE XVI.

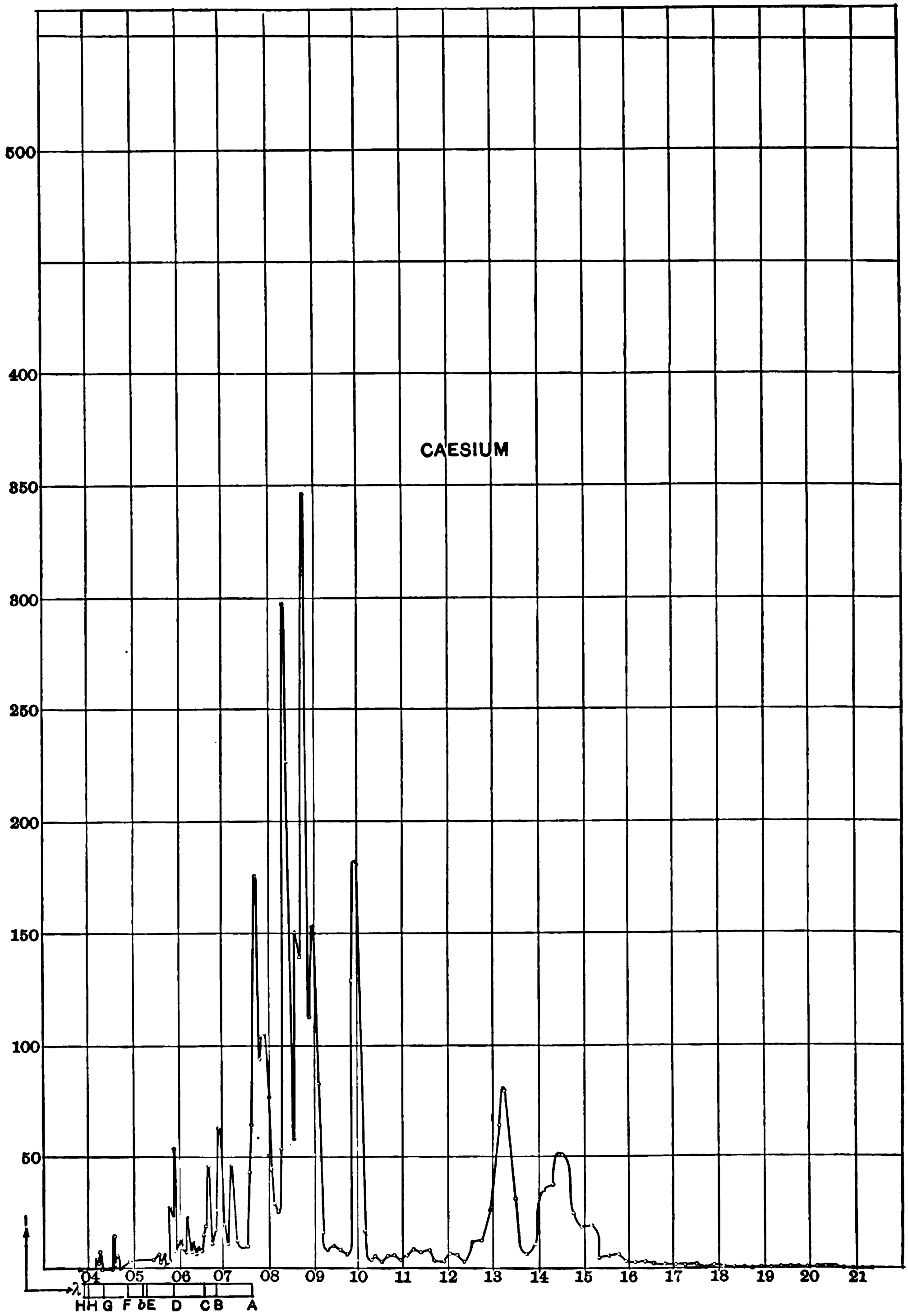
	λ			
Li	0.819 μ			
Na	1.150	1.148 μ		
K	1.266	1.257	1.253	1.244 μ
Rb	1.718	1.653 μ		
Cs	0.922	0.877	0.867	0.828 μ

In a number of cases, for instance with lithium and sodium, the coincidence with the values here obtained is as close as could have been expected. With the other metals, however, noticeably greater discrepancies occur, so that these observations in their present form cannot be regarded as a full confirmation of the formulæ of Kayser and Runge.

I desire in conclusion to express to Professor August Kundt my warmest thanks, not only for the cordial support he so freely gave, but also for his many suggestions and for the kindly interest which he took in the entire investigation.

PHYSICAL LABORATORY, UNIVERSITY OF BERLIN,
June, 1892.

Fig. 8.


CAESIUM
500
400
350
300
250
200
150
100
50
I
λ
04
05
06
07
08
09
10
11
12
13
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16
17
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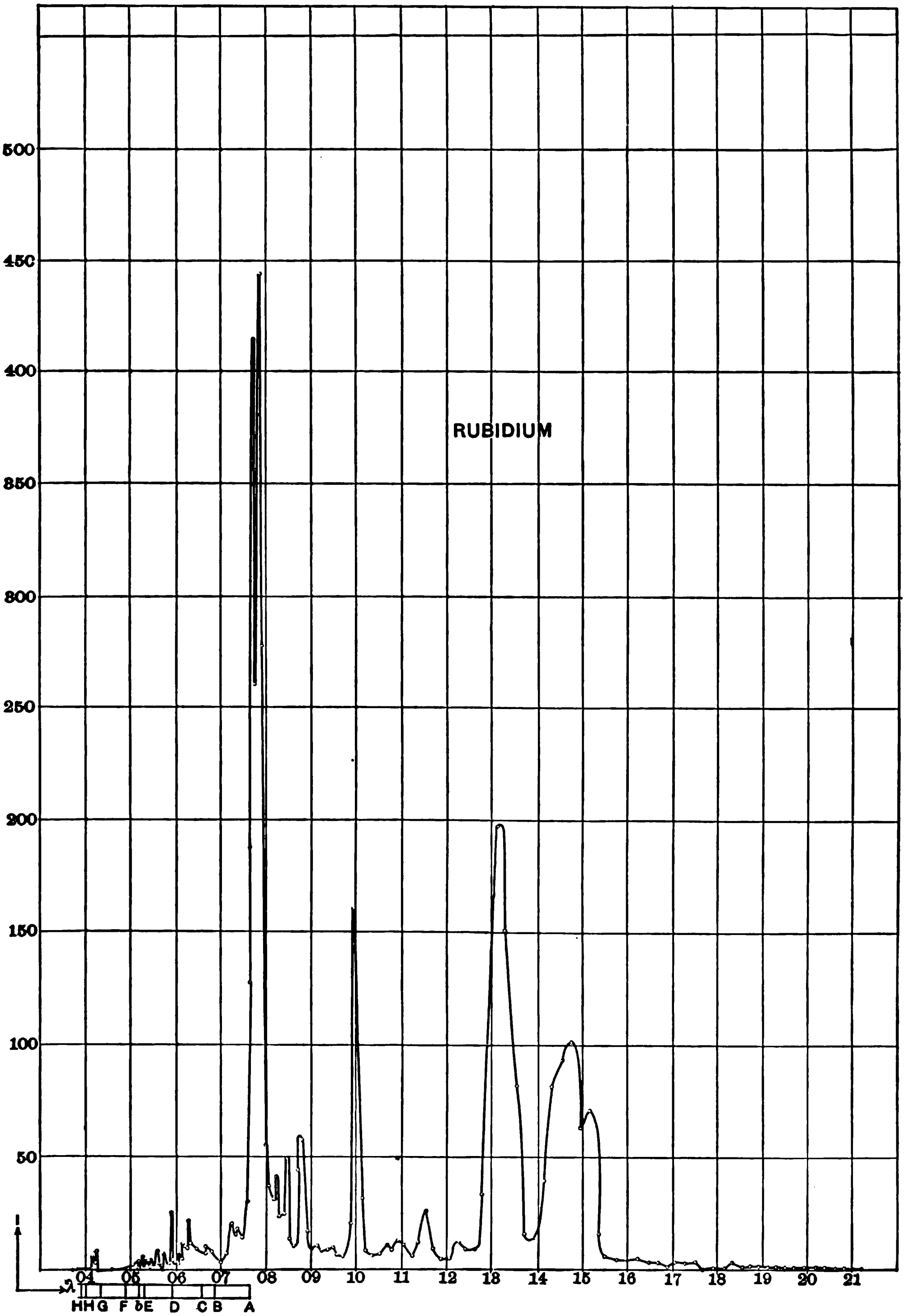
20
21
H H G F b E D C B A

B. W.

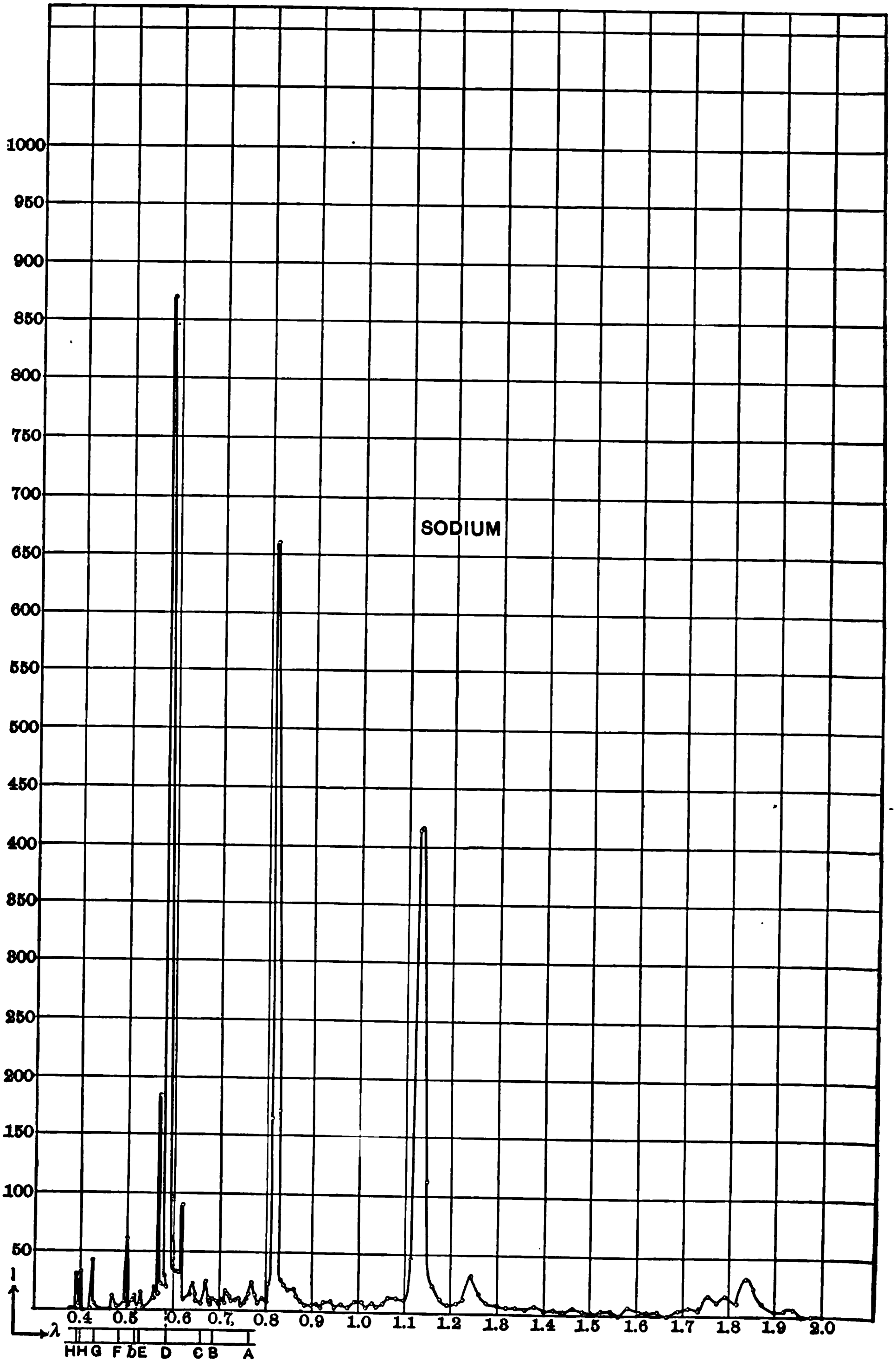
Fig. 9.

PLATE III.



NOW.

Fig. 5.


SODIUM
1000
950
900
850
800
750
700
650
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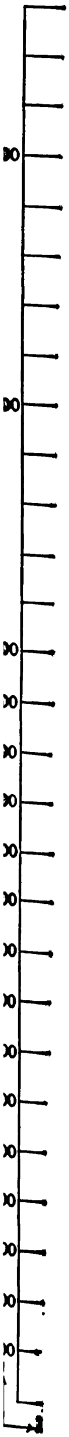
550
500
450
400
350
300
250
200
150
100
50
I
λ
0.4
0.5
0.6
0.7
0.8
0.9
1.0
1.1
1.2
1.3
1.4
1.5
1.6
1.7
1.8
1.9
2.0
H H G F b E D C B A



GENERAL DISCUSSION OF THE CURRENT FLOW IN TWO MUTUALLY RELATED CIRCUITS CONTAINING CAPACITY.[1]

By Frederick Bedell and Albert C. Crehore.

IN a paper presented at the General Meeting of the American Institute of Electrical Engineers[2] at Chicago, June 7, 1892, the problem of finding the current which flows in a single circuit having resistance, self-induction, and capacity in series was considered. A general solution was obtained for the current which flows when the impressed electromotive force is any function of the time whatever. This solution was applied to four cases, which arise according to the nature of the particular electromotive force considered, and curves were drawn to illustrate the current in each case in the particular yet representative examples assumed.

It is our present object to study the more general case of two independent circuits, each of which contains resistance, self-induction, and capacity, which are connected only by means of their common magnetic field.

Throughout the discussion the following symbols will be used:

R = resistance.
L = coefficient of self-induction (a constant).
M = coefficient of mutual induction (a constant).
C = capacity.
E = (*a*) constant electromotive force; or
(*b*) maximum value of harmonic electromotive force.

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[1] A paper presented before the World's Congress of Electricians, Chicago, Ill., August, 1893.

[2] Transactions A. I. E. E., Vol. IX., p. 303.

e = instantaneous value of electromotive force.

I = (a) constant current; or

(b) maximum value of harmonic current.

i = instantaneous value of current.

Q = constant charge of condenser.

q = instantaneous value of charge of condenser.

$\omega = 2\pi n = 2\pi \times$ frequency = angular velocity.

$+\theta$ = angle of advance.

$-\theta$ = angle of lag.

T = (a) period; or (b) time-constant.

$j = \sqrt{-1}$.

ϵ = base of Naperian logarithms = 2.71828.

c = arbitrary constant of integration.

The meaning of other letters when used will be evident.

It has been shown that the differential equation of energy for a single circuit is

$$(1) \qquad e i dt = \frac{i dt \int i dt}{C} + R i^2 dt + L i \frac{di}{dt} dt,$$

in which the first member represents the total energy imparted to the circuit by the impressed electromotive force. The second member consists of three terms which represent respectively the three ways in which the energy is used in the circuit. The first term is the energy required to charge the condenser; the second is that expended in heat; the third is that required to produce the magnetic field.

If a second circuit is now placed in mutual relation to the first, so that they possess a common magnetic field, a fourth term must be added to equation (1) depending upon the coefficient of mutual induction of the two circuits. The equation of energy for the primary coil may be written,

$$(2) \qquad e i_1 dt = \frac{i_1 dt \int i_1 dt}{C_1} + R_1 i_1^2 dt + L_1 i_1 \frac{di_1}{dt} dt + M i_1 \frac{di_2}{dt} dt,$$

where the subscripts one and two refer to the first and second coil or to the "primary" and "secondary" respectively. The equation of energy for the second coil which has no impressed electromotive force is

$$(3)\qquad 0=\frac{i_2dt\int i_2dt}{C_2}+R_2i_2^2dt+L_2i_2\frac{di_2}{dt}dt+Mi_2\frac{di_1}{dt}dt.$$

If we divide equation (2) by i_1dt and equation (3) by i_2dt, we obtain equations of electromotive forces

$$(4)\qquad e=f(t)=\frac{\int i_1dt}{C_1}+R_1i_1+L_1\frac{di_1}{dt}+M\frac{di_2}{dt};$$

$$(5)\qquad 0=\frac{\int i_2dt}{C_2}+R_2i_2+L_2\frac{di_2}{dt}+M\frac{di_1}{dt}.$$

In these equations the electromotive force is expressed as a function of the time. The solution of these differential equations will be the complete solution of the problem of circuits with constant coefficients of mutual and self induction, having resistance and capacity, and it will give us the current that flows in either the primary or secondary at any time due to any impressed electromotive force. In order to solve the equations we may eliminate from the pair either i_1 or i_2 and obtain a single equation containing but one independent variable. To perform this elimination we may differentiate (4) and (5) enough times to obtain one more equation than we have variables to eliminate. We may then form a determinant of the coefficients of the variables to be eliminated and proceed in the usual way in elimination by determinants.

If we use the symbols $D=\frac{d}{dt}$, and $D^2=\frac{d^2}{dt^2}$, etc., equations (4) and (5) may be written

$$(6)\qquad f(t)=\frac{\int i_1dt}{C_1}+R_1i_1+L_1Di_1+MDi_2;$$

$$(7)\qquad 0=\frac{\int i_2dt}{C_2}+R_2i_2+L_2Di_2+MDi_1.$$

By differentiation we obtain the six equations

$$(8)\qquad f'(t)=\frac{1}{C_1}i_1 + R_1Di_1 + L_1D^2i_1 + MD^2i_2.$$

$$(9)\qquad f''(t)=\frac{1}{C_1}Di_1 + R_1D^2i_1 + L_1D^3i_1 + MD^3i_2.$$

$$(10)\qquad f'''(t)=\frac{1}{C_1}D^2i_1 + R_1D^3i_1 + L_1D^4i_1 + MD^4i_2.$$

$$(11)\qquad 0=\frac{1}{C_2}i_2 + R_2Di_2 + L_2D^2i_2 + MD^2i_1.$$

$$(12)\qquad 0=\frac{1}{C_2}Di_2 + R_2D^2i_2 + L_2D^3i_2 + MD^3i_1.$$

$$(13)\qquad 0=\frac{1}{C_2}D^2i_2 + R_2D^3i_2 + L_2D^4i_2 + MD^4i_1.$$

In these six equations there are five quantities which it is required to eliminate, viz., i_1, Di_1, D^2i_1, D^3i_1, and D^4i_1, in order to obtain an equation containing only i_2 and the derivatives of i_2. To obtain a single equation containing i_1 alone, it is necessary to eliminate i_2, Di_2, D^2i_2, D^3i_2, and D^4i_2.

To obtain the equation for primary current, we form a determinant consisting of six columns, five of which contain the coefficients of i_2, Di_2, D^2i_2, etc., in the equations, while the sixth column contains all quantities in the equations independent of i_2, Di_2, or D^2i_2, etc. Thus, to eliminate i_2, we may write the determinant,

$$(14)\qquad \begin{array}{|cccccc|}
i_2 & Di_2 & D^2i_2 & D^3i_2 & D^4i_2 & \text{Remaining Terms.} \\
\hline
0 & 0 & M & 0 & 0 & \frac{1}{C_1}i_1 + R_1Di_1 + L_1D^2i_1 - f'(t) \\
0 & 0 & 0 & M & 0 & \frac{1}{C_1}Di_1 + R_1D^2i_1 + L_1D^3i_1 - f''(t) \\
0 & 0 & 0 & 0 & M & \frac{1}{C_1}D^2i_1 + R_1D^3i_1 + L_1D^4i_1 - f'''(t) \\
\frac{1}{C_2} & R_2 & L_2 & 0 & 0 & MD^2i_1 \\
0 & \frac{1}{C_2} & R_2 & L_2 & 0 & MD^3i_1 \\
0 & 0 & \frac{1}{C_2} & R_2 & L_2 & MD^4i_1 \\
\hline
\end{array} = 0.$$

Since the first column is composed entirely of zeros except the fourth row, the determinant reduces to $\frac{1}{C_2}$ times its minor determinant. One column of this minor determinant is all zeros except one row, so that the determinant reduces to one of the fourth order. By erasing the first and second columns and the fourth and fifth rows of the determinant of the sixth order, it becomes

$$(15) \qquad \begin{vmatrix} M & 0 & 0 & \frac{1}{C_1}i_1 + R_1Di_1 + L_1D^2i_1 - f'(t) \\ 0 & M & 0 & \frac{1}{C_1}Di_1 + R_1D^2i_1 + L_1D^3i_1 - f''(t) \\ 0 & 0 & M & \frac{1}{C_1}D^2i_1 + R_1D^3i_1 + L_1D^4i_1 - f'''(t) \\ \frac{1}{C_2} & R_2 & L_2 & MD^4i_1 \end{vmatrix} = 0.$$

Multiplied out and expanded, this determinant gives

$$(16) \quad \Big[(L_1L_2 - M^2)D^4 + (R_1L_2 + R_2L_1)D + \left(\frac{L_1}{C_2} + \frac{L_2}{C_1} + R_1R_2\right)D^2 + \left(\frac{R_1}{C_2} + \frac{R_2}{C_1}\right)D + \frac{1}{C_1C_2}\Big]i_1 = \frac{1}{C_2}f'(t) + R_2f''(t) + L_2f'''(t).$$

Here we have a differential equation of the fourth order containing but two variables i_1 and t, the solution of which gives the desired primary current in terms of the impressed electromotive force.

The differential equation for the secondary current may be obtained in a similar manner by eliminating i_1 and its derivatives from equations (8) to (13) inclusive. In order to perform the elimination, we may write as before, the determinant

$$(17) \qquad \begin{array}{cccccl} i_1 & Di_1 & D^2i_1 & D^3i_1 & D^4i_1 & \text{Remaining Terms.} \\ \hline \frac{1}{C_1} & R_1 & L_1 & 0 & 0 & MD^2i_2 - f'(t) \\ 0 & \frac{1}{C_1} & R_1 & L_1 & 0 & MD^3i_2 - f''(t) \\ 0 & 0 & \frac{1}{C_1} & R_1 & L_1 & MD^4i_2 - f'''(t) \\ 0 & 0 & M & 0 & 0 & \frac{1}{C_2}i_2 + R_2Di_2 + L_2D^2i_2 \\ 0 & 0 & 0 & M & 0 & \frac{1}{C_2}Di_2 + R_2D^2i_2 + L_2D^3i_2 \\ 0 & 0 & 0 & 0 & M & \frac{1}{C_2}D^2i_2 + R_2D^3i_2 + L_2D^4i_2 \end{array} = 0.$$

Since the first column of this determinant is composed entirely of zeros except the first row, it reduces to the minor of the fifth order found by erasing the first row and first column. Since the first column of this minor determinant is composed entirely of zeros except its first row, the whole reduces to the determinant of the fourth order found by striking out both the first and second rows and columns. Thus,

$$(18)\qquad \begin{vmatrix} \frac{1}{C_1} & R_1 & L_1 & MD^4 i_2 - f'''(t) \\ M & 0 & 0 & \frac{1}{C_2} i_2 + R_2 D i_2 + L_2 D^2 i_2 \\ 0 & M & 0 & \frac{1}{C_2} D i_2 + R_2 D^2 i_2 + L_2 D^3 i_2 \\ 0 & 0 & M & \frac{1}{C_2} D^2 i_2 + R_2 D^3 i_2 + L_2 D^4 i_2 \end{vmatrix} = 0.$$

Multiplying out and expanding, we obtain the desired differential equation for the secondary current.

$$(19)\quad \Big[(L_1 L_2 - M^2) D^4 + (R_1 L_2 + R_2 L_1) D^3 + \left(\frac{L_1}{C_2} + \frac{L_2}{C_1} + R_1 R_2\right) D^2 + \left(\frac{R_1}{C_2} + \frac{R_2}{C_1}\right) D + \frac{1}{C_1 C_2}\Big] i_2 = -Mf'''(t).$$

It is noticeable that these differential equations (16) and (19) for the primary and secondary currents are very similar. The first members are alike if we write i_2 for i_1. The second members show a marked difference. The equation for primary current contains R_2, L_2, and C_2, with the three derivatives of $f(t)$, while the equation for secondary current contains only M, with the third derivative of $f(t)$.

The general solutions of these equations of the fourth order, (16) and (19), would give the value of the primary and secondary currents at any time, where the impressed electromotive force is any function whatsoever of the time, and they would involve the literal solution of the general bi-quadratic equation, that we might find four factors of the bi-quadratic expression in order to resolve the inverse operator into four partial fractions. In their full generality, the equations are too cumbersome for practical purposes, but readily admit of solution, as they now stand, if we assume the impressed electromotive force to be harmonic, as will be discussed later in this paper. However, the solutions may be obtained for any impressed electromotive force whatsoever, by the introduction of certain modifications which reduce the equations to an order lower than the fourth. If the equation can be reduced to the second order, it is comparatively easy to find its general solution ; but we will not attempt to write the solution of equations of the third and fourth orders, except in the case of an harmonic impressed electromotive force, which is to be discussed later.

If we omit one of the condensers from the circuit, it will reduce the equations (16) and (19) to the third order, and if we omit both condensers, the equations reduce to the second order. The resulting equations in these particular cases may be written down from (16) and (19) by omitting terms and reducing the accents and the order by the required amount. To see how the order is reduced, we will form the equation in the case where both condensers are omitted. To do this let us return to (6) and (7). Omitting the condenser terms, these equations of electromotive forces become

$$(20) \qquad f(t) = R_1 i_1 + L_1 D i_1 + M D i_2.$$

$$(21) \qquad 0 = R_2 i_2 + L_2 D i_2 + M D i_1.$$

We need differentiate each equation only once to obtain enough equations to eliminate either of the variables i_1 or i_2.

By differentiation we have

$$(22) \qquad f'(t) = R_1 D i_1 + L_1 D^2 i_1 + M D^2 i_2.$$

$$(23) \qquad 0 = R_2 D i_2 + L_2 D^2 i_2 + M D^2 i_1.$$

Forming a determinant of these four equations to eliminate i_2, we have

$$(24) \qquad \begin{array}{cccc} i_2 & D i_2 & D^2 i_2 & \text{Remaining Terms.} \\ \left|\begin{array}{c} 0 \\ 0 \\ R_2 \\ 0 \end{array}\right. & \begin{array}{c} M \\ 0 \\ L_2 \\ R_2 \end{array} & \begin{array}{c} 0 \\ M \\ 0 \\ L_2 \end{array} & \left.\begin{array}{l} R_1 i_1 + L_1 D i_1 - f(t) \\ R_1 D i_1 + L_1 D^2 i_1 - f'(t) \\ M D i_1 \\ M D^2 i_1 \end{array}\right| \end{array} = 0.$$

Since the first column is all zeros except the third row, this determinant reduces to

$$(25)\qquad \begin{vmatrix} M & 0 & R_1 i_1 + L_1 D i_1 - f(t) \\ 0 & M & R_1 D i_1 + L_1 D^2 i_1 - f'(t) \\ R_2 & L_2 & M D^2 i_1 \end{vmatrix} = 0;$$

and this expanded gives

$$(26)\quad [(L_1 L_2 - M^2) D^2 + (R_1 L_2 + R_2 L_1) D + R_1 R_2] i_1 = R_2 f(t) + L_2 f'(t).$$

Forming the determinant to eliminate i_2, we have

$$(27)\qquad \begin{array}{cccc} i_1 & Di_1 & D^2 i_1 & \text{REMAINING TERMS.} \\ \hline R_1 & L_1 & 0 & M D i_2 - f(t) \\ 0 & R_1 & L_1 & M D^2 i_2 - f'(t) \\ 0 & M & 0 & R_2 i_2 + L_2 D i_2 \\ 0 & 0 & M & R_2 D i_2 + L_2 D^2 i_2 \end{array} = 0.$$

This reduces to

$$(28)\qquad \begin{vmatrix} R_1 & L_1 & M D^2 i_2 - f'(t) \\ M & 0 & R_2 i_2 + L_2 D i_2 \\ 0 & M & R_2 D i_2 + L_2 D^2 i_2 \end{vmatrix} = 0;$$

and, when expanded, gives

$$(29)\quad [(L_1 L_2 - M^2) D^2 + (R_1 L_2 + R_2 L_1) D + R_1 R_2]\, i_2 = - M f'(t).$$

Upon comparing (26) and (29) with (16) and (19) we see that we might have written both (26) and (29) immediately from the more general forms, by reducing each exponent and accent by two and omitting all terms containing C_1 or C_2.

In a similar manner we may find the equations of the third order from the general by omitting one condenser only. If we omit the primary condenser only, we may write from the general the equation in this particular case by reducing all exponents and accents by unity and omitting terms containing C_1. Thus, omitting C_1 from the primary, we have for the primary current

$$(30)\quad \left[(L_1L_2-M^2)D^3+(R_1L_2+R_2L_1)D^2+\left(\frac{L_1}{C_2}+R_1R_2\right)D+\frac{R_1}{C_2}\right]i_1$$
$$=\frac{1}{C_2}f(t)+R_2f'(t)+L_2f''(t).$$

The secondary current equation becomes

$$(31)\quad [(L_1L_2-M^2)D^3+(R_1L_2+R_2L_1)D^2+\left(\frac{L_1}{C_2}+R_1R_2\right)D+\frac{R_1}{C_2}\Big]i_2$$
$$=-Mf''(t).$$

If the secondary condenser is omitted, the equations (16) and (19) become

$$(32)\quad \left[(L_1L_2-M^2)D^3+(R_1L_2+R_2L_1)D^2+\left(\frac{L_2}{C_1}+R_1R_2\right)D+\frac{R_2}{C_1}\right]i_1$$
$$=R_2f'(t)+L_2f''(t);$$

and

$$(33)\quad \left[(L_1L_2-M^2)D^3+(R_1L_2+R_2L_1)D^2+\left(\frac{L_2}{C_1}+R_1R_2\right)D+\frac{R_2}{C_1}\right]i_2$$
$$=-Mf''(t).$$

Another consideration which will enable us to reduce the order of our equations is that of no magnetic leakage. If we consider that all the lines of induction generated by the primary circuit thread the secondary, it is well known that the product of the coefficients of self-induction equals the square of the coefficient of mutual induction of the two circuits; that is, $L_1L_2=M^2$, or $L_1L_2-M^2=0$. This approximation is so nearly realized in most of the closed magnetic circuit transformers that it is worth while to consider how it simplifies the equations. The coefficient of the terms of the highest order in any of the differential equations yet written is this quantity $L_1L_2-M^2$. If we consider therefore that there is no magnetic leakage in the transformer, we may reduce the general equations (16) and (19) to the third order. It will not be attempted here to write down the general solution of the equation of the third order. However, if we wish to consider the case where only one condenser is in circuit and there is no magnetic leakage, equations (30) to (33) inclusive reduce to the second order and can be readily solved. When there is no condenser in either circuit, equations (26) and (29) reduce to very simple forms, being of the first order when the consideration of no magnetic leakage is introduced. Upon the solution of these equations of the first order, in the case of an harmonic electromotive force, may be established the well-known transformer diagram usually built up geometrically by a synthetic process.

It is now proposed to obtain the general solutions of equations (26) and (29) which give the complete solution of the problem of two mutually related circuits without considering condensers, but with no limiting assumptions in regard to the magnetic leakage. Then will be given the solution of the same case simplified by the assumption that there is no magnetic leakage. Lastly, the cases will be considered where there is no magnetic leakage, and one of the circuits contains a condenser.

[*To be concluded.*]

THE STUDY OF THE DISTRIBUTION OF STRAINS BY POLARIZED LIGHT.

By A. Marston.

A FEW months since the writer became interested in the determination of the safe loads for a certain class of members of bridges. There was no good theory as to the distribution of the strains in those members, and it was suggested to him to attempt its study by means of polarized light. The method to be described in this paper was worked out in the course of the investigation then undertaken.

The fact that this method, which is mainly of interest to the physicist, was thus the result of the study of an engineering problem is an interesting illustration of the interdependence of the various professions and sciences. It is this interdependence which makes the writer willing to venture to present the results of his experiments in a physical journal, although he is aware that the apparatus at his command was crude, and that his methods of research were approximate. He is the more willing to do this because the interpretation of the results of these experiments depends wholly upon a few elementary principles of optics, well known to all scientists.

When any transparent isotropic substance, such as glass, is subjected to strain, the relative positions of its particles are changed, and, corresponding to the ellipsoid of elasticity of a bi-refractive crystal, we have the ellipsoid of strain. Hence, we should expect that when light falls upon it, only those components of the vibrations would be transmitted which are parallel to the axes of the diametral section of the ellipsoid of strain made by a plane perpendicular to the ray of light, just as in the case of the bi-refractive crystal.

As a matter of fact this property of glass has long been known. When a plate of it is subjected to strain, and interposed between the polarizer and analyzer brilliant color effects are produced, just as with thin crystalline plates.

This property of glass can be taken advantage of to determine exactly the intensity and direction of the strain at any given point in it. Since glass is an isotropic body, whose elastic constants are known, we are able to test any theoretical solution of the general problem of determining the distribution of the strain in an isotropic body by comparing the results of theory with the strain in a similar body of glass, subjected to the action of similar external forces. In this line something has already been done. Wertheim,[1] whose work is referred to in Tyndall's "Lectures on Light," used the colors of polarization in the study of the elasticity of glass. He established the fact that the retardation of the ray is proportional to the load. He described a device, which he designated the "chromatic dynamometer," for the study of stress uniformly applied. In the Transactions of the American Society of Civil Engineers, Vol. III., Nickerson has published an account of some experiments to determine the position of the neutral axis in beams by this method. The theory on which he based the interpretation of his results was, however, incorrect, and it seems strange that his work was not criticized at once.

There are two things to determine in order to specify the strain at any point in a solid; one, the *directions* of its axes, and the other, the *intensities* of the strains along those axes. Corresponding to these two things we have two sets of effects when a glass body, strained so that one of the axes of strain has a known constant direction for all points of the body, is placed between two Nicol's prisms and examined by polarized light passed through it parallel to the known axis. First, if the body be strained in such a way that the lines of strain vary in direction from point to point, we shall find dark bands traversing the glass body, which will vary in position as the prisms rotate relative to the glass body or to each other. In the second place, we shall observe color bands varying in quality and position, with similar rotation of the prisms. The dark bands depend upon the directions of the lines of strain, while the color effects are due to the relative intensities of those two principal strains whose axes are perpendicular to the direction of light. For the sake of clearness we will consider these effects separately in the discussion which follows, and separate them in the figures illustrating the appearances observed in actual cases.

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[1] Wertheim: Comptes Reudus 32, p. 289 (1851). Also, Phil. Mag. (4) 8, p. 241 (1854).

THE DIRECTIONS OF THE LINES OF STRAIN.

We will take up first, then, the study of the directions of the lines of strain throughout the body. In this case, the interpretation of the meaning of the dark bands observed in the field of view of the analyzer will be much simplified if we keep the prisms crossed. Let the polarizing plane of the polarizer be set at any given angle with the vertical. Then at all points where the lines of strain are parallel or perpendicular to the plane of polarization, the light will not have the direction of its vibrations changed in passing through the glass, and consequently cannot pass through the analyzer. Hence at those points the surface of the glass will remain dark. At any other point, however, the plane of polarization will be rotated so as to contain a tangent to the line of strain. Light from that point will, therefore, be able to pass more or less completely through the analyzer.

The result will be that the surface of the glass, as seen through the analyzer, will appear to be crossed by one or more dark bands, which will give the loci of the points where the lines of strain are parallel or perpendicular to the plane of polarization of the polarizing prism.

By setting the plane of polarization at any desired angle with the vertical, we may thus determine the direction of the lines of strain at a number of points in the glass body, and may draw elementary arcs through those points. By varying the angle of inclination of the polarizing plane and taking a series of observations, we may obtain the elementary arcs at points as close together as desired. By connecting these elementary arcs, we shall then have an approximate sketch of the lines of strain throughout the body, and sometimes may be able to deduce at once the true theory of their location.

To illustrate the application of this method let us take the case shown in Plate I. Here a one-inch cube of optical glass was pressed by a one-inch roller of optical glass loaded with 400 lbs. The cube had two opposite faces polished, and the others carefully ground. The polished faces were the ones observed, and were set at right angles to the rays of polarized light. The roller was one inch in diameter and one inch long, and was carefully ground cylindrical. Its ends were polished. It was placed with its axis parallel to the rays of light, and the line of contact was a medial line of the upper surface of the cube. The cube rested on the upper face of a piece of plate glass $\frac{1}{8}''$ thick, $1''$ wide in the direction of the rays of light, and $1\frac{1}{2}''$ long. The plate glass rested on a block of hard wood. The applied force was measured by spring balances.

The dark bands shown in Figs. 1 to 8 were sketched by dividing the observed face of the cube into $\frac{1}{10}''$ squares by placing against it a reticule of fine threads stretched in a brass frame. The sketches were made on one-inch squares similarly divided, as shown in the figures.

The polarizer was a Nicol's prism having the distance between parallel sides about $1\frac{1}{4}''$. The analyzer was a similar prism having about $1''$ between parallel sides. These prisms were set in brass tubes in the usual way, and were mounted on wooden stands in such a way that the plane of polarization of each could be set at any desired angle with the vertical, and the angle measured within a degree or two.

When the strained cube was placed between the polarizer and analyzer, with the plane of polarization inclined to the right of the vertical, the amount indicated on each figure, the dark bands shown on the figures were observed. It is to be noted that in addition to these dark bands, color effects were also observed similar to those shown on Plate II. These are omitted from the figures for reasons already given.

In interpreting the results of this experiment, we notice first that there are two regions, in the right and left upper corners

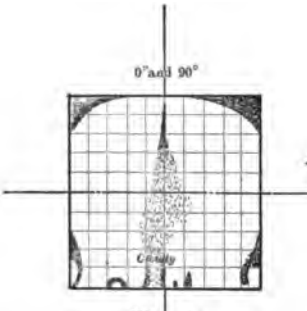

Fig. 1.

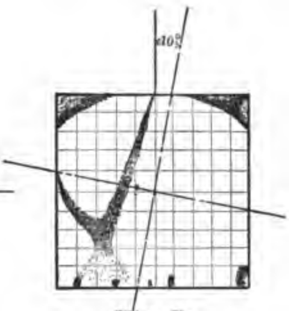

Fig. 2.

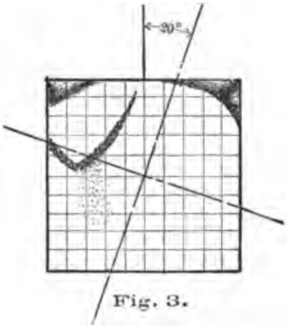

Fig. 3.

PLATE I.

DETERMINATION OF LINES OF STRAIN BY POLARIZED LIGHT.

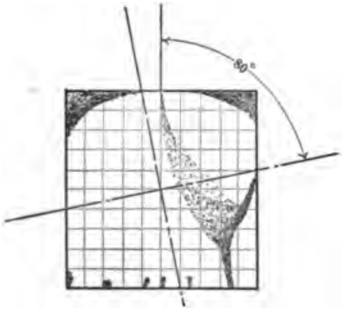

Fig. 8.

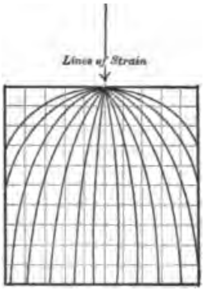

Fig. 9.

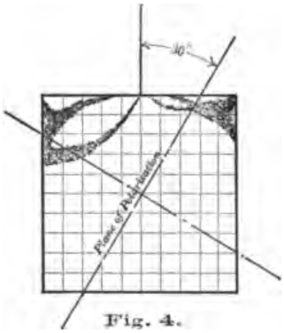

Fig. 4.

1 inch Cube Pressed by 1 inch Roller Loaded with 400 Lbs. The Color Bands are Omitted from the Figures. Prisms Crossed.

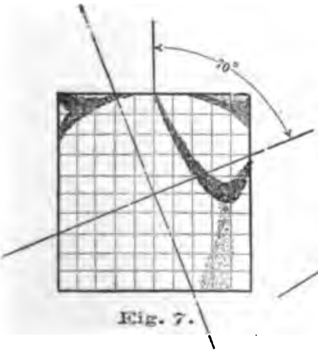

Fig. 7.

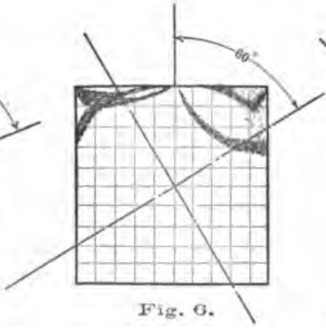

Fig. 6.

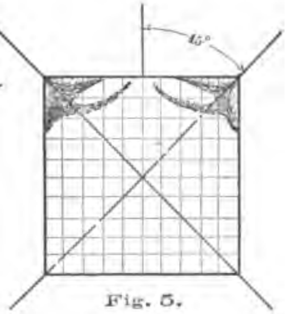

Fig. 5.

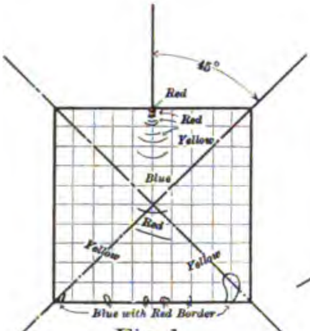

Fig. 1.

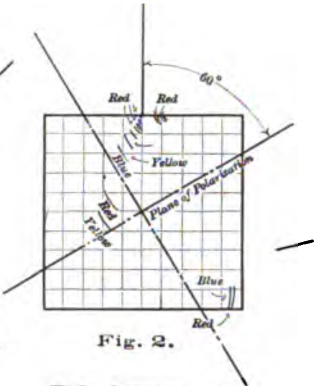

Fig. 2.

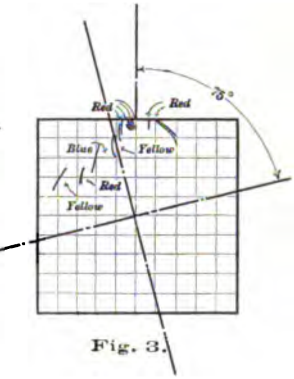

Fig. 3.

PLATE II.

DIFFERENCE OF PRINCIPAL STRAINS BY POLARIZED LIGHT.

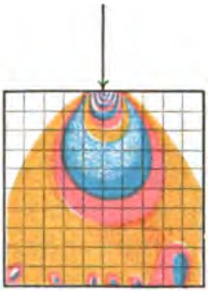

Fig. 7.

Note. — There were six bands of red plainly visible, with a seventh probably just on.

1 inch Cube Pressed by 1 inch Roller Loaded with 400 Lbs. Prisms Parallel.

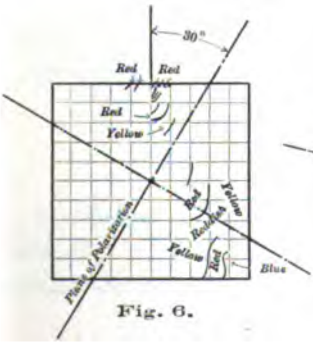

Fig. 6.

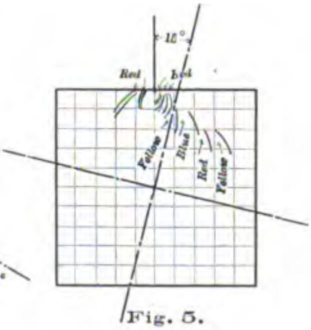

Fig. 5.

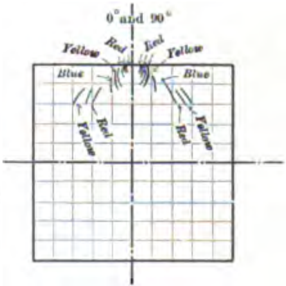

Fig. 4.

respectively, which were dark for nearly all positions of the prisms. In those figures in which these regions are not dark, the light may have been reflected from the sides of the prism in such a way as to produce the appearances seen. It seems probable that the stress was there too feeble to give the glass any bi-refractive qualities.

Aside from this, certain of the dark bands indicate the regions where the lines of strain are inclined in the direction of the planes of polarization, and the others the regions where the lines of strain are perpendicular to those planes.

Where these dark bands, and the dark regions already mentioned in the upper right and left corners overlap, we have the curious appearances seen in Figs. 4, 5, and 6.

By drawing elementary arcs, inclined at the proper angles to the vertical, through the points indicated by the dark bands, and connecting them, we obtain the sketch of the lines of strain shown in Fig. 9.

The dark places along the bottom of the cube, seen in Figs. 1, 2, and 8, indicate points of local intensity of pressure and distortion of the lines of strain, due to the unevenness of the bearing surface. By comparing closely with Fig. 7, Plate II., we can see how the lines of strain converge from each side to these points of local intensity of pressure.

In this case the equations of the lines of strain have not been worked out, but in a series of experiments, which the writer has in progress, he was able to decide at once upon the true theory of their directions. In making a complete study of the theory of the strain in the cube under consideration, it would be necessary to make a set of experiments similar to the one illustrated, for different loads and for different dimensions of rollers and cubes.

THE INTENSITY OF THE STRAIN.

Let us consider next the study of the intensity of the strain by means of the color-effects already referred to. It may be said, in advance, that no such complete solution of this problem can be obtained as of that of the directions of the lines of strain. Nevertheless, we get very interesting and instructive results.

In what follows, it must be understood that the discussion applies only to cases where one of the axes of strain has a known direction, constant for all points of the glass, and that the polarized light is passed through the glass parallel to this known axis.

Reasoning by analogy from bi-refractive crystals, we should expect that the velocities of the two rays of light in the case under consideration would be inversely proportional to those two axes of the strain ellipsoid which are at right angles to the direction of the polarized light, *i.e.* to the intensities of the principal strains in those directions. Actual experiments, whose results will be given hereafter, approximately verify this assumption, at least for small strains. Hence the relative retardation will depend upon the relative intensities of the principal strains named, and in the colors observed in the strained glass we have a measure of the difference between those principal strains at each point of the body.

Hereafter, unless otherwise specified, by principal strains will be meant those two principal strains at right angles to the ray of light.

When the difference between the principal strains at any point in the glass gradually increases from 0, the first color waves in the polarized light to be suppressed will be the shortest ones, *i.e.* those in the vicinity of violet. We should therefore expect that the first color to make its appearance would be the complementary color, yellow, and that following this we would have red, and then blue. When the difference becomes equal to one and one-half times that causing yellow we will again have yellow, the relative retardation in this case being one and one-half times the wave-length of violet. Following this second yellow will come second bands of red and blue, and so on.

To show how completely this theory is borne out by experiment, we give the results obtained by subjecting the glass cube, already experimented with, to strains of known character. In an ordinary Olsen testing machine of 50,000 lbs. capacity, the cube was subjected to a series of uniformly distributed vertical compressive stresses, gradually increasing from 0 to 7050 lbs. The appearance when polarized light was passed through it was observed by means of the Nicol's prisms, already described. The prisms were set with their polarizing planes parallel, and inclined at an angle of 45° to the vertical. The resulting colors are shown in the following table: —

Table No. 1.

SHOWING COLORS OBSERVED IN 1″ CUBE OF OPTICAL GLASS WHEN SUBJECTED TO SIMPLE VERTICAL COMPRESSIVE STRESSES.

Load.	Color.	Load.	Color.
100 lbs.	Clear	2100 lbs.	Greenish-yellow
200 "	Cloudy	2550 "	3d Red
300 "	Faint yellow	3000 "	Green
400 "	Bright yellow	3650 "	4th Red
525 "	1st Red	4050 "	Green
600 to 900 "	Blue	4750 "	5th Red
		5250 "	Green
1200 "	Yellow	5900 "	6th Red
1550 "	2d Red	6450 "	Green
1800 "	Greenish-blue	7050 "	7th Red

In making this experiment it was found that the delicacy of the test was such that it was very difficult to get rid of the effects of local pressures, due to slight inequalities in the bearing surfaces. However, by using steel bearing blocks one inch square, having the bearing surfaces carefully surfaced, and by interposing a few thicknesses of paper between them and the cube, the resulting colors were so evenly distributed over the face of the cube that no doubt was left in the mind of the observer that such slight unevenness as still remained was due entirely to inequalities in the distribution of the applied load.

In this experiment, since both of the principal stresses other than the vertical were 0, the difference between the principal strains is proportional to the load. The results may be regarded as a calibration of the observed colors for the glass cube when it is subjected to strain.

It was observed that the range of the loads corresponding to any particular band of red was less than for either of the other colors. As near as could be observed the loads corresponding to the middle lines of the different bands of red were as follows:— 1st, 525 lbs.; 2d, 1550 lbs.; 3d, 2550 lbs.; 4th, 3650 lbs.; 5th, 4750 lbs.; 6th, 5900 lbs.; 7th, 7050 lbs.

In interpreting these results it is to be noticed that the loads given are approximate, because each is for a *band* of color, which corresponds to a certain *range* of loads. Moreover, the testing machine used was not very sensitive to increments of load less than 100 lbs., and no means were available for testing its accuracy. To remove as far as possible inaccuracies due to the testing machine, a similar series of observations was made on a piece of optical glass one inch thick by two inches square, finished similarly to the glass cube before tested. The loads for the same strains and colors as in the cube should have been twice those already given. The results were as follows:— 1st, 1100 lbs.; 2d, 3175 lbs.; 3d, 5300 lbs.; 4th, 7500 lbs.; 5th, 9600 lbs.; 6th, 12,000 lbs.; 7th, 14,100 lbs.

These results establish the law that, as far as the experiments extended, the retardation varies approximately directly with the strain. They indicate that the stress necessary to produce a retardation of one wave-length of greenish blue, the complementary color of red, is about 1065 lbs.

The experiments could not be carried further because, as the loads increased, the colors became pale, and after the seventh series were hardly distinguishable from each other or from white light. After the second series we have green in place of the yellow and blue. This increasing paleness of the colors is due to the well-known fact that, for retardations of several half wave-lengths, the interference effects in the case of white light become indistinct. This difficulty could be overcome by using monochromatic light.

To determine the effect of compound stress upon the colors, a small hydraulic press was constructed, by which horizontal pressures as high as 300 lbs. could be applied to the inch cube at the same time that it was subjected to vertical pressure in the Olsen machine. The following will serve as an illustration of the results :

TABLE No. 2.

SHOWING COLORS OBSERVED WHEN POLARIZED LIGHT IS PASSED THROUGH A 1″ CUBE OF OPTICAL GLASS SUBJECTED TO COMPOUND STRESSES.

Pressures.		Color.	Pressures.		Color.
Vertical.	Horizontal.		Vertical.	Horizontal.	
0	0	Clear	400	300	Nearly clear
100	0	Cloudy	500	0	Blue and red
200	0	Cloudy	500	100	Bright yellow (and a little red at center)
300	0	Faint yellow	500	200	Faint yellow
300	100	Cloudy	500	300	Cloudy
300	200	Cloudy	600	0	Blue
300	300	Clear	600	100	Blue and red
400	0	Bright yellow	600	200	Yellow (red at center)
400	100	Faint yellow	600	300	Yellow
400	200	Cloudy			

Similar observations were made for a series of vertical loads varying up to 4400 lbs. In the above experiments the lack of uniformity in the coloring in different parts of the field indicates some irregularity in the distribution of the loads. Allowing for this, however, the results show just what our theory would lead us to expect, viz. that the colors observed correspond to those caused by simple stresses equal to the *difference* between the components of the compound stress.

To further determine the effect of compound stress, a few experiments were made with a slip of plate glass $1\frac{1}{8}'' \times \frac{5}{16}'' \times 3\frac{1}{2}''$, which was subjected to vertical tensions in the direction of its length of from 0 to 250 lbs. in the Olsen machine, and at the same time to horizontal pressures by the hydraulic press. The horizontal pressures were applied to the middle of the edges of the slip, over a space about one inch long, and varied from 0 to 300 lbs. The results showed in general that the resulting colors corresponded to simple stresses, equal to the numerical *sum* of the components of the compound stresses.

The preceding experiments show that, for small strains at least, *the colors seen in a strained glass body, when polarized light is passed through it in a direction parallel to one of the axes of strain, are measures of the algebraic differences of the intensities of those two principal strains whose directions are perpendicular to the direction of the polarized light.*

Since the axes of the principal stresses coincide with those of the principal strains, and since the difference of any two principal strains is directly proportional to the difference of the corresponding principal stresses, it follows that the word *stress* might have been substituted for *strain* throughout the preceding statement.

Let us now apply this method of studying the intensity of the stresses to the case of the one-inch cube of optical glass, for which the lines of stress have already been determined. [See Plate I. and the accompanying discussion.]

The manner of applying the vertical load of 400 lbs. to this cube has already been described. The polarizer and analyzer for the experiment were the Nicol's prisms already described. In this case, however, their principal planes were set parallel, so that the cube in its unstrained condition was clear. By this means we get rid of all dark bands such as those shown on Plate I., and the colors make their appearance for smaller loads than do the complementary colors seen when the prisms are crossed.

When we look at the glass cube through the analyzer we see a series of bands of yellow, red, and blue, instead of the uniform tint seen in the preceding experiments. Each of these bands indicates a region where the difference between the intensities of the principal stresses falls within the range of stress shown in Table I. for the corresponding color. We observe, however, that these color bands are not well defined in all parts of the cube for any one position of the prisms, and that when the prisms are rotated, the region in which the color bands are well defined changes.

To understand the reason for this, we must look at what takes place when the ray of polarized light passes through the glass at any point. As has already been said, it is divided into two rays, polarized in the directions of the axes of the stress. Now if one of the axes of the stress be parallel to the plane of polarization, the ray polarized in the direction of the other axis cannot pass through the analyzer at all, and consequently no interference can take place. On the other hand, when the axes of the stress make angles of 45° with the plane of polarization, the rays pass through the analyzer in equal amounts, and interference is complete. For all other angles than 45°, 0°, or 90° between the axes of stress and the plane of polarization, partial interference can take place. For this reason, in the experiments with uniform simple and compound stresses on the cube, the prisms were set at 45° to the vertical. Since the axes of strain were vertical and horizontal, the angle of 45° gave the position for complete interference. In those experiments the glass became colorless when the prisms were set with their planes of polarization vertical or horizontal, and nearly so for any angle with the vertical differing much from 45°.

In the case at present under consideration, since the lines of stress vary in direction at different points in the cube, in order to map out the color bands throughout their whole extent, it will be necessary to take a series of observations with the prisms set at different angles with the vertical, and to combine the results. This is what was done in the experiments, the results of which are given in Plate II. Fig. 1 shows where the bands were sharply defined when the prisms were set at 45° with the vertical. Evidently this will be where the lines of stress are nearly vertical or horizontal. Similarly, Fig. 2 marks the bands plainly where the lines of stress are inclined 15° or 75° to the vertical, Fig. 3 where they are inclined 30° or 60°, and so on. By comparing with Fig. 9, Plate I., it will be seen how completely the theory is borne out.

By combining the results shown in Figs. 1 to 6, we obtain the complete outlines of the several color bands as shown in Fig. 7. We thus obtain a map, as it were, of the difference of the intensities of the principal stresses. By the aid of Table I., we can assign an approximate numerical value to this difference at any point of the cube. Thus, at the line of contact between the roller and the cube, the difference between the horizontal and vertical components of the stress is about 7000 lbs. per square inch, and since both the stresses are compressive at this point, the maximum pressure between the roller and the cube must be greater than 7000 lbs. per square inch, though the vertical load was only 400 lbs.

The stresses themselves are in this case probably greater at points where their difference is greater, so that Fig. 7 gives in a general way an idea of the distribution of the stress throughout the cube. Thus, a quarter of an inch from the bottom we see that the stress is quite uniformly distributed throughout the whole cube. The fact that the color here is bright yellow, like that caused by a uniformly distributed load of 400 lbs. [see Table No. I.], shows that the horizontal stresses there are nearly zero.

The small areas of red and blue along the bottom of the cube show points of local intensity of pressure, due to inequalities in the bearing surfaces. It is evident that, for some reason, the pressure on the right side of the cube was greater than elsewhere.

If we were attempting to express the stress at any given point in the cube by a formula, we could make a series of experiments like that whose results are given in Plate II. A careful study of all the results would give checks for any proposed formula, or might suggest an approximate solution of the problem, which would answer for practical purposes.

In making any such series of experiments, the writer believes that it would be practicable, with suitable apparatus, to throw an image of the cube, or other glass body experimented with, upon a screen, on which the dark bands and color bands could then be easily and accurately traced. To obtain the best results, the experiments should be conducted with monochromatic light.

AMES, Iowa, June 12, 1893.

MINOR CONTRIBUTIONS.

EDITORIAL NOTE.

THE physicist has occasion to give thought to many matters which are of considerable scientific interest, but which, for one reason or another, he does not embody in a memoir. To this class of subjects belong details of laboratory manipulation or demonstration, observations of isolated facts, etc., etc. Phenomena of great interest and beauty, indeed, such as one meets with in the every-day routine of the laboratory, often go undescribed. The contemplation of such phenomena is one of the chief charms in the pursuit of physics, and yet we are apt to forego mention of them, either because they seem to be matters too obvious to deserve publication, or because their relations are obscure.

It is for topics of the classes just indicated that the editors of the *Physical Review* have planned the department of *Minor Contributions.* It is not intended to make it a department of minor interest, nor for the discussion of matters of merely minor importance. It is believed, indeed, that the communications to be grouped under this head will be full of valuable suggestion, that they will be read with much interest, and that writers whose productions appear as "minor contributions" will find themselves in as good company as elsewhere in the *Review.*

It is the fashion in these crowded times, especially among the men of science who write in English, to cultivate a condensed style. All details not essential are omitted, and many of the very features which in the course of the research had most delighted the investigator, and which, if allowed expression, would have been a source of pleasure to his readers, are refused a place.

It is the presence of these minor elements which gives to the memoirs of some of the masters in the early part of the century their charm, and which renders especially attractive the reading of the works of certain of the French physicists even of a later day. In such side-dissertations, embodied in the classics of physics, we find many an intimation of the discoveries of a later day, many an interesting glimpse into the future of the science. If through this department of the *Review* an opportunity is made and taken advantage of, whereby expression may be had of these minutiæ of physics, which, suggestive and ofttimes important though they be, seem to be in danger of being dropped from our literature, the purpose of the editors will have been fully accomplished.

AN ACOUSTIC PYROMETER.

FERNANDO SANFORD.

IN devising a piece of apparatus for showing the increase of velocity of a sound wave in air for an increase of temperature, the idea occurred to me that the same apparatus could be used for measuring high temperatures in the interior of furnaces or factory chimneys. I remembered also the suggestion of Mr. Preston in the *Philosophical Magazine* of July, 1891, that a resonant tube and tuning fork be used for this purpose, but believing that the apparatus which I had prepared had some advantages over the one proposed by Mr. Preston, I have thought that it might be of sufficient interest to warrant its publication.

The apparatus which I had prepared consists of a brass tube 5½ inches long, of ⅝-inch bore, closed at one end by a brass plug. To this tube is fastened by a movable clamp another brass tube about a foot long of ¼-inch bore, so bent and flattened at the end next to the larger tube that in blowing through it a current of air is driven across the open end of the larger tube, and causes it to give forth a musical note.

Upon heating the sounding tube the pitch of the note rises, increasing with the square root of the absolute temperature. The tube can be immersed almost to its mouth in a liquid, or can be placed entirely within a furnace or chimney with only the mouthpiece projecting, and can be blown by a bellows, or by air from the lungs.

I made a series of measurements with the tube in water at different temperatures from zero to eighty degrees, determining the pitch of the note by Appun's "*Ton Messer*," and found that my determinations varied by three or four degrees from the temperature of the water as determined by a thermometer. This variation seemed to me to be mostly due to a difference in temperature between the air inside the tube and the water outside. A part of it was due, however, to the coercive power of the reeds in the "*Ton Messer*" upon the column of air in the tube. At the temperature of the experiment a difference of one degree in temperature would make something less than one vibration a second difference in the pitch of the note given by a column of air of this length. With the series of reeds differing by four vibrations a second from each other, a difference of one vibration a second in the pitch of the note could be determined; but when the tube was nearly in unison with the reed it would make only a few beats at starting, and would then vibrate in exact unison with it. This difficulty would be still greater in attempting to use a resonance tube with a tuning fork, as in that case the sound of the fork is reinforced by a tube varying considerably in length from the one giving the greatest reinforcement.

I was unable to compare my measurements made at higher temperatures with any other standard; but I was able to determine the pitch with the same degree of accuracy up to the highest temperature to which I could heat the tube in a gas flame.

It goes without saying that the tube could be as well made of platinum or porcelain for measuring high temperatures, and that it could be used equally well for measuring very low temperatures.

In some experiments made to determine what form of mouthpiece was best adapted to the tube, I found that the open tube sounded by blowing across the open end gave fewer overtones than when it was provided with a mouthpiece like an organ pipe.

I have thought that the melting point of very refractory substances could be determined by making the tube of the substance to be tested, and finding the highest pitch which it could be made to give before melting; but I have at present no source of sufficiently high temperature available for the test.

LELAND-STANFORD UNIVERSITY,
March, 1893.

METHOD FOR DETERMINING FOCAL LENGTHS OF MICROSCOPE LENSES.

W. S. FRANKLIN.

THE following is a method for determining the focal lengths of the lenses of a compound microscope *in situ*. It depends upon measurements of the magnifying power with short tube, the magnifying power with long tube, the change in the length of the tube, and the movement of the objective in refocussing. The method has been occasionally assigned by the author as a laboratory exercise since 1889.

Let m be the magnifying power of the microscope with short tube, and m' the magnifying power with long tube; magnifying power here signifying the ratio of apparent size of an object as seen with the microscope to its apparent size as seen with the naked eye at a distance of D cm.

Let p be the equivalent focal length of the eyepiece; p' the focal length of the objective; a the distance from the object to the center of the objective; and b the distance of the image from the center of the objective. Let k be the movement of the objective in refocussing, and l the change in the length of the tube. Let D be the distance of the auxiliary scale used in determining magnifying power.

Then,

$$m = \left(\frac{D}{p} + 1\right)\frac{b}{a}, \tag{1}$$

$$m' = \left(\frac{D}{p} + 1\right)\frac{b + l}{a - k}, \tag{2}$$

$$\frac{1}{p'} = \frac{1}{a} + \frac{1}{b} \tag{3}$$

$$\frac{1}{p'} = \frac{1}{a - k} + \frac{1}{b + l}, \tag{4}$$

From these equations we easily obtain :—

$$a^2 lm - b^2 km' = 0 \tag{5}$$

$$a^2 - \frac{2\,km'}{m' - m}a - \frac{klmm' - k^2 m'^2}{(m' - m)^2} = 0. \tag{6}$$

Equation (6) enables the calculation of a. The value of b may then be determined from (5), and p and p' from (1) or (2) and (3) or (4) respectively.

The magnifying power is most easily determined by making use of an unsilvered slip of glass affixed to the eyepiece so as to enable an auxiliary scale to be seen in the field of the microscope. The power m will then be given as the apparent length on this scale of a unit length on a stage micrometer.

The distance, D, is to be taken as the distance from a certain one of the two principal, focal planes of the eyepiece, to the image (in the glass slip) of the auxiliary scale. In the case of the negative (Huyghens') eyepiece, D is the distance from a point midway between the two lenses to the image, in the glass slip, of the auxiliary scale. In case of the positive (Ramsden's) eyepiece this focal plane is between the two lenses of the combination at a distance from the eye lens equal to three-fourths of the distance between the lenses.

The change of length of the microscope tube may be easily measured directly. The movement k, in refocussing may be conveniently measured by means of a micrometer caliper mounted temporarily and playing against an arm fastened to the tube just above the objective.

The most serious source of error is that due to distortion unless, indeed, a very small object be used, when the determination of power will be inaccurate. However, errors due to distortion are of the same order of magnitude as the uncertainty in focal lengths due to spherical and chromatic aberration, and cannot in strictness be considered as detracting from the accuracy of the method.

The method has been carried out in the physical laboratory at this place upon a Beck microscope. A Brown and Sharpe micrometer caliper was used for measuring k. A Zeiss stage micrometer divided to $\frac{1}{10}$ mm. was used as the object, and l was measured directly. The distance D was chosen rather large so as to enable it to be measured with greater certainty.

The mean of three sets of observations gave $m = 184$, $m' = 289$, $k = .094$ cm., $l = 6.67$ cm., and $D = 88$ cm. Whence $a = 2.00$ cm., $b = 13.45$ cm., $p' = 1.74$ cm., $p = 3.33$ cm. With an objective of shorter focal length the mean of two sets of observations gave $p' = .507$ cm. and $p = 3.36$ cm. Duplicate sets of observations furnished values for p and p' which differed from the above by about one per cent.

AMES, IOWA, June 10, 1893.

SOME APPLICATIONS OF ELECTRIC HEATING IN PHYSICAL LABORATORY PRACTICE.

EDWARD L. NICHOLS.

IT is strange that so little attention has been given by physicists to the advantages of electric heating in calorimetric and other similar work. Professor Henry Crew[1] quite recently described "a new method of obtaining a constant temperature" in which a coil of wire is used to heat an air bath with admirable effect. It would seem that devices more or less closely related to the one due to his ingenuity must have occurred to others engaged in laboratory operations; but I recall no published account of such methods. The application of electric heating for temperature control has been in use in the physical laboratory of Cornell University for a considerable time, and it gives me pleasure to confirm Mr. Crew's statements as regards the excellent results which may be obtained by means of it.

It will be seen from what follows that the methods of procedure to be described differ from his chiefly in three particulars: (1) the use of naked wire, whereby the range of temperature attainable is much increased; (2) the absence of the ice bath, it having been found that remarkable constancy of temperature within a heated coil occurred, under almost all ordinary conditions of the surrounding space; (3) the measurement of temperatures by means of a calibrated coil of copper wire.

The method to be described consists in heating the body or apparatus in question by means of a spiral coil of uncovered wire surrounding it. The wire, which may be of German silver, is traversed by strong electric currents, and thus brought to a high temperature. The best material for this purpose is some metal of high specific resistance and small temperature coefficient. It should be capable of withstanding high temperatures without melting. The method under consideration has been applied under my direction in a variety of laboratory experiments. I propose to describe several of these briefly for the purpose of illustrating its advantages.

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[1] Crew, Philosophical Magazine, 33 (5), p. 89, 1892.

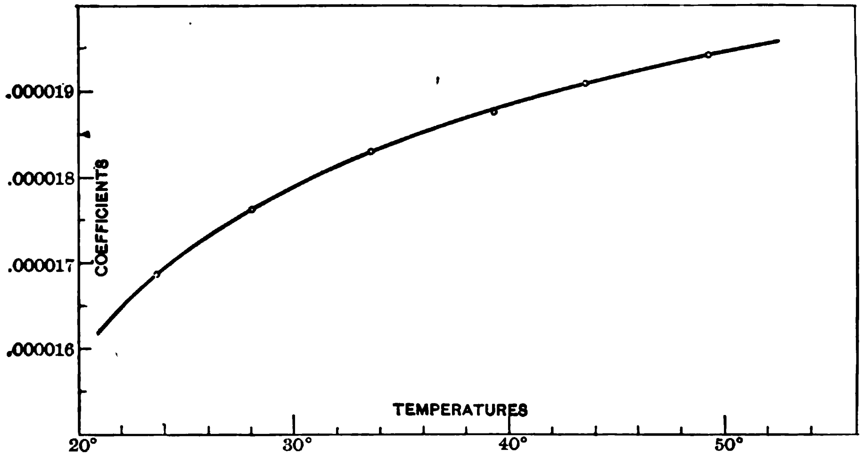
I. The first of these experiments was suggested by a paper read by Professor William A. Rogers at the Cleveland meeting of the American Association for the Advancement of Science. He called attention in that paper to the fact that thermometers when subjected to a fall of temperature frequently indicate apparent discontinuity in the curve of cooling, the mercury column falling stepwise, and in some cases even retracing its course in the tube. This observation having been verified in the case of several thermometers of the best French and American manufacture, it seemed of interest to determine whether the phenomenon might not be due in part to some peculiarity in the law of the contraction of glass. The experiments were made by Mr. Ernest F. Nichols. Rods of glass were mounted upon the bed of a comparator. One end of the rod was fixed, and to the free end a mirror was attached in such a manner that the changes in the length of the rod would produce a motion of rotation. The image of a vertical scale set up at a distance of about three meters was observed by means of a reading telescope. In this way every change in the length of the rod was greatly magnified. The rod was surrounded by an open spiral coil of iron wire reaching from end to end. Sufficient current was sent through the wire to heat it, and this current, which was supplied by a storage battery, was kept constant until the movement of the image of the scale had ceased. The circuit was then broken, and the rod was allowed to cool. A curve of cooling was obtained by noting the movement of the image at frequent intervals until the rod had reached the temperature of the room again. Upon plotting this curve it was found that the discontinuities in the rate of cooling if present in the case of the specimen of glass tested were extremely small. The curves were not perfectly true, but the irregularities were so small as to make it uncertain whether they were not to be ascribed to errors of observation or to imperfections in the apparatus. Rods of aluminium and brass were treated in the same manner for the purpose of determining whether these metals would give smoother curves of cooling than had been obtained in the case of glass. These curves also showed slight irregularities. The outcome of the experiment, therefore, was a negative one; but that it was evident from the curves obtained that the phenomenon noticed in thermometers could not be ascribed altogether to the suspected peculiarity in the law of the cooling of glass. From the point of view of the present paper, however, the experiment was of considerable interest. It made it evident that this means of heating is an advantageous one in the study of the linear dilation of rods of considerable length.

II. In the same year, 1891, the method was applied to a determination of the coefficient of expansion of a standard meter bar of brass. This experiment was carried on by Mr. Joseph Le Conte. For the purpose of determining the temperature of the bar the same was wrapped from end to end with several turns of fine silk-covered copper wire. The resistance of this longitudinal coil, the temperature coefficient of which had been previously determined with a good deal of care, was measured by the fall of potential method, using a sensitive galvanometer, a single cell of battery, and a compensated standard made by combining German silver and copper. Thus the temperature of the bar was averaged for its entire length. This bar was mounted upon a comparator, and was surrounded by a coil of large uncovered German silver wire in the manner described in Experiment I. The length of the bar was measured by means of reading microscopes attached to the comparator. The bed of the instrument was protected from the radiation of the coil by means of a layer of asbestos paper. It was found by trial that the rise of temperature of the massive iron bed of the comparator was almost unappreciable even when the coil was heated to a temperature of 50° C. In this experiment the measurements were taken with the circuit closed, the bar having been brought to the desired constant temperature by long-continued application of the current. It was found that any temperature within wide range could be reached by simple adjustment of the resistance of the circuit through which the heating current flowed, and that no troublesome fluctuations took place. The coefficient of the bar was found to be in close agreement with determinations for this metal by the best known methods. It became evident in the course of the experiment that this method of applying heat in the determination of the coefficient of expansion of specimens of considerable size was very much more convenient and under much better control than any of those which have been used in the past. The method of measuring temperature by means of an integrating coil extending over the entire length of the bar was also found to be superior to any procedure involving the use of a mercury thermometer. How very difficult it is to place thermometers so as to obtain true indications of the temperature of a bar in process of being tested is well known to all who have attempted work of this description. The calibration of the coil of copper wire for temperature measurements showed that the temperature indicated by it represented very closely indeed the temperatures of the mass of metal which it surrounded. I shall have occasion to speak of this more in detail in the discussion of another experiment.

III. The next illustration of the use of this method of heating is taken from an investigation which has been in progress during the past year at the hands of Mr. J. S. Shearer. A detailed account of his experiments is now in preparation, and will be published shortly. Mr. Shearer studied the influence of temperature upon the rate of a tuning fork. In his work the method under consideration found double application. In the first place, for the purpose of measuring the rate of the fork while hot, he substituted for the rather cumbersome device employed by Kayser, in his well-known work upon this subject, the use of a spiral coil similar to those which have already been described. In this case, however, the fork was mounted vertically, and was driven by means of an electromagnetic contact device. The whole fork was surrounded by a cylindrical coil. This consisted of German silver wire wound upon a drum of asbestos. At the base of the fork other asbestos packing was used so as to prevent convection currents. It was found possible to bring the fork to any desired temperature up to 100° C., and to maintain it at that temperature for as long a time as one wished. The rate was determined by the method of Lissajous' figures. The distribution of temperatures in this case was less satisfactory than where the metal to be heated was mounted in a horizontal position, but the temperature differences within the coil were not sufficiently great to interfere seriously with the accuracy of the determinations. Here, again, the method of electric heating showed itself incomparably superior to those previously in vogue. Since the results are to be given elsewhere, I need only allude to those features of the experiment which touch upon the question before us. Having determined the law of change in the rate of vibration of the fork, it became important to ascertain in how far the change was due to the influences of temperature upon Young's modulus of elasticity. In previous work of this kind the values for Young's modulus, determined by experiments upon steel wires at the hands of Wertheim and others, had been made use of in the computation. The classical determinations of Wertheim for the elasticity of steel leave a point of great importance in the application of the theory of the vibration of tuning forks in doubt; some varieties showing the continued diminution in the value of the modulus, others, on the contrary, showing a rise in Young's modulus to a maximum with subsequent decrease. Under such circumstances it is not possible to make use of the data already at hand, and it becomes necessary to determine this constant in the case of the particular specimen of metal under investigation. By clamping the base of the fork securely to the bed of the comparator, and surrounding one prong with a spiral German silver coil, in the manner described in the first two experiments alluded to, it was found possible to get a very good determination of Young's modulus by the method of deflections, applying weights up to five kilograms to the free end of the fork, and noting the deflections by means of a mirror as in Experiment I. Changes in the modulus amounting to one per cent could have been detected with certainty. I do not know of any other means of heating a tuning fork and maintaining its temperature constant during the time necessary to make such observations as these, which can be compared with ease of application and in nicety of control with the one which it is the purpose of this paper to describe; nor do I any method so convenient for determining the modulus of elasticity.

IV. An experiment illustrating this method of heating, in application to work of another character, is as follows: This is also taken from an investigation now in progress in the Physical Laboratory of Cornell University. It was desired to determine the effect of heating sugar solutions upon the rotation of the plane of polarization. In the Soleil saccharimeter the tube containing the fluid, the rotational coefficient of which was to be determined, was surrounded by a coil of German silver wire precisely in the method already indicated. It was found possible in this way to bring about temperature changes within the entire range below the boiling point of the solution, and to maintain them during the measurement of the angle of rotation. The entire apparatus consisted simply of a German silver helix, surrounding the brass tube of the saccharimeter, in series with suitable resistances, and a galvanometer for the measurement of current. Since the temperature itself could not be readily determined directly at the time of observation, a calibration test was made with the thermometer inserted in the liquid, the temperature being measured as a function of the current flowing through the coil. The relation between temperature and current having been ascertained once for all, it was therefore only necessary to note the readings of the galvanometer in order to find the temperature to which the measurement of the angle of rotation applied. A graduate student, Miss A. S. Baxter, has completed some preliminary experiments with this apparatus, and there is every reason to think that the method will lead to excellent results.

V. Another illustration of the use of this method occurs in some measurements similar to those of Mr. Le Conte (Experiment II.). In this case, however, the bar of metal to be studied was a piece of white speculum metal, one decimeter in length, ruled for use under the microscope. It was desired to determine the temperature coefficient, and the change of that coefficient with change of temperature. Under my direction Mr. C. H. Sharp has recently completed such measurements. The piece of metal being short, it was not deemed necessary to integrate the temperatures by means of a copper coil. Thermometer readings, taken with the thermometer bulb in various positions, showed that the range of temperatures inside the heated coil was very small, and it seemed certain that the average of these readings represented quite accurately the temperature of the bar itself. The results obtained in this case are shown by means of the accompanying figure in which abscissæ are temperatures and ordinates are coefficients of linear expansion corresponding thereto. It will be seen that the curve is not a straight line, but that, as has been shown by many early experimenters upon the expansion of solids, the coefficient increases notably with rise of temperature.



VI. My final example is drawn from some investigations upon the thermal conductivity of a copper bar. As the results will probably be described in this *Review*, I will merely say that the bar in question, which is one of those distributed by Professor W. A. Rogers in 1891 for the purpose, having been twice measured after established methods, was taken in hand by the present experimenters, Messrs. Child and Quick, who, in their final determinations, substituted for the bath at the hot end of the bar a closely wound, snug-fitting coil of large wire carrying heavy currents. The control of this coil was found to be much more complete and very much simpler than that of any high temperature bath. Finally, to obtain the curve of cooling, the entire bar, one meter long, was surrounded with an open spiral and heated to nearly 300° C. It was found practicable to secure great uniformity of temperature for the entire length of the bar, no simple matter by the usual methods, to produce and maintain the heat without vitiating the surface, and finally to begin readings for the cooling curve immediately upon removal of the heating coil, thus extending the upward range of the measurements.

Temperatures were determined throughout the investigation by means of a closely fitting collar, consisting of one layer of smaller copper wire, the position of which upon the bar could be readily changed.

Enough has been stated in the account of these six experiments to indicate that electric heating has a wide application in the physical laboratory. To those who have occasion to direct or plan experimental work, the usefulness of such methods as have been described will be obvious.

Physical Laboratory of Cornell University,
June 23, 1893.

NEW BOOKS.

Electrical Papers. By Oliver Heaviside. 2 vols. London, Macmillan & Co., 1892.

The two volumes before us contain a reprint of fifty articles on electrical subjects, which appeared originally in *The English Mechanic*, *The Philosophical Magazine*, *The Journal of the Society of Telegraph Engineers*, *The Electrician*, *The Philosophical Transactions*, and *Nature* during the interval from July, 1872, to July, 1892. They contain in addition three papers which were written within that interval, but are now published for the first time. The first twelve articles are on matters dealing principally with telegraphy, and are but loosely connected. The next eight articles deal mainly with the theory of the propagation of variations of current along wires, beginning with applications of the simple electrostatic theory of Sir William Thomson to cables under different circumstances, and followed by extensions to include self-induction, or the influence of the inertia of the magnetic medium, and the mutual influence both electrostatic and magnetic of parallel wires. We next come to a series of papers relating to electrical theory in general, first published in *The Electrician* between 1882 and 1887; then to a study of the theory of the propagation of induction and electric current in round wires; and lastly to a more comprehensive treatment of electromagnetism, based upon Maxwell's theory in a long series of papers entitled "Electromagnetic Induction and its Propagation." The remaining articles consist of applications growing out of the preceding theory, namely, the self-induction of wires, resistance and conductance operators, electromagnetic waves, the forces, stresses, and fluxes of energy in the electromagnetic field, etc.

Thus it will be seen that the work, though a reprint of papers, has a continuity of treatment, and is suitable for the electrician or the advanced student, though not adapted for use as a manual. Mr. Heaviside is, I believe, now engaged putting the electromagnetic theory in a form suitable for a student.

One who reads these papers cannot fail to be impressed with the philosophic earnestness and philanthropic motive of the author. He is a lover of wisdom and a searcher after truth. In this connection it is interesting to read how some of his most important papers, namely, those on the self-induction of wires, were first received. Because his views were contradictory of those then accepted by the post-office officials, an attempt was made to prevent their discussion and publication. These are the papers, which are now published for the first time. We cannot but agree with the author when he says "there seemed to be an idea that official views, in virtue of their official nature, should not be controverted or criticized. But there seems something wrong here, as the later evidence in support of my views has shown. For what other object have scientific men than to get at the truth, and how is it to be done without free discussion?"

The articles on electric and electromagnetic theory form the best commentary that has been written on Maxwell's "Electricity and Magnetism." The fundamental ideas are explained and elucidated, and the author is not content unless he places before the reader the true inwardness of the analytical expressions and equations. But they are more than a commentary: they constitute a development of Maxwell's theory.

In these volumes we have a full account of the author's system of rational electric units. In many cases the true relation between electric quantities is clouded by the appearance of an extraneous 4π; for example, the resultant force outwards at a charged surface is not equal to the density, but to 4π times the density. This is due to the manner in which the unit of electricity or the unit magnetic pole is defined. If the unit pole were defined as that from which unit amount of force emanated, the force at distance r would be $\frac{1}{4\pi r^2}$ instead of $\frac{1}{r^2}$. The advantage would be that 4π would appear only where its appearance is rational; all the equations would then be seen to be truths instead of appearing as mere formulæ. It is likely, therefore, that the use of rational units would be beneficial in theoretical investigations; but whether the practical units already defined should be made to change with every improvement in the theory of units is a different question.

The author makes (Vol. II., p. 201) some highly important observations on the relation of physical science to mathematical analysis. It will be, perhaps, in the recollection of some readers that Professor Sylvester, a few years since, in the course of his learned paper on Bipotential, poked fun at Professor Maxwell for having, in his investigation of the conjugate properties possessed by complete spherical-surface harmonics, made use of Green's theorem concerning the mutual energy of two electrified systems. He said (in effect, for the quotation is from memory) that one might as well prove the rule of three by the laws of hydrostatics, or something similar to that. In the second edition of his treatise, Professor Maxwell made some remarks that appear to be meant for a reply to this; to the effect that although names involving physical ideas are given to certain quantities, yet, as the reasoning is purely mathematical, the physicist has a right to assist himself by the physical ideas. Certainly; but there is much more in it than that. For not only the conjugate properties of spherical harmonics, but those of all other functions of the fluctuating character which present themselves in physical problems, including the infinitely undiscoverable, are involved in the principle of energy, and are most simply and immediately proved by it, and predicted beforehand. We may indeed get rid of the principle of energy, and treat the matter as a question of the properties of quadratic functions; a method which may commend itself to the pure mathematician. But by the use of the principle of energy, and assisted by the physical ideas involved, we are enabled to go straight to the mark at once and avoid the unnecessary complexities connected with the use of the special functions in question, which may be so great as to wholly prevent the recognition of the properties which, through the principle of energy, are necessitated. It seems to me that we may go even further, and hold that mathematical analysis should be based not on arbitrary laws, but on physical principles. Certain it is that the great developments of analysis have been due to an infusion of physical ideas and principles.

The author goes much further than Clerk Maxwell in the use of space-analysis. The position of Clerk Maxwell was paradoxical; he considered that the notation of quaternions was good, but the method itself bad; he made use of the former, but not of the latter. Mr. Heaviside uses a method which he thus describes (Vol. II., p. 521): "As electromagnetism swarms with vectors, the proper language for its expression and investigation is the Algebra of Vectors. An account is therefore given of the method employed by the author for some years past. The quaternionic basis is rejected, and the algebra is based upon a few definitions of notation merely. It may be regarded as quaternions without quaternions, and simplified to the uttermost; or else as being merely a conveniently condensed expression of the Cartesian mathematics, understandable by all who are acquainted with Cartesian methods, and with which the vectorial algebra is made to harmonize. It is confidently recommended as a practical working system."

It strikes me that to base a method upon a few definitions of notation is a highly illogical procedure, and indeed cannot be done. Let us see how it is attempted. A vector is denoted by a Clarendon letter as **A**, its magnitude by the corresponding Italic capital as A, and the magnitudes of its three rectangular components by A_1, A_2, A_3. This is a matter of notation, and the notation is real, in the sense of corresponding to something in nature. But observe the next step. "The scalar product of a pair of vectors **A** and **B** is denoted by **AB**, and is defined to be

$$\mathbf{AB} = A_1B_1 + A_2B_2 + A_3B_3 = AB\cos\widehat{\mathbf{AB}} = \mathbf{BA}".$$

This is not a definition merely. If we define $AB\cos\widehat{\mathbf{AB}}$ as the scalar product of **A** and **B**, and denote it by the simple juxtaposition of its factors, then A_1B_1 is the scalar product of A_1i and B_1i; A_2B_2 is the scalar product of A_2j and B_2j, and A_3B_3 is the scalar product of A_3K and B_3K. That $AB \cos \widehat{\mathbf{AB}}$ is equal to $A_1B_1 \cos ii + A_2B_2 \cos jj + A_3B_3 \cos kk$ is not a matter of definition, but is a geometrical truth which may be demonstrated. It is a principle, not a definition. The author's introduction of the vector product contains a similar confusion of definition and principles. He says, "The vector product of a pair of vectors is denoted by $V\mathbf{AB}$, and is defined to be the vector whose tensor is $AB \sin \widehat{\mathbf{AB}}$, and whose direction is perpendicular to the plane of **A** and **B**, thus

$$\mathbf{VAB} = i(A_2B_3 - A_3B_2) + j(A_3B_1 - A_1B_3) + k(A_1B_2 - A_2B_1)$$

where i, j, k are any three mutually rectangular unit vectors." Now the definition refers merely to the meaning of **V**, and applies to all the terms of the above equation; while the equation itself asserts the geometrical truth, that the directed area formed by **A** and **B** is equal to the geometrical sum of the three areas specified, Mr. Heaviside, in avoiding the Scylla of metaphysics, has been drawn into the Charybdis of definition. It seems to me that the true basis for the principles of space-analysis is to be found in geometry, and more generally in physical science.

Mr. Heaviside opposes the quaternionic practice of introducing a minus before the scalar product; he holds that

$$\mathbf{SAB} = A_1B_1 + A_2B_2 + A_3B_3,$$

while according to Hamilton

$$\mathbf{SAB} = -(A_1B_1 + A_2B_2 + A_3B_3).$$

The quaternionic practice is a consequence of making Nature more simple than she really is, namely, the identifying of vectors and versors. This principle logically carried out would land us in believing that there is no difference between direction and change of direction, or between a translation and a rotation.

The square of a vector being positive, it follows that the reciprocal of a vector has the same direction as the vector but the reciprocal magnitude. This also agrees much better with the principles of analysis than does the quarternionic doctrine that the direction of the reciprocal is the opposite of that of the vector. The author is fully impressed with the importance of preserving harmony with the Cartesian analysis — logical harmony, so that we may pass from the one to the other with as much ease as we pass from algebra to arithmetic.

Mr. Heaviside in these volumes displays some wit, and that may be the reason why he describes his practical working system as "Quaternions without quarternions." The phrase is suggestive of the play of Hamlet with the character of Hamlet left out. To keep the quarternion out of the play he certainly resorts to extreme measures. For instance, in an early paper (Vol. I., p. 271) he gets a product which is the sum of a scalar product and a vector product; namely, if

$$\nabla = i\frac{d}{dx} + j\frac{d}{dy} + k\frac{d}{dr} \text{ and } \mathbf{R} = Xi + Yj + Zk,$$

then

$$\nabla\mathbf{R} = -\left(\frac{dX}{dx} + \frac{dY}{dy} + \frac{dZ}{dz}\right) + i\left(\frac{dZ}{dy} - \frac{dY}{dz}\right) + j\left(\frac{dX}{dz} - \frac{dZ}{dx}\right) + k\left(\frac{dY}{dx} - \frac{dX}{dy}\right).$$

In a foot-note now added he says: "As the above is the only paper in which I have used the quaternionic ideas and notation, it is perhaps desirable to emphasize the fact that the use was parenthetical. There is great advantage in most practical work in ignoring the quarternion altogether, and also the double signification of a vector above referred to, and in abolishing the quaternionic minus signs." But in the same paper (p. 269) we find the following admission: "Now it appears that when ∇ is applied to a vector, it gives its curl and its convergence respectively. This extraordinary effect of ∇ is not easily to be understood — although symbolically it works out very well, for there is undeniably a certain amount of mystery about the rules for vector multiplication."

The explanation which I have published of this mystery is, that the products of vectors and the products of versors are distinct, forming parts of analysis complementary to one another; that the *complete* product of two vectors consists of two partial products, the one scalar and the other vector; and that this complete product is a quaternion in the sense of being such a sum, but not in the original and proper sense of the word "the ratio of two vectors."

Physical analysis is not all a play of vectors, nor is it all a play of quarternions; there are many actors beside. Our author in revolting from the one extreme has gone to the other; but in securing fair play for the vectors he has furthered the development of that space-analysis which at the present time is the greatest want of the physicist and especially of the electrician.

ALEXANDER MACFARLANE.

Handbuch der Physiologischen Optik. H. VON HELMHOLTZ. 2d edition.

Since the appearance of the first edition of Helmholtz's classic work on physiological optics, investigation in this subject has brought out many new truths, and hence physicists will give a hearty reception to this new edition by the recognized authority on the subject. About two-thirds of the entire work has already appeared. The general arrangement of the book is about the same as in the original edition; but several improvements are noticeable. All new paragraphs are distinguished by an *n* in the margin, and the double page numbering allows easy reference to be made to the original edition. The bibliography is very full, and the references are at the bottom of the page instead of at the end of the section, as was the case in the original.

The anatomical description of the eye is very complete, and exhibits the professor's early surgical training. The description of the retina is especially full, and includes an account of the recent work of Max Schultz and others. New paragraphs are also added under Dioptrics of the Eye, Accommodation, and Astigmatism. The most important additions, however, are on The Intensity of the Sensation of Light and Color-Blindness. Under the former heading there are fully seventy pages of new matter. It is principally devoted to luminosity (Helligkeit) in its various relations to color, retinal sensitiveness, persistence of vision, etc.

Much space is devoted to the subject of color-sensation and color-blindness. The author reviews the Young hypothesis of color-sensation as given in the first edition, points out the inconsistencies in it, and proposes a new modification. The new hypothesis postulates three primary color-sensations, as did the original Young-Helmholtz theory, but these elements are very different. They are carmine red, green of wave length about 560, and blue of wave length about 482, — and all much more highly saturated than any spectral colors. The author's new hypothesis to explain color-blindness supposes the two primary sensations of the dichroic eye to be produced by two colors derived from the three normal fundamental colors, but not necessarily identical with them. The method of the derivation of the dichroic colors from the normal fundamental is not indicated, and there seems no necessary manner of derivation. This new view accords better with the recent work of Koenig and Dieterici than the original Young-Helmholtz theory, from the fact that it requires no distinction of dichromatism into red-blindness and green-blindness.

The Hering theory is very clearly enunciated and compared with that of the author. This theory he considers as simply a modification of the Young theory, which explains the facts of color mixtures as well as, but not better than, the original. It differs principally in the choice of the elementary sensations. In the Hering theory there are three fundamental color-sensations of which at least two exhibit both positive and negative characteristics. One of them gives white under excitation, and black when at rest; another gives blue and yellow, and the third gives green and its complementary red. White light excites only the white-black visual component; yellow excites this and also the blue-yellow component. Blue does the same, but in the opposite sense. When the blue and the green excitations are in equilibrium, they exert no effect on the blue-yellow component. Also, according to this theory the sensation of luminosity depends solely upon the excitation of the white-black component.

E. S. FERRY.

Matter, Ether, and Motion. By A. E. DOLBEAR. 8vo, pp. 334. Boston, Lee and Shepard, 1892.

This little book is written for the use of the "large body of persons desirous of knowing more of natural philosophy, and especially of its trend, who have neither time nor opportunity to read and digest monographs on a thousand topics." The book will be found to treat of the whole domain of physics as fully as could be expected in so small a volume, and abounds in mechanical analogies and numerical illustrations. The necessity of the ether in modern physical theories is strongly emphasized, while the views of the author on the vortex atom theory are indicated by the remarks (page 43) that "there is no other theory that has any degree of probability at all."

Hurried writing and proof-reading are doubtless to blame for the misprints and inaccurate statements that occur at frequent intervals throughout the book. In most cases these errors are merely a source of annoyance, but with readers who are unfamiliar with the subject they might often be the cause of serious misconceptions. As an example of the numerous misprints may be mentioned the omission of the word "thousand" on pages 80 and 81, which makes it appear that the velocity of light is only 185 miles per second. The occurrence of this error on two consecutive pages can scarcely fail to prove confusing. At the top of page 64 will be found a numerical error which cannot be explained by a single misprint. In fact, I have been unable to discover how the result, which is wrong, could have been obtained by any ordinary mistake. Accuracy in numerical work in a book of this kind is doubtless of secondary importance; yet numerical illustrations constitute so large a part of the contents that it seems essential that the reader's confidence in the results should not be shaken. An error of a more serious character appears in the discussion of the thickness of a soap film on page 11, where the interference effects are explained with no mention of the change of phase introduced by reflection. A result is thus obtained which is half what it should be. The statement made on page 68 that a weight resting upon a table requires energy for its support is extremely unfortunate. In this, as in several other cases, the explanation is so brief that many will fail to appreciate the author's real meaning, and without suitable interpretation the statement is certainly in the highest degree misleading.

There will be noticed throughout the book an indiscriminant mingling of well established theory and more or less plausible speculation, with no means by which the beginner can distinguish between then. This renders the book to a certain extent unsuitable for the class of readers for whom it is intended. But the numerous mechanical analogies, which in many cases are extremely ingenious, together with the easy conversational style in which the book is written, will make it interesting and suggestive reading for those who are well grounded in physical principles.

Ernest Merritt.

Recent Text-books in Physics.[1] — Mr. Selby's text-book on Mechanics is intended for students who are familiar with the elements of geometry and algebra, but whose mathematical preparation does not extend beyond this point. In addition to a discussion of the mechanics of the particle the book treats of the motion of an extended body, and also contains chapters on Gravitation, Elasticity, Hydrostatics, and Capillarity. In spite of the restrictions imposed by the necessity of elementary mathematical methods, the treatment in the earlier portions of the book is highly satisfactory, and it is not until the chapters on Gravitation and Elasticity are reached that the complexity of the proofs becomes annoying. A commendable feature of the book is the frequent insertion of numerical problems. The use of the pound as the unit of mass may cause some confusion to engineering students in this country, but the employment of the various systems of units is in all cases perfectly consistent.

In the new edition of the well-known laboratory manual of Glazebrook and Shaw no change has been made in the general method of treatment. Important additions, usually in the form of new practise experiments, have however been introduced throughout the book, the result being an increase of almost 150 pages over the edition of 1889.

In the chapters on Mechanics new experiments will be noticed, dealing with falling bodies, the fly-wheel, the ballistic pendulum, graphical statics, and capillarity. The chapter on Magnetism has been enlarged by the addition of four experiments, the most important of these being upon the study of magnetic induction by means of the magnetometer. The ballistic method, so common in this country, has unfortunately not been mentioned, although a satisfactory discussion of the ballistic galvanometer itself is given in connection with the subject of capacity measurements. Among the additions to the chapters on Electricity may be mentioned experiments on electromagnetic induction, the silver voltameter, the use of the Clark cell, and the measurement of coefficients of induction. Two experiments have been added to the chapter on Calorimetry, and three to the chapters on Light. In fact, new matter will be found in almost every part of the book.

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[1] *Elementary Mechanics of Solids and Fluids.* By A. L. Selby. 8vo, pp. 299. Oxford, Clarendon Press, 1893.

Practical Physics. By R. T. Glazebrook and W. N. Shaw. Fourth edition. 8vo, pp. 633. London, Longmans, Green & Co., 1893.

Lehrbuch der Experimentalphysik. By E. von Lommel. 8vo, pp. 643. Leipzig, Johann Ambrosius Barth, 1893.

It is doubtless unnecessary to refer to the many excellent features of a book which is familiar to all teachers of physics. Suffice it to say that the matter that has been added is presented with the same care and clearness that have characterized the earlier editions.

Professor Lommel's text-book, while intended for the use of students desiring to review the subject of Experimental Physics, is yet arranged in such form as to be available as a text-book for beginners. The style is direct and clear, and illustrated diagrams are numerous. So far as possible the arrangement of matter has been made to correspond with the historical development of the science, and the names of the principal investigators have been mentioned in connection with each phenomenon described.

As is usual in the case of elementary text-books, the mathematical treatment suffers from the necessity of avoiding calculus methods. In cases where the subject cannot be properly treated without the use of the calculus Professor Lommel has, however, omitted the mathematical discussion altogether, and has thus avoided most of the unsatisfactory proofs which encumber so many of our text-books. The only departure from this rule which I have noticed is in the discussion of the value of the potential at a point, on page 278. The proof there given of the fact that the potential is universally proportional to the distance from the acting mass, although common, is by no means rigorous.

The value of Professor Lommel's book is increased by the explanations that are frequently given of recent developments in experimental physics. Thus there will be found a brief but intelligible statement of Lippmann's method of color photography, and of Wiener's experiments in stationary light waves. The principle of the "drehstrom" motor is also explained. The distinctly modern character of the work, together with the general excellence of the methods of presentation, make it a matter of especial regret that the book will not be generally available for American students.

Ernest Merritt.

Volume I. *November–December, 1893.* *Number 3.*

THE

PHYSICAL REVIEW.

ON THE ELECTRIC STRENGTH OF SOLID, LIQUID, AND GASEOUS DIELECTRICS.

By Alexander Macfarlane and G. W. Pierce.

IN the course of a research on "The Disruptive Discharge of Electricity,"[1] Dr. Macfarlane obtained the result that the electrostatic gradient necessary to force a spark through a thin stratum of dielectric diminishes as the thickness increases, when air or other gas is the dielectric; but remains constant when turpentine or other insulating liquid is the dielectric. Mr. Steinmetz has recently obtained the same result,[2] and in addition has shown that a solid dielectric, such as paraffined paper, behaves in the same manner as a liquid dielectric. In order to obtain further independent data, Mr. Pierce has made a series of measurements for paraffined paper, beeswaxed paper, and kerosene oil. We had not the means of making absolute measurements, but this was immaterial, as our object was to compare these solid and liquid dielectrics with air in the most direct manner possible. To effect the comparison two discharging tables were used. On each table a pair of brass discs, about four inches in diameter, were supported parallel to one another, and the connecting-rods were attached to the poles of a

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[1] Trans. Roy. Soc. Edinb., Vol. XXVIII., p. 633, and Vol. XXIX., p. 561. Proc. R. S. E., Vol. X., p. 555; Phil. Mag., Dec., 1880; Trans. Amer. Inst. El. Eng., March, 1893.

[2] Trans. Amer. Inst. El. Eng., Feb., 1893.

Toepler-Holtz machine, so as to form two alternative paths for the discharge. One of the tables was provided with a micrometer-millimeter scale, by which the distance between the plates could be read off accurately. It was used for the air gap. The sheet, or sheets, of paraffined or beeswaxed paper was placed between the other pairs of discs, and the discs pressed together. The air interval was at the beginning made too small, so that the spark should pass through it; for the passage of a number of sparks does not alter the air stratum, while a single spark through the solid damages it permanently. The air stratum was gradually increased in thickness until the spark preferred to pass through the solid. The thickness of the air interval was obtained by reading the micrometer; the equivalent thickness of the solid was ascertained by cutting out a small piece round the hole pierced by the discharge, and measuring the thickness by means of the spherometer. The sheets of paper used were prepared by taking sheets of linen typewriter paper, dipping them into the melted paraffin or beeswax, and drawing them out so as to make the saturation as uniform as possible. In the case of the kerosene oil, the plates were fixed horizontally in a glass vessel, the lower one supported on a conducting rod passing through the bottom of the vessel, the upper one suspended from a graduated rod which moved tight in the cover of the vessel. The oil was filtered before

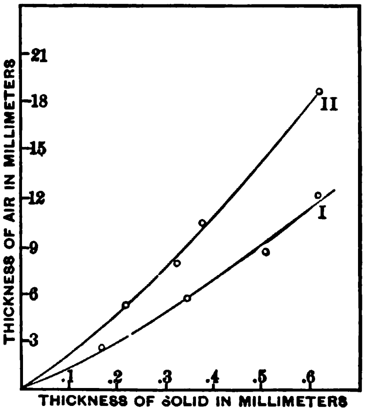

Diagram 1.

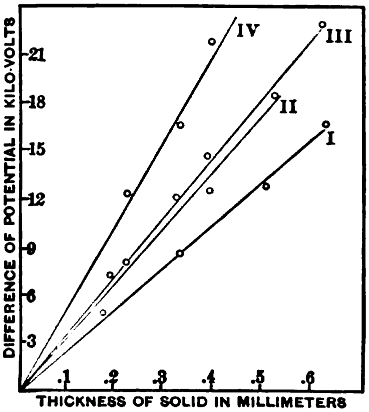

Diagram 2.

use, and the bubbles of gas formed by a discharge removed before proceeding to another comparison.

Table I. appended gives the results for paraffined paper. The value given in the second column for the thickness of the paper, and that in the third column for the equivalent thickness of air, are the averages of all the determinations for the specified number of sheets. The fourth and fifth columns give the difference of potential in kilovolts required to pass a spark through the corresponding stratum of air, that of the fourth column being obtained from the papers of Dr. Macfarlane, while that in the fifth column is from the paper of Mr. Steinmetz.

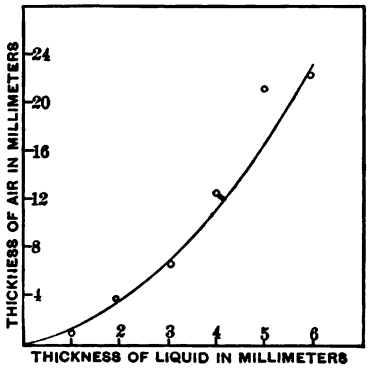

Diagram 3.

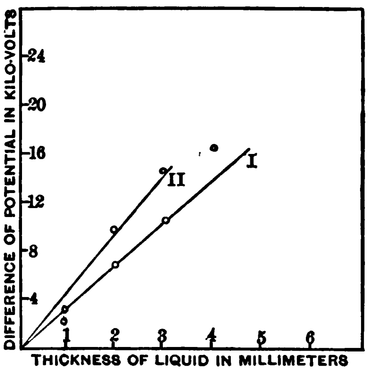

Diagram 4.

In Table II. the results for beeswaxed paper are tabulated in a similar manner.

Table III. gives the results for kerosene oil. The value entered for the equivalent stratum of air is the average of all the determinations made for the given stratum of oil.

The diagrams appended exhibit the results graphically. In Diagram 1, curve I. is for paraffined paper, and curve II. for beeswaxed paper; in Diagram 3 the curve is for kerosene oil.

The equivalent thickness of air is not proportional to the thickness of the solid or liquid, but increases more rapidly as the stratum increases. The curves are concave towards the axis of ordinates. Now the curve giving the differences of potential for different thicknesses of air is concave towards the axis of abscissæ. Diagrams 2 and 4 are designed to test whether these opposite bendings neutralize one another. In Diagram 2, curve I. gives the difference of potential for paraffined paper, and curve III. for beeswaxed paper, using Steinmetz's values of the air equivalent; while curves II. and IV. give the same, using Macfarlane's values of the air-equivalent. In Diagram 4, curve I. is for Steinmetz's values, and curve II. for Macfarlane's values. All these curves are very approximately straight lines passing through the origin.

From these and the other results mentioned above we may conclude that thin strata of a solid or liquid dielectric are equally strong whatever the thickness, while thin strata of a gaseous dielectric grow weaker as the thickness is increased. Viewed in the light of the mathematical theory of electricity in equilibrium, it is the gaseous dielectric which is anomalous. The cause does not appear to be any surface phenomenon, but rather to be due to the greater rarity of a gas which allows discharge by convection to be more readily set up. For the greater thicknesses convection is sometimes started in a liquid dielectric, and it is observed that the discharge then takes place at a lower difference of potential.

A table of electrostatic gradients is appended, Table IV., which has been compiled from this and the other investigations mentioned. Where Steinmetz has a value for the same dielectric, it is placed in a parallel column.

TABLE I.

PARAFFINED PAPER.

Number of sheets.	Thickness of paper in millimeters.	Thickness of air in millimeters.	Diff. of Potential in kilovolts (Macfarlane).	Diff. of Potential in kilovolts (Steinmetz).
1	.174	2.63	7.2	5.
2	.339	5.42	12.6	8.9
3	.506	8.47	18.6	13.
4	.620	11.76	—	16.5

TABLE II.

BEESWAXED PAPER.

Number of sheets.	Thickness of paper in millimeters.	Thickness of air in millimeters.	Diff. in Potential in kilovolts (Macfarlane).	Diff. of Potential in kilovolts (Steinmetz).
1	.214	5.18	12.3	8.2
2	.326	8.05	16.7	12.1
3	.390	10.5	22.2	14.9
4	.606	18.3	—	22.3

TABLE III.

KEROSENE OIL.

Thickness of oil in millimeters.	Thickness of air in millimeters.	Diff. of Potential in kilovolts (Macfarlane).	Diff. of Potential in kilovolts (Steinmetz).
1	1.0	3.6	2.5
2	4.0	9.9	6.9
3	6.8	15.	10.5
4	12.5	—	17.
5	20.8	—	—

TABLE IV.

ELECTRIC STRENGTH.

Dielectric.	Electrostatic gradient in kilovolts per centimeter.		Dielectric.	Electrostatic gradients in kilovolts per centimeter.	
	Macfarlane.	Steinmetz.		Macfarlane.	Steinmetz.
Oil of turpentine	94	64	Paraffined paper .	360	339
Paraffin oil . .	87	—	Beeswaxed paper .	540	—
Olive oil . . .	82	—	Air(thickness, 5 mm.)	23.8	16
Paraffin (melted)	56	81	CO_2 " "	22.7	—
Kerosene oil. .	50	—	O " "	22.2	—
Paraffin (solid) .	130	—	H " "	15.1	—
			Coal gas	22.3	—

ON A METHOD OF PHOTOGRAPHING THE MANOMETRIC FLAME WITH APPLICATIONS TO THE STUDY OF THE VOWEL *A*.

BY ERNEST MERRITT.

THE manometric flame, with its attached capsule, has long been recognized as an important aid in the study of sound. Its many applications to the investigation of stationary waves, where the sensitiveness of the flame to small changes of pressure has made it of especial service, are too well known to require mention here. In the study of the quality of sounds the manometric flame has also been found to possess many advantages, since the extreme lightness of the moving parts makes it possible for them to follow the vibrations of complex sounds with almost perfect accuracy. The simplicity and elegance of the method which is thus afforded of detecting the presence of overtones have led to its wide use for purposes of demonstration, but the applications of the method have been almost wholly confined to experiments of an illustrative character. Koenig's investigation of the vowel sounds[1] is, I believe, the only example of the use of the manometric flame in the quantitative study of timbre.

The chief difficulty which has stood in the way of a more extensive use of the manometric flame undoubtedly lies in the transient character of the images that are seen in the revolving mirror. To obtain accurate drawings of these images for use in further study is so laborious a task that few are willing to undertake it, and the drawings by Koenig, which are found in all our text-books, will probably long remain as monuments to the skill and patience of this great student of sound.

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[1] Rudolph Koenig, On Manometric Flames, Phil. Mag., Series iv., 45, p. 1; Pogg. Ann., cxlvi., p. 161. Also, Sur les notes fixes caractéristiques des diverses voyelles, Comptes Rendus, 70, p. 931.

The possibility of photographing the manometric flame, and thus obtaining a permanent and accurate record of its vibrations, does not appear to have occurred to any of the earlier experimenters. The reason for this is doubtless to be found in the comparatively undeveloped state of the photographic art at that time. With the slow and cumbersome wet-plate processes of twenty years ago success in instantaneous photography could scarcely be expected. The improvements that have been made in photographic processes during recent years are, however, of a most radical nature, and many problems are now successfully attacked by photographic methods which but a few years ago would have been deemed incapable of solution. When the attention of the writer was called to the problem of photographing the manometric flame,[1] it therefore seemed to him not improbable that the use of modern dry-plates would lead to satisfactory results. It was with this hope that the work here described was begun.

METHODS AND APPARATUS.

To obtain a photograph of the manometric flame which shall be available as a means of analysis requires (1) a lens by which an image of the flame may be thrown upon the sensitive plate, and (2) some device for producing a uniform movement of the image in a line at right angles to the direction of its vibration. This movement might be brought about by the use of a revolving mirror, or by a movement of the lens, either of which methods would doubtless have proven satisfactory. With the apparatus at hand, however, it appeared more convenient to keep both lens and image stationary, and to move the plate itself. The plate-holder, mounted in a suitable carriage, was therefore arranged to slide between guides across the back of the camera. With high speeds it was found necessary to extend these guides for about 50 cm.

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[1] By Professor O. F. Emerson, who was desirous of making use of the manometric flame in connection with certain studies in phonetics. Although the results have proven satisfactory from the standpoint of the physicist, the method has been found to be unavailable for the purpose first considered.

on either side of the camera in order to avoid the jar of sudden starting and stopping. To obtain approximately the same speed in different experiments the carriage was usually set in motion by means of a rubber band, whose tension, when once adjusted, was left unaltered. To guard against stray light all parts of the apparatus were painted black, and the experiments were performed in a room which was partially darkened.

The difficulty from a photographic standpoint in obtaining good negatives of the manometric flame will be appreciated when it is remembered that the plate must move during each vibration through a distance at least as great as the width of the flame. If the motion is slower than this the vibrations will not be sharply separated, and the negative will be valueless so far as its use in analysis is concerned. For a sound whose pitch is 500 vibrations per second the exposure at any one point of the plate must therefore be less than .002 sec., while if wider separation is desired, or if the tone studied is of higher pitch, the time of exposure must be correspondingly less. It is not surprising therefore that all attempts to photograph manometric flames of the ordinary form proved unsuccessful. When gas was used which had been highly enriched with benzine and naphthalene it was indeed possible to obtain faint negatives, after a long and tedious development, and by the use of the most sensitive plates; but all attempts to make these negatives sufficiently dense for convenient measurement proved utterly fruitless. The use of large lenses and concave mirrors to increase the brightness of the image, and the employment of a variety of substances to enrich the gas, were alike without avail.

A sufficiently brilliant flame was finally obtained, however, by the adoption of a form of jet which enabled the gas to be burned in pure oxygen. I have since found that a somewhat similar method of producing an actinic flame has been used by M. Doumer,[1] who succeeded in photographing such flames in 1886, and afterwards applied his method[2] to the study of the French vowels *i* and *u*. Since, however, the description of M. Doumer's work appears to be confined to the brief notes in the Comptes Rendus which are referred to above, I have thought that a detailed description of the apparatus which I have found most serviceable, together with a presentation of some of the results obtained, may not be devoid of interest.

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[1] Mesure de la hauteur des sons par les flammes manométriques, Comptes Rendus, 103, p. 340.

[2] Des voyelles dont le caractère est très-aigu, Comptes Rendus, 105, p. 1247.

In Fig. 1 is shown a sectional diagram of the manometric capsule and jet which were finally employed. It will be seen that the burner differs from those commonly used only in the presence of the outer tube *T*, which surrounds the jet proper and extends as high as the bottom of the flame. Oxygen was admitted to this tube at *O*, while gas entered the capsule at *G* and proceeded through the short brass tubing *AA* to the flame *F*. The flame was thus surrounded by an atmosphere of oxygen, and was found to burn with a brilliant white light, strikingly different from that of the ordinary gas flame. By adjusting the pressure of the gas and oxygen, the height and brilliancy of the flame could be kept under perfect control. On account of the intense heat it was found necessary to make the tip of the burner of platinum, and the best results were obtained when the aperture at *B* was considerably larger than is customary in manometric jets. In the burner here described the opening at *B* was a little over 1 mm. in diameter.

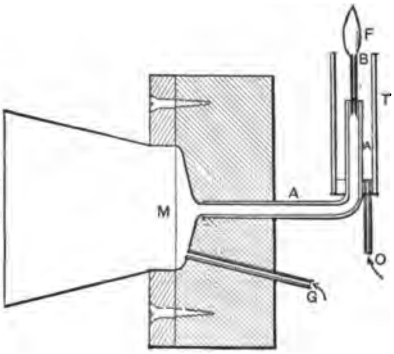

Fig. 1.

The manometric capsule, as will be seen in the figure, did not differ in any essential particular from those ordinarily used. Its size, however, was somewhat greater than is common, the diameter of the membrane being about 5 cm. The tubes leading to the jet were made as short as possible in order to reduce the effects of friction, and sudden changes in diameter were avoided. The membrane *M* was made of thin dentist's rubber. A metallic mouth-piece of the form shown served to concentrate the sound waves upon the surface of the membrane, and so intensify the effect.

With the form of apparatus described above, the flame was found to be not only extremely brilliant, and of high actinic power, but also fully as sensitive to sound vibrations as manometric flames of the ordinary form. An adjustment of pressures which made the flame about 15 mm. high was found to give the best results. No difficulty was met with in obtaining sharp negatives of the vibrating flame with any of the ordinary instantaneous plates. By enriching the gas with benzine the actinic effect of the light was considerably increased, but with the plate moving at the rate of about two meters per second, which was the speed used throughout most of these experiments, the results obtained with ordinary gas were quite satisfactory. It should be mentioned that the camera was placed at such a distance from the jet as to make the image on the plate almost equal in size to the flame itself. The lens was used with full aperture, about 4 cm., and had a focal length of 40 cm.

In making use of the above apparatus for the study of vowel sounds, two flames were ordinarily used, one being placed vertically above the other. The upper flame was set in vibration by the sound to be studied, while the lower was connected with an organ pipe of known pitch. Both flames were photographed at the same time. A means of determining the pitch of the sound studied was thus afforded which was susceptible of a high degree of accuracy, the advantage of the method lying in the fact that its results were unaffected by a change in the speed of the moving plate. Prints from two of the negatives thus obtained are shown in Figs. 1 and 2 of Plate IV.

Results.

A few of the photographs that have so far been taken will be found reproduced in Figs. 3 to 8, Plate IV., and will serve to indicate the general character of the results obtained. Figs. 3 to 6 show the forms assumed by the manometric flame when set in vibration by the vowel sounds of *a* and *o* as they occur in the words *father*, *law*, *hat*, and *know*. In each case the vowel has been sung at four different pitches, corresponding respectively to $C_2(128)$, E_2, G_2, and $C_3(256)$. In Fig. 7 the four vowels are represented upon the same plate for convenience in comparison, the pitch being 128 in each case. Fig. 8 shows the effect on the flame of the humming sound of *m*, also sung at four different pitches. It should be mentioned that the photographs shown in Figs. 3 to 8 were taken only to illustrate the results obtained. Examples of photographs actually used for measurement are shown in Figs. 1 and 2.

Although the flame was plainly visible to the eye throughout its entire length, the actinic effect appears to be confined to the upper portions. In some of the photographs a faint band of light may be detected which extends for some distance below the brighter portions of the image and indicates the position of the bottom of the flame. But in most cases it is only the light from the upper half of the flame that has left any trace upon the plate.

It will be seen that in the majority of cases an inspection of the photograph is sufficient to determine the pitch of the most prominent harmonic. In Fig. 1, for example, the fundamental is evidently accompanied by a harmonic of the seventh order, while in other cases the frequency of the overtone is three, four, or five times that of the fundamental. It is clear, also, that the overtones are always harmonics; a fact which has indeed been doubted, but which scarcely requires experimental verification. In some cases, however, peculiarities in the form assumed by the flame lead to considerable uncertainty as to the order of the principal harmonic. Examples of such cases are seen in Figs. 3 and 6. If the indistinct form of the image is due to an improper adjustment of the flame, as in the upper image of Fig. 7, it can easily be remedied. But in several instances the difficulty appears to be inherent in the character of the sound itself, and no adjustment of sensitiveness has resulted in marked improvement.

It will be evident, upon a careful inspection of the figures, that the results cannot in general be explained by the presence of a single overtone. According to the ordinary theory of vowel formation,[1] we should, in fact, expect that at least two harmonics would usually be present; namely, those which are most nearly identical in pitch with the characteristic tone of the vowel considered. The relative amplitudes of the two will depend upon the amounts by which they differ in pitch from the characteristic, and a complete study of the vowel sounds would require not only a determination of these amplitudes, but also a knowledge of the phases of the various components. The manometric flame is unfortunately not suited for such determinations, for we can feel no certainty that the height of the flame bears a constant ratio to the pressure on the membrane. M. Doumer is indeed of the opinion that the manometric flame may be used to determine differences of phase.[2] His experiments were, however, confined to the case where the only overtone present was the octave of the fundamental; and although the results under these conditions were good, I doubt whether the method would prove satisfactory with more complex sounds.

In the case of vowels which have but a single characteristic tone, the manometric flame may nevertheless be of considerable use. It enables the pitch of the most prominent overtone to be determined in almost all cases, and so indicates which of the harmonics lies nearest to the characteristic tone of the vowel. By a series of experiments in which the same vowel is sung at a variety of different pitches, the ordinary theory may thus be tested, and the characteristic tone determined. With this object in view some fifty negatives were taken when the manometric flame was set in vibration by the vowel *a* as pronounced in the word *father*. The results of these experiments are shown in the table on the following page.

It will be seen that seven different voices have been studied, varying in character from bass to high soprano, while the range of pitches that is represented extends over two and a half octaves. The negatives from which these results are computed were taken at irregular intervals during a period of about two years.

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[1] Helmholtz, Tonempfindungen.

[2] Comptes Rendus, 105, p. 222.

THE VOWEL *A* AS IN *FATHER*.

Character of voice.	Pitch of fundamental.	Order of harmonic.	Pitch of harmonic.	Deviation from 736.
(A.) Bass	102	7	714	— 22
“ “	196	4	768	+ 32
(B.) Baritone	113*	7	791	+ 55
“ “	115*	7	805	+ 69
(C.) Baritone	202	4	808	+ 72
“ “	257	3	771	+ 35
(D.) High baritone	125	6	750	+ 14
“ “ “	127	6	762	+ 26
“ “ “	128	6	768	+ 32
“ “ “	131	6	786	+ 50
“ “ “	135	6	810	+ 74
“ “ “	148*	5	740	+ 4
“ “ “	148*	5	740	+ 4
“ “ “	150	5	750	+ 14
“ “ “	150*	5	750	+ 14
“ “ “	156	5	780	+ 44
“ “ “	158	5 – 4(?)	790, 632	+ 50, – 104
“ “ “	165	4	660	– 76
“ “ “	169	4	676	– 60
“ “ “	183*	4	732	– 4
“ “ “	188	4	752	+ 16
“ “ “	189	4	756	+ 20
“ “ “	191	4	764	+ 28
“ “ “	193	4	772	+ 36
“ “ “	198	4	792	+ 56
“ “ “	199	4	796	+ 60
“ “ “	200	4	800	+ 64
“ “ “	208	4 – 3(?)	832, 624	+ 96, – 112
“ “ “	217*	3	651	– 85
“ “ “	226	3	678	– 58
“ “ “	240	3	720	– 16
“ “ “	241	3	723	– 13
“ “ “	247*	3	741	+ 5
“ “ “	252*	3	756	+ 20
“ “ “	250	3	750	+ 14
“ “ “	292	3	876	+ 140
“ “ “	322	2	644	– 92
“ “ “	379	2	758	+ 22
(E.) High tenor	201	4	804	+ 68
“ “ “	222	4	888	+ 152
“ “ “	263	3	789	+ 53
“ “ “	295	3	885	+ 149
“ “ “	333	2	666	– 70
“ “ “	402	2	804	+ 68
“ “ “	539	2	1078	+ 342
(F.) Soprano	197	5	985	+ 249
“ “	262	3	786	+ 50
“ “	267	3	801	+ 65
“ “	434	2	868	+ 132
“ “	518	2	1036	+ 300
“ “	667	1	667	– 69
(G.) A child of eight years	275	3	825	+ 89
“ “ “ “ “ “	394	2	788	+ 52
“ “ “ “ “ “	440	2	880	+ 44

* Observations marked with a star are from negatives which were considered especially sharp and distinct.

No attempt was made to sing the vowel at defined pitches, the only object being to obtain as great a variety as possible. It will thus be seen that observations have sometimes been duplicated on different days. The uniform agreement of the results in all such cases affords convincing evidence of the reliability of the method. It should be mentioned in this connection that the possibility of selective resonance in the apparatus was tested in a variety of ways. Two manometric capsules, differing widely in size and proportions, were found to give identical results, while changes in the pressure of the gas, and in the burner itself, were also without effect. Although the distinctness of the images was considerably altered by changes in the adjustment of the flame, the essential features remained in all cases the same.

A consideration of the results in the case of the voices marked *A*, *B*, *C*, and *D* in the table, has led to the adoption of the number 736 as the most probable value of the pitch of the characteristic tone. In the last column of the table will be found in each case the difference between this number and the pitch of the most prominent harmonic. It will be seen that in the observations on these four voices the difference in no case exceeds half the fundamental. In those cases where the order of the harmonic is in doubt, the difference reaches its greatest values, while the results obtained from negatives that were considered especially distinct show a close agreement in pitch between the harmonic and the characteristic tone. The results in the case of these voices are thus seen to be in complete accord with the ordinary theory.

The characteristic tone of the last three voices appears to be higher than that of the first four, although the observations are not sufficient to determine its pitch accurately. Thus with a fundamental whose pitch is 197, the harmonic in the case of the soprano voice is 985, while the baritone voice *D*, with practically the same fundamental, has an overtone whose frequency is only 792. In the observations on the voices *E* and *G* there are also indications of a higher characteristic.

It will be remembered that the results obtained by Helmholtz[1] and Willis in the case of the vowel *a* indicate a characteristic whose pitch is about 990, while the pitch assigned by Koenig[1] is 896. The low value obtained from my own observations seems to show that the American pronunciation of this vowel is noticeably different from that customary in European languages. The differences in the results obtained with so common a vowel call attention in a striking manner to the necessity for some means of indicating vowel quality which shall be more definite than the use of a letter of the alphabet.

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[1] Tonempfindungen.

It may not be out of place to point out a few of the applications of the manometric flame which are rendered possible by the use of photography. M. Doumer has already called attention[2] to the advantages of the manometric flame as a means of measuring pitch. Two flames, set in vibration by the sounds whose pitches are to be compared, may be photographed upon the same plate, and a measurement of the negative leads at once to a determination of the relative frequencies. Although this method is probably not capable of the high degree of refinement that is reached by the use of Lissajous' figures, it has the advantage of being applicable to any vibrating body which produces an audible sound. Used in connection with a standard fork, the manometric flame also forms a simple optical chronograph, by which short-time intervals may be measured with great accuracy. The advantages due to the absence of friction will at once be appreciated.

For use in the lecture-room the form of flame here described is found to be a great improvement over those usually employed. With a height of 10 cm., the flame is still highly sensitive to vocal sounds, and is so brilliant that it may be projected, by the aid of a lens and revolving mirror, so as to be clearly seen by an audience of two hundred. It is by no means necessary to construct a special burner for experiments of this kind, since an ordinary oxygen blast lamp, whose central jet was connected with a manometric capsule, has been found to give excellent results.

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[1] Comptes Rendus, 70, p. 931. Also Phil Mag., Series IV., Vol. 45.

[2] Comptes Rendus, 103, p. 340.

Summary.

The more important conclusions that have been reached in the foregoing paper may be summarized as follows: —

1. The actinic effect of the manometric flame may be greatly increased by the use of a burner in which the flame is surrounded by pure oxygen. With this form of burner photographs of the vibrating flame may readily be obtained upon a moving plate.

2. The brilliancy of the flame obtained in this way increases its value in lecture experiments, while its actinic effect enables it to be used for the measurement of small intervals of time, as well as for the determination of pitch and timbre.

3. By the aid of photographs of the manometric flame, the conclusion is reached that the characteristic tone of the vowel *a*, as it occurs in the word *father*, has a frequency which is not far from 736 vibrations per second.

PHYSICAL LABORATORY OF CORNELL UNIVERSITY,
June, 1893.

PLATE IV.

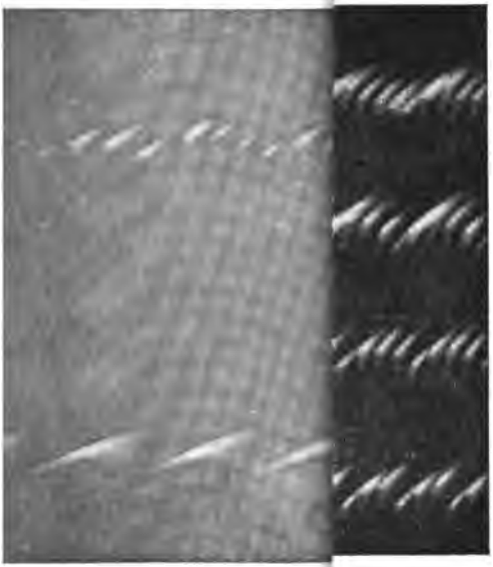

Fig. 1.

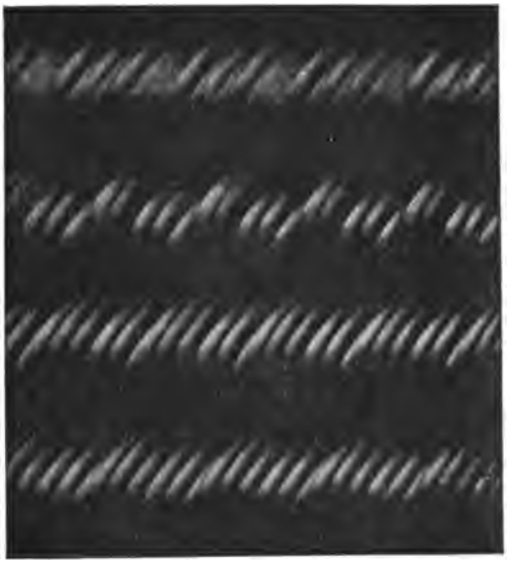

Fig. 7.
Four vowels at the pitch 128.

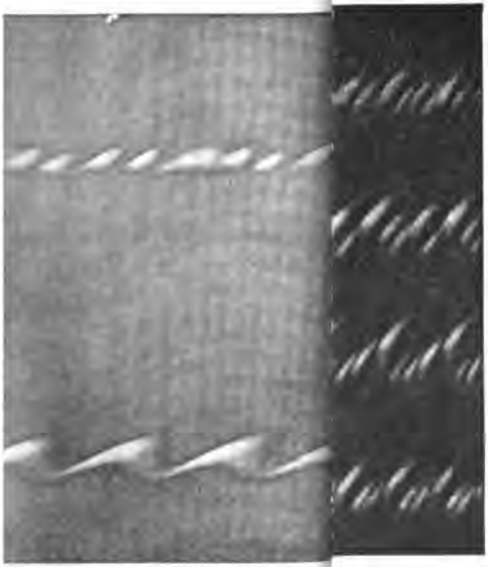

Fig. 2.

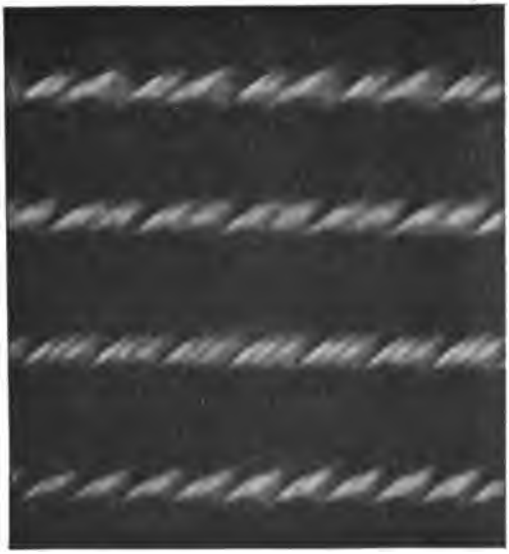

Fig. 8.
M.

GENERAL DISCUSSION OF THE CURRENT FLOW IN TWO MUTUALLY RELATED CIRCUITS CONTAINING CAPACITY.[1]

II.

By Frederick Bedell and Albert C. Crehore.

General Solution for the Currents in Two Mutually Related Circuits with no Condenser.

In symbolic notation (see Johnson's Differential Equations, Chap. V.), equations (26) and (29) may be written

$$(34)\quad i_1 = \frac{1}{D^2 + \dfrac{R_1L_2 + R_2L_1}{L_1L_2 - M^2}D + \dfrac{R_1R_2}{L_1L_2 - M^2}}\,\frac{[R_2f(t) + L_2f'(t)]}{L_1L_2 - M^2},$$

and

$$(35)\quad i_2 = \frac{-1}{D^2 + \dfrac{R_1L_2 + R_2L_1}{L_1L_2 - M^2}D + \dfrac{R_1R_2}{L_1L_2 - M^2}}\,\frac{Mf'(t)}{L_1L_2 - M^2}.$$

Resolving the inverse operator into partial fractions, we have the identical equation

$$(36)\quad \frac{1}{D^2 + \dfrac{R_1L_2 + R_2L_1}{L_1L_2 - M^2}D + \dfrac{R_1R_2}{L_1L_2 - M^2}}$$

$$= \frac{L_1L_2 - M^2}{\sqrt{(R_1L_2 + R_2L_1)^2 - 4R_1R_2(L_1L_2 - M^2)}}\left\{\frac{1}{D + r_1} - \frac{1}{D + r_2}\right\}.$$

The radical expression which occurs in equation (36) may be written

$$\sqrt{(R_1L_2 + R_2L_1)^2 - 4R_1R_2(L_1L_2 - M^2)} = \sqrt{(R_1L_2 - R_2L_1)^2 + 4R_1R_2M^2}.$$

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[1] A paper presented before the World's Congress of Electricians, Chicago, Ill., August, 1893.

For simplification, the abbreviations have been used:

$$(37)\qquad \tau_1=\frac{R_1L_2+R_2L_1-\sqrt{(R_1L_2-R_2L_1)^2+4R_1R_2M^2}}{2(L_1L_2-M^2)},$$

and

$$(38)\qquad \tau_2=\frac{R_1L_2+R_2L_1+\sqrt{(R_1L_2-R_2L_1)^2+4R_1R_2M^2}}{2(L_1L_2-M^2)}.$$

Placing (36) in (34), we obtain

$$(40)\qquad i_1=\frac{1}{\sqrt{(R_1L_2-R_2L_1)^2+4R_1R_2M^2}}\left\{\frac{R_2f(t)+L_2f'(t)}{D+\tau_1}-\frac{R_2f(t)+L_2f'(t)}{D+\tau_2}\right\}.$$

Similarly (35) becomes

$$(41)\quad i_2=\frac{M}{\sqrt{(R_1L_2-R_2L_1)^2+4R_1R_2M_2}}\left\{\frac{f'(t)}{D+\tau_1}-\frac{f'(t)}{D+\tau_2}\right\}.$$

Now the linear equation of the first order may be written

$$y=\frac{1}{D+a}f(x),$$

and its solution is known to be

$$y=\epsilon^{-ax}\int\epsilon^{ax}f(x)\,dx+c\epsilon^{-ax}.$$

Hence we have

$$(42)\qquad \frac{f(x)}{D+a}=\epsilon^{-ax}\int\epsilon^{-ax}f(x)\,dx+c\epsilon^{-ax}.$$

Replacing $f(x)$ by $R_2f(t)$, and a by τ_1 in this general formula, we have

$$(43)\qquad \frac{R_2f(t)}{D+\tau_1}=R_2\epsilon^{-\tau_1t}\int\epsilon^{\tau_1t}f(t)\,dt+c\epsilon^{-\tau_1t}.$$

Similarly we should find

$$(44)\qquad \frac{L_2f'(t)}{D+\tau_1}=L_2\epsilon^{-\tau_1t}\int\epsilon^{\tau_1t}f'(t)\,dt+c\epsilon^{-\tau_1t},$$

and also

$$(45)\qquad \frac{Mf'(t)}{D+\tau_1}=M\epsilon^{-\tau_1t}\int\epsilon^{\tau\,t}f'(t)\,dt+c\epsilon^{-\tau_1t}.$$

Substituting these values in (40) and (41), we obtain for the primary current,

$$(46)\quad i_1 = \frac{1}{\sqrt{(R_1L_2-R_2L_1)^2+4R_1R_2M^2}}\left\{\epsilon^{-\tau_1 t}\int\epsilon^{\tau_1 t}[R_2f(t)+L_2f'(t)]dt - \epsilon^{-\tau_2 t}\int\epsilon^{\tau_2 t}[R_2f(t)+L_2f'(t)]dt\right\} + c_1\epsilon^{-\tau_1 t}+c_2\epsilon^{-\tau_2 t};$$

and for the secondary current

$$(47)\quad i_2 = \frac{-M}{\sqrt{(R_1L_2-R_2L_1)^2+4R_1R_2M^2}}\left\{\epsilon^{-\tau_1 t}\int\epsilon^{\tau_1 t}f'(t)dt - \epsilon^{-\tau_2 t}\int\epsilon^{\tau_2 t}f'(t)dt\right\} + c_3\epsilon^{-\tau_1 t}+c_4\epsilon^{-\tau_2 t}.$$

These equations, (46) and (47), are the complete solutions for the current flowing in two mutually related circuits, containing no condenser, with no assumption in regard to magnetic leakage. It is noticed, (37) and (38), that there are two time constants, as in the case of capacity and self-induction in a single circuit; but no oscillatory effect can be obtained in this case, because the expression under the radical is always real for all values of the constants R, M, or L. Further discussion will be deferred until the equations for the other cases are found.

GENERAL SOLUTION FOR TWO MUTUALLY RELATED CIRCUITS IN WHICH THERE IS NO CONDENSER, ASSUMING NO MAGNETIC LEAKAGE.

Upon the introduction of the condition of no leak in equations (26) and (29), *i.e.* equating $L_1L_2-M^2$ to zero, they become

$$(48)\qquad \left[D+\frac{R_1R_2}{R_1L_2+R_2L_1}\right]i_1 = \frac{R_2f(t)+L_2f'(t)}{R_1L_2+R_2L_1},$$

and

$$(49)\qquad \left[D+\frac{R_1R_2}{R_1L_2+R_2L_1}\right]i_2 = \frac{-Mf'(t)}{R_1L_2+R_2L_1}.$$

The solution of these linear equations of the first order may be written by means of the general formula (42), and we have

$$(50)\quad i_1 = \frac{\epsilon^{-\tau t}}{R_1L_2 + R_2L_1}\int \epsilon^{\tau t}[R_2f(t) + L_2f'(t)]\,dt + c_1\epsilon^{-\tau t},$$

and

$$(51)\quad i_2 = -\frac{M}{R_1L_2 + R_2L_1}\epsilon^{-\tau t}\int \epsilon^{\tau t}f'(t)\,dt + c_2\epsilon^{-\tau t},$$

$$(52)\quad \text{where} \qquad \tau = \frac{R_1R_2}{L_1R_2 + L_2R_1}.$$

These equations might have been written from the general solutions (46) and (47), but it is easier to derive them independently from the differential equations. It is remarkable how much the consideration of no magnetic leakage simplifies the mathematical expression of the results.

Before entering upon the discussion of these results, it is thought best to obtain the equations for the case in which there is a condenser in one of the circuits, when there is no magnetic leakage.

GENERAL SOLUTION FOR TWO MUTUALLY RELATED CIRCUITS WITH A CONDENSER IN ONE CIRCUIT, ASSUMING NO MAGNETIC LEAKAGE.

The solutions may be obtained from the differential equations (30) to (33), after equating $L_1L_2 - M^2$ to zero. With this supposition, when we have the secondary condenser only, the equations become

(53)

$$i_1 = \frac{1}{D^2 + \dfrac{\left(\dfrac{L_1}{C_2} + R_1R_2\right)}{R_1L_2 + R_2L_1}D + \dfrac{R_1}{C_2(R_1L_2 + R_2L_1)}}\;\frac{\dfrac{1}{C_2}f(t) + R_2f'(t) + L_2f''(t)}{R_1L_2 + R_2L_1},$$

(54)

$$i_2 = \frac{-1}{D^2 + \dfrac{\dfrac{L_1}{C_2} + R_1R_2}{R_1L_2 + R_2L_1}D + \dfrac{R_1}{C_2(R_1L_2 + R_2L_1)}}\;\frac{Mf''(t)}{R_1L_2 + R_2L_1}.$$

With the primary condenser only, they become

$$(55)\quad i_1 = \frac{1}{D^2 + \dfrac{\dfrac{L_2}{C_1} + R_1R_2}{R_1L_2 + R_2L_1}D + \dfrac{R_2}{C_1(R_1L_2 + R_2L_1)}} \, \frac{R_2f'(t) + L_2f''(t)}{R_1L_2 + R_2L_1},$$

and

$$(56)\quad i_2 = \frac{-1}{D^2 + \dfrac{\dfrac{L_2}{C_1} + R_1R_2}{R_1L_2 + R_2L_1}D + \dfrac{R_2}{C_1(R_1L_2 + R_2L_1)}} \, \frac{Mf''(t)}{R_1L_2 + R_2L_1}.$$

Resolving the inverse operator in (53) and (54) into partial fractions, we have the identity

$$(57)\quad \frac{1}{D^2 + \dfrac{\left(\dfrac{L_1}{C_2} + R_1R_2\right)}{R_1L_2 + R_2L_1}D + \dfrac{R_1}{R_1L_2 + R_2L_1}}$$

$$= \frac{R_1L_2 + R_2L_1}{\sqrt{\left(\dfrac{L_1}{C_2} + R_1R_2\right)^2 - 4R_1(R_1L_2 + R_2L_1)}} \left\{ \frac{1}{D + \tau_1''} - \frac{1}{D + \tau_2''} \right\},$$

where, for abbreviation,

$$(58)\quad \tau_1'' = \frac{\dfrac{L_1}{C_2} + R_1R_2 + \sqrt{\left(\dfrac{L_1}{C_2} + R_1R_2\right)^2 - 4R_1(R_1L_2 + R_2L_1)}}{2(R_1L_2 + R_2L_1)},$$

$$(59)\quad \tau_2'' = \frac{\dfrac{L_1}{C_2} + R_1R_2 - \sqrt{\left(\dfrac{L_1}{C_2} + R_1R_2\right)^2 - 4R_1(R_1L_2 + R_2L_1)}}{2(R_1L_2 + R_2L_1)}.$$

Equations (55) and (56), for the case of the primary condenser only, may be similarly treated by interchanging R_1, L_1, C_1, τ_1', τ_2', with R_2, L_2, C_2, τ_2'', τ_2'', where, for the primary condenser, we have the abbreviations

$$(60)\quad \tau_1' = \frac{\dfrac{L_1}{C_2} + R_1R_2 + \sqrt{\left(\dfrac{L_2}{C_1} + R_1R_2\right)^2 - 4R_2(R_1L_2 + R_2L_1)}}{2(R_1L_2 + R_2L_1)}.$$

$$(61)\quad \tau_2' = \frac{\dfrac{L_2}{C_1} + R_1R_2 - \sqrt{\left(\dfrac{L_2}{C_1} + R_1R_2\right)^2 - 4R_2(R_1L_2 + R_2L_1)}}{2(R_1L_2 + R_2L_1)}.$$

This method of obtaining the solution which has previously been given, (34) to (47), finally gives the integral equations in the case of one condenser with the consideration of no leak.

For the case in which there is a condenser in the secondary and no leak, the solutions are

$$(62)\ i_1 = \frac{1}{\sqrt{\left(\frac{L_1}{C_2} + R_1R_2\right)^2 - 4R_1(R_1L_2 + R_2L_1)}} \left\{ \epsilon^{-\tau_2''t} \int {}^{\tau_2''t} \left[\frac{1}{C_2} f(t) \right.\right.$$

$$\left.\left. + R_2 f'(t) + L_2 f''(t) \right] dt + \epsilon^{-\tau_1''t} \int \epsilon^{\tau_1''t} \left[\frac{1}{C_2} f(t) + R_2 f'(t) + L_2 f''(t) \right] dt \right\}$$

$$+ c_1 \epsilon^{-\tau_1''t} + c_2 \epsilon^{-\tau_2''t}.$$

$$(63)\ i_2 = \frac{M}{\sqrt{\left(\frac{L_1}{C_2} + R_1R_2\right)^2 - 4R_1(R_1L_2 + R_2L_1)}}$$

$$\left\{ \epsilon^{-\tau''t} \int \epsilon^{\tau_1''t} f''(t)\, dt - \epsilon^{-\tau_2''t} \int \epsilon^{\tau_2''t} f''(t)\, dt \right\} + c_3 \epsilon^{-\tau_1''t} + c_4 \epsilon^{-\tau_2''t}.$$

When there is a condenser in the primary alone, and no leak, the solutions are

$$(64)\ i_1 = \frac{1}{\sqrt{\left(\frac{L_2}{C_1} + R_1R_2\right)^2 - 4R_2(R_1L_2 + R_2L_1)}}$$

$$\left\{ \epsilon^{-\tau_2't} \int \epsilon^{\tau_2't} [R_2 f'(t) + L_2 f''(t)]\, dt - \epsilon^{-\tau_1't} \int \epsilon^{\tau_1't} [R_2 f'(t) + L_2 f''(t)]\, dt \right\}$$

$$+ c_5 \epsilon^{-\tau_1't} + c_6 \epsilon^{-\tau_2't}.$$

$$(65)\ i_2 = \frac{M}{\sqrt{\left(\frac{L_2}{C_1} + R_1R_2\right)^2 - 4R_2(R_1L_2 + R_2L_1)}}$$

$$\left\{ \epsilon^{-\tau_1't} \int \epsilon^{\tau_2't} f''(t)\, dt - \epsilon^{-\tau_2't} \int \epsilon^{\tau_2't} f''(t)\, dt \right\} + c_7 \epsilon^{-\tau_1't} + c_8 \epsilon^{-\tau_2't}.$$

We have now obtained the equations for current flow under various conditions in regard to magnetic leakage and the location of condensers. The solutions have thus far been general; that is, there have been no limitations in regard to the nature of the impressed electromotive force, which may be any function whatsoever of the time. In (46) and (47) we have the expressions for current flow when there are no condensers, without limitation in regard to there being no magnetic leakage. In (50) and (51) we have the same simplified by the assumption of no leakage. Equations (62) and (63) are the solutions in case of no leak and a secondary condenser; and (64) and (65) are the same for a primary condenser. These general solutions will now be interpreted in turn for certain particular impressed electromotive forces, after which the solution in case of two condensers and no assumption as to absence of magnetic leak will be taken up for an harmonic impressed electromotive force.

DISCUSSION OF THE GENERAL SOLUTION FOR THE CURRENTS IN TWO MUTUALLY RELATED CIRCUITS WITH NO CONDENSER.

Equations (46) and (47) are the general expressions for the current flowing in two mutually related circuits due to any electromotive force whatsoever impressed upon one of them. Each equation consists of a particular integral and a complimentary function containing two arbitrary constants of integration to be determined according to the imposed conditions. In order to perform the operations indicated in the particular integrals, in which the electromotive force is expressed as $f(t)$, it is necessary to assume the electromotive force to be some particular function of the time.

GENERAL CASE OF "MAKE" OR "BREAK."

Ordinarily this would be the case in which initially the two currents are zero and a certain electromotive force is suddenly impressed; or, the case in which initially the two currents have certain assigned values, and the electromotive force is suddenly reduced to zero. All the cases of make or break, that is, the introduction or the removal of the electromotive force, may be generally stated thus: the initial conditions are a primary current I' and secondary current I'', due to some primary impressed electromotive force, the exact nature of which is immaterial; this electromotive force is suddenly changed to a certain known electromotive force. The initial or final values of any of these electromotive forces or currents may be zero.

Let us suppose that the final value of the impressed electromotive force is a constant, $e=f(t)=E'$. Substituting $f(t)=E'$, and $f'(t)=0$ in the general equations (46) and (47), and performing the indicated integrations, we obtain for the primary and secondary currents,

$$i_1=\frac{E'}{R_1}+c_1\epsilon^{-\tau_1 t}+c_2\epsilon^{-\tau_2 t}, \tag{66}$$

$$i_2=c_3\epsilon^{-\tau_1 t}+c_4\epsilon^{-\tau_2 t}. \tag{67}$$

The arbitrary constants of integration in these complementary functions are to be obtained according to the conditions of the problem. Let I stand for the final steady value $\frac{E'}{R_1}$ of the primary current. Counting time from the time of alteration of the primary electromotive force, we have

(68) When $t=0$, $\quad i_1=I'=I+c_1+c_2$,
and $\quad i_2=I''=c_3+c_4$.

This gives two equations in which there are four unknown arbitrary constants to be determined, and evidently two more equations must be obtained before they can be found. These equations may be formed from the consideration of the quantities of electricity which will flow in the two circuits while the magnetic field is changing, on account of the change in the impressed electromotive force. The number of lines initially threading the primary circuit is L_1I' due to the primary current, plus MI'' lines due to the secondary; finally, when the primary current has the steady value I, the number of lines will be L_1I. The quantity of electricity (in C. G. S. units) which will flow in the primary, *due to the change in the magnetic field,* will be equal to the change in lines divided by the resistance, or

$$Q_1=\frac{L_1I'+MI''-L_1I}{R_1}.$$

Similarly, the initial number of lines threading the secondary will be $L_2I''+MI'$, which will change to the final value MI; whence

$$Q_2=\frac{L_2I''+MI'-MI}{R_2}.$$

Now these values of the quantities of electricity, in primary and secondary, may be obtained from the integrals of the current equations. The integrals of (66) and (67) between the limits zero and infinity, will give the quantities of electricity which will flow during an infinite time from the time of changing the impressed electromotive force; thus, from (66),

$$\int_{t=0}^{t=\infty} i_1 dt = It_\infty + \frac{c_1}{\tau_1} + \frac{c_2}{\tau_2}.$$

Now the first term in the second member is the quantity which will flow, due to the final steady current $I = \frac{E'}{R_1}$, and the remaining two terms represent the quantity which will flow, due to the change in the magnetic field, or

$$Q_1 = \frac{c_1}{\tau_1} + \frac{c_2}{\tau_2}.$$

The secondary flow, from (67), is

$$Q_2 = \int_{t=0}^{t=\infty} i_2 dt = \frac{c_3}{\tau_1} + \frac{c_4}{\tau_2}.$$

This gives us the two additional equations containing the unknown arbitrary constants. From these two equations and the two before obtained (68), the four constants are found to be

$$c_1 = \frac{\tau_1(\tau_2 Q_1 + I - I')}{\tau_2 - \tau_1},$$

$$c_2 = \frac{\tau_2(I' - I - \tau_1 Q_1)}{\tau_2 - \tau_1},$$

$$c_3 = \frac{\tau_1(\tau_2 Q_2 - I'')}{\tau_2 - \tau_1},$$

$$c_4 = \frac{\tau_2(I'' - \tau_1 Q_2)}{\tau_2 - \tau_1}.$$

Equations (66) and (67), with these values substituted for the arbitrary constants of integration, give the values of primary and secondary currents at any time after the alteration of the primary impressed electromotive force. The values of τ_1 and τ_2 so depend upon the constants of the circuits that they are always real, and so the change of the currents from their initial to final values is gradual and non-oscillatory.

The nature of this change in the currents is shown by the typical curves in Fig. 1 representing a case in which the primary current is changed from a value *OA* to a final value *OB*, the secondary current rising from zero to a maximum, and gradually dying away to zero again. In all cases where the primary is either increased or decreased from one steady value to another by make or by break, the secondary current curve would have a shape similar to that shown. The primary current curve as shown is typical for any case of decrease, either to a finite steady value or to zero; if inverted, it would show the change for a corresponding increase.

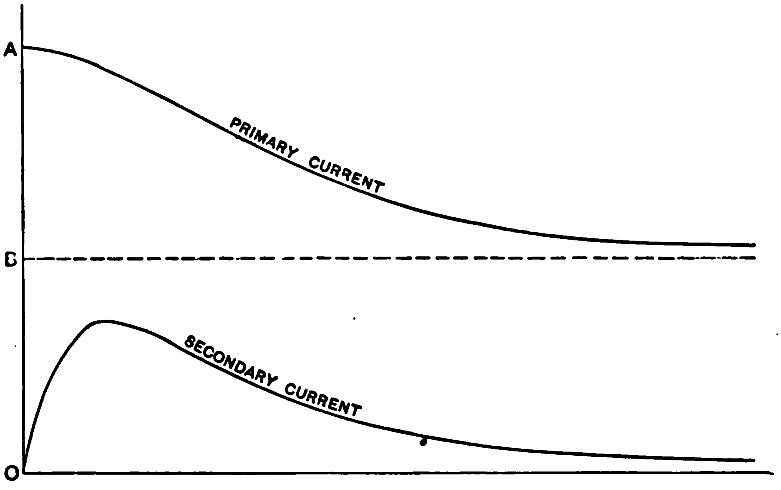

Fig. 1.

CASE OF NO MAGNETIC LEAKAGE.

In the hypothetical case of no magnetic leakage the equations take the simpler forms of (50) and (51). According to these equations, the change of the primary and secondary currents from initial to final values, due to a change of the primary impressed electromotive force from its initial to a final steady value, would be represented by exponential curves. Fig. 2 typically represents such a change. The primary current is changed from a value *OA* to a final steady value *OB* in the opposite direction, the total change being represented by *AB*. The secondary current is represented by an exponential curve with initial value *OC*, and final value zero. Now in the case supposed, the secondary current will be initially zero, and to follow this exponential law it would have to immediately assume the value *OC*. Evidently this is impossible, and it shows that the case of absolutely no magnetic leakage is hypothetical. The dotted line shows the nature of the rise from zero, giving a curve as that shown in Fig. 1.

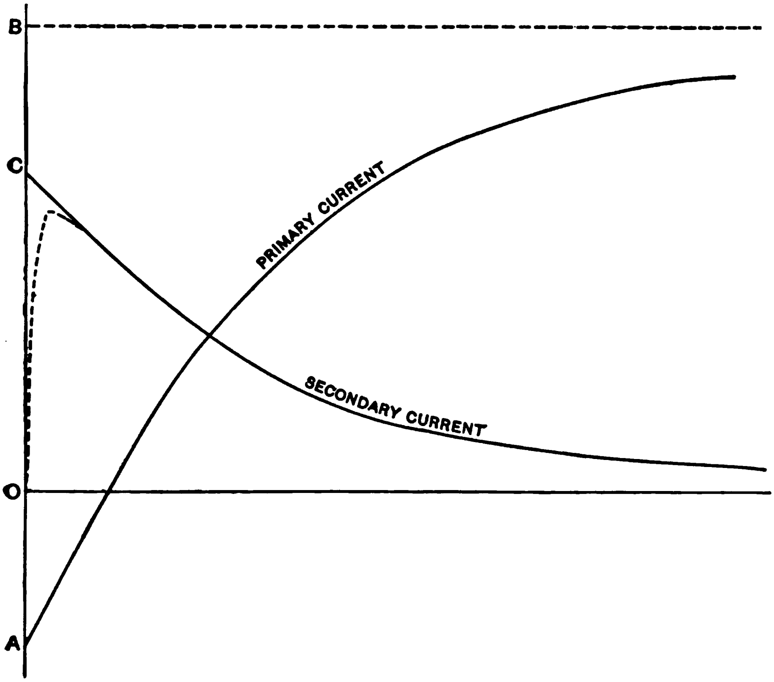

Fig. 2.

Discussion of Case of Mutually Related Circuits containing a Condenser.

Let us first consider the case in which there is a condenser in the secondary circuit. The general case of make or break will be treated as before by assuming the primary and secondary currents to be initially I' and I'' respectively, and the impressed electromotive force to be altered to a constant value E'. The expressions for the currents in the primary and secondary at any time t after the change, may be found directly from equations (62) and (63) by substituting $f(t)=E'$, $f'(t)=0$, and $f''(t)=0$. Making these substitutions and performing the integrations indicated, we obtain

$$(69) \qquad i_1 = \frac{E'}{R_1} + c_1\epsilon^{-\tau_1''t} + c_2\epsilon^{-\tau_2''t},$$

$$(70) \qquad i_2 = c_3\epsilon^{-\tau_1''t} + c_4\epsilon^{-\tau_2''t}.$$

These equations are similar to (66) and (67) already discussed, and when the constants τ_1'' and τ_2'' are real, the phenomena attending the make or break do not differ from those already referred to, illustrated in Fig. 1, and need no further explanation.

OSCILLATORY CASE.

The constants of the two circuits may have such values, however, that the constants τ_1'' and τ_2'' are imaginary, in which case the equations (69) and (70) may be transformed, by means of the exponential values of the sine and cosine, into a real form. By referring to the values of the constants τ_1'' and τ_2 given in (58) and (59), we can note whether these values are real or imaginary, and whether or not a transformation is necessary. When $\left(\frac{L_1}{C_2} + R_1R_2\right)^2$ is greater than $4\,R_1(R_1L_2+R_2L_1)$, the values of τ_1'' and τ_2'' are real, and the equations (69) and (70) may be interpreted as (66) and (67); that is, there is no oscillation. When $\left(\frac{L_1}{C_2}+R_1R_2\right)^2$ is less than $4\,R_1(R_1L_2+R_2L_1)$, the values of τ_1'' and τ_2'' become imaginary. Equations (69) and (70) can be transformed, however, into the real forms,

$$(71) \qquad i_1 = \frac{E'}{R_1} + A_1\epsilon^{-pt}\sin(\alpha t + \Phi_1),$$

$$(72) \qquad i_2 = A_2\epsilon^{-pt}\sin(\alpha t + \Phi_2).$$

In the equations A_1, A_2, Φ_1, Φ_2, are constants of integration, which may be determined from the supposed conditions; p and α are constants depending upon the constants of the circuit, thus:

$$p = \frac{\frac{L_1}{C_2} + R_1R_2}{2\,(R_1L_2 + R_2L_1)}, \tag{73}$$

$$\alpha = \frac{\sqrt{\left(\frac{L_1}{C_2} + R_1R_2\right)^2 - 4R_1(R_1L_2 + R_2L_1)}}{2\,(R_1L_2 + R_2L_1)}. \tag{74}$$

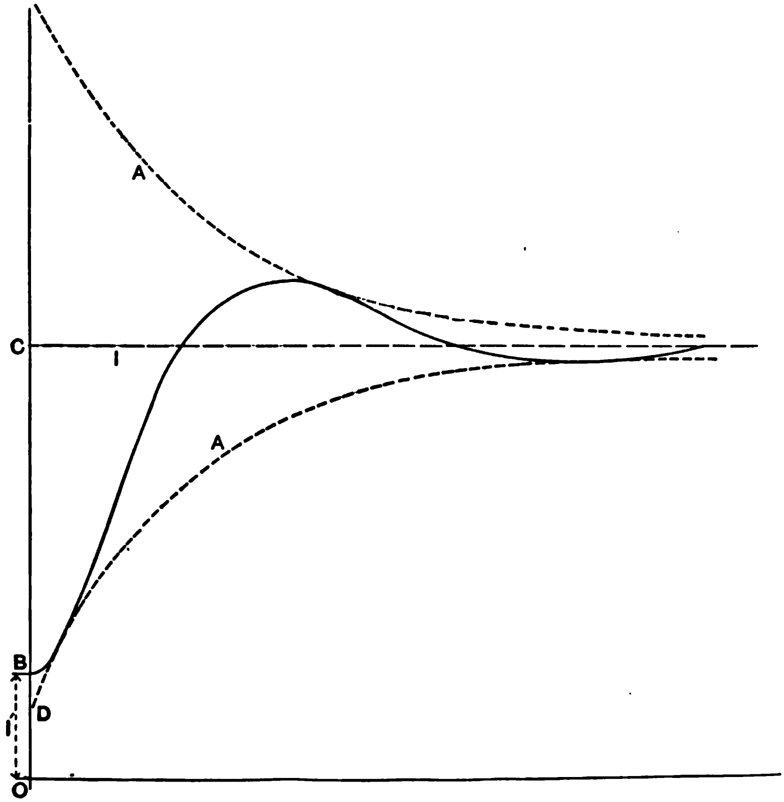

Fig. 3.

Equations (71) and (72) show that both primary and secondary currents will oscillate harmonically about their final values with a period equal to $\frac{2\pi}{\alpha}$, the maximum values of the oscillations decreasing rapidly with a logarithmic decrement depending upon the value of p. The final steady value of the primary current will be $\frac{E'}{R_1}$, and the secondary will become zero after a short interval of time. The amplitude of the oscillations depends upon the values of A_1 and A_2; their relative phase upon the values of Φ_1 and Φ_2.

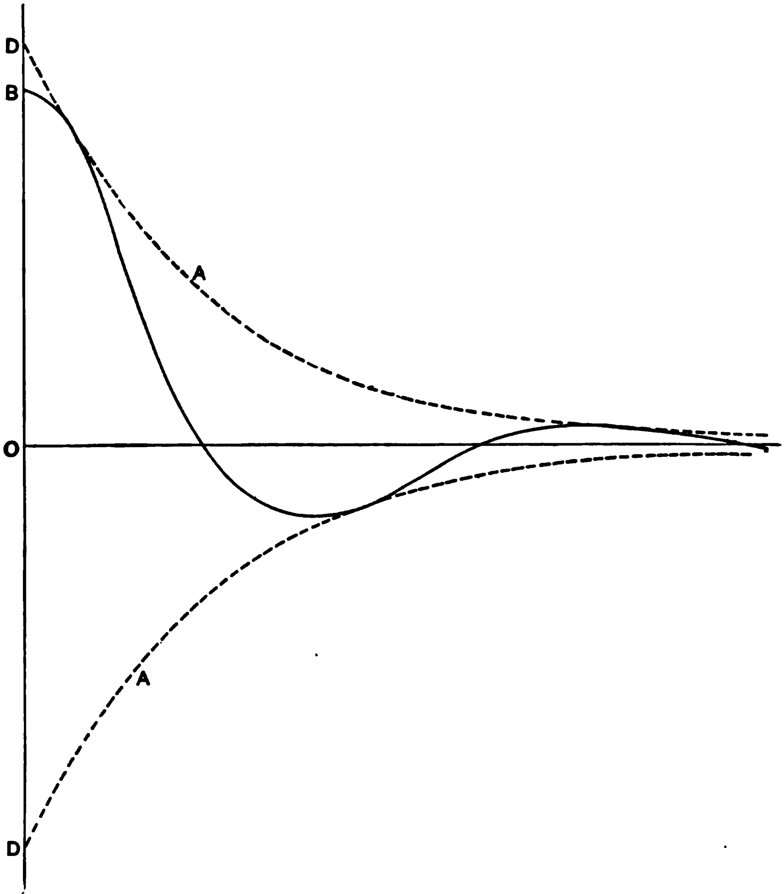

Fig. 4.

The nature of these oscillations will be more clearly understood by inspection of the typical curves in Figs. 3, 4, and 5. Fig. 3 represents the value of the primary current at each point of time as it changes from an initial value I' to a final value I. The distance BC represents the difference between the initial and final currents. The curve lies between two logarithmic envelopes AA, the initial value of which is equal to A_1 in equation (71), the rate of decay depending upon p. The relation of I' and I to the origin is immaterial; either may be the greater or may be zero.

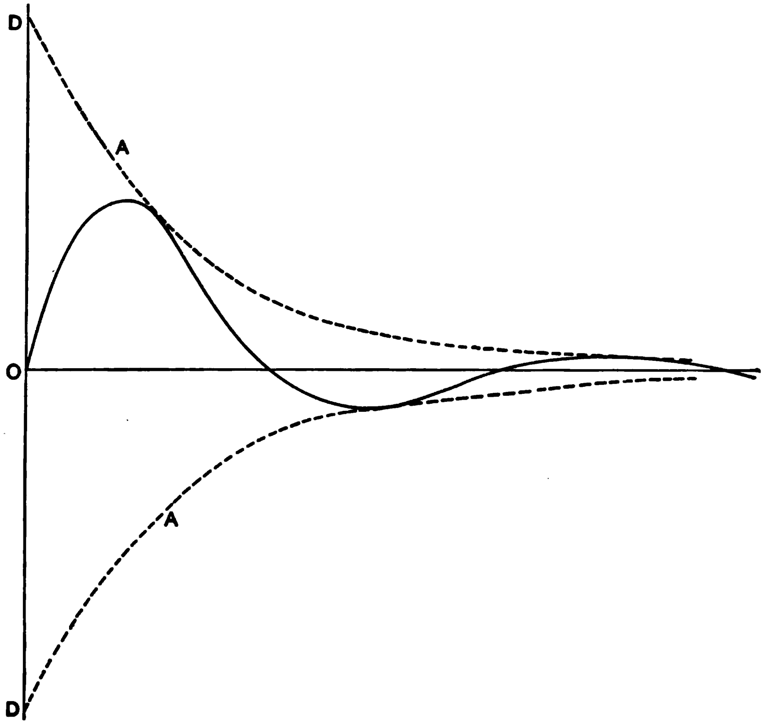

Fig. 5.

If the initial value I' is zero, the origin should be moved from O to B. If the initial value is not zero and the final value is zero, the oscillations are as shown in Fig. 4. Where both initial and final values are zero, the curve in Fig. 5 shows the instantaneous values of the primary current.

The oscillations in the secondary circuit are similar in general character to those in the primary. The nature of the oscillations in the secondary current, as it changes from an initial value OB

to a final steady value, is shown by Fig. 4. Fig. 5 shows the same with the secondary current initially zero.

The foregoing explanation of the oscillations caused by make or break, when the secondary circuit contains a condenser, is simply the interpretation of equations (62) and (63). If the condenser be placed in the primary circuit instead of the secondary, the phenomena will in many respects be the same as those just described, inasmuch as the equations (64) and (65) for the currents in this case are similar to (62) and (63) for the secondary condenser, and a detailed discussion is accordingly unnecessary.

HARMONIC IMPRESSED ELECTROMOTIVE FORCE.

PRIMARY CURRENT.

In taking up the discussion of current flow when the electromotive force impressed upon the primary is harmonic, let us return to the most general expressions for the primary and secondary currents, — the differential equations (16) and (19), — and assume the electromotive force to have a value $E \sin \omega t$.

Substituting in (16),

$$f(t) = E \sin \omega t; \qquad f''(t) = -E\omega^2 \sin \omega t;$$
$$f'(t) = E\omega \cos \omega t; \qquad f'''(t) = -E\omega^3 \cos \omega t;$$

the expression for primary current is obtained:

$$(75) \qquad i_1 = \frac{\frac{E\omega}{C_2} \cos \omega t - R_2 E\omega^2 \sin \omega t - L_2 E\omega^3 \cos \omega t}{(L_1L_2 - M^2)D^4 + (R_1L_2 + R_2L_1)D^3 + \left(\frac{L_1}{C_2} + \frac{L_2}{C_1} + R_1R_2\right)D^2 + \left(\frac{R_1}{C_2} + \frac{R_2}{C_1}\right)D + \frac{1}{C_1C_2}}.$$

Now $\qquad D \sin \omega t = \omega \cos \omega t,$ and $D^2 \sin \omega t = -\omega^2 \sin \omega t$

whence $\qquad D^2 = -\omega^2,$ and $D^4 = \omega^4.$

The numerator in (75) may be written as one term by the trigonometric formula,

$$(76) \qquad A \sin \theta + B \cos \theta = \sqrt{A^2 + B^2} \sin\left(\theta + \tan^{-1}\frac{B}{A}\right).$$

Combining the numerator in this manner, substituting $-\omega^2$ and ω^4 for D^2 and D^4, and multiplying by D, we may write (75) thus:

$$(77)\quad i_1=\frac{DE\omega^2\sqrt{R_2^2+\left(\frac{1}{C_2\omega}-L_2\omega\right)^2}\sin\left\{\omega t-\tan^{-1}\left(\frac{1}{C_2R_2\omega}-\frac{L_2\omega}{R_2}\right)\right\}}{\alpha D+\omega\beta},$$

where

$$(78)\qquad \alpha=\omega^4(L_1L_2-M^2)-\omega^2\left(\frac{L_1}{C_2}+\frac{L_2}{C_1}+R_1R_2\right)+\frac{1}{C_1C_2};$$

$$(79)\qquad \beta=\omega^3(R_1L_2+R_2L_1)-\omega\left(\frac{R_1}{C_2}+\frac{R_2}{C_1}\right).$$

To free (77) from the operator D, operate upon the numerator as indicated, and multiply numerator and denominator by $\alpha D-\omega\beta$. Substitute $-\omega^2$ for D^2, and perform the operation D in the numerator. Having now become rid of the operator D, we have the required integral, which expresses the value of the primary current at any time. Thus:

$$(80)\qquad i_1=I_1\sin(\omega t+\Phi),$$

where

$$(81)\qquad I_1=\frac{E\omega^2\sqrt{R_2^2+\left(\frac{1}{C_2\omega}-L_2\omega\right)^2}}{\sqrt{\alpha^2+\beta^2}},$$

and

$$(82)\qquad \Phi=\tan^{-1}\frac{\beta}{\alpha}-\tan^{-1}\left(\frac{1}{C_2R_2\omega}-\frac{L_2\omega}{R_2}\right).$$

Here Φ is the angle between the primary current and the impressed electromotive force.

SECONDARY CURRENT.

The secondary current is similarly obtained by referring to the general equation (19), and substituting $f'''(t)=-E\omega^3\cos\omega t$; $D^2=-\omega^2$; $D^4=\omega^4$. Making these substitutions, and multiplying numerator and denominator by D, we obtain

$$(83)\qquad i_2=\frac{DME\omega\cos\omega t}{\alpha D+\omega\beta}.$$

Operating upon the numerator by D, then multiplying both numerator and denominator by $\alpha D-\omega\beta$, and again operating as indicated, we free the equation from the operator D, and obtain the final integral

$$(84)\qquad i_2=I_2\sin\left(\omega t-90^\circ+\tan^{-1}\frac{\beta}{\alpha}\right),$$

where α and β stand for the expressions given in (78) and (79), and

$$(85)\qquad I_2=-\frac{ME\omega^3}{\sqrt{\alpha^2+\beta^2}}.$$

These equations just obtained for the primary and secondary currents are the general equations for a transformer, subjected to an harmonic impressed electromotive force, when we have a condenser in each circuit, and make no assumption as to the absence of magnetic leakage; but neglect the change in coefficient of self-induction due to the presence of iron. Their complete discussion would be beyond the limits of the present paper. The generality of the equations may be modified by various limitations, — as, for instance, by the omission of one or both condensers, and by the assumption of no magnetic leakage. The subject can be treated in the inverse order, and the more general obtained synthetically from the simpler cases, the results being the same as those obtained analytically. The identification of the results obtained for a certain case from the general solutions, and those obtained for the same case by a process of building up from simpler cases, is not always readily shown. Suffice it to illustrate in the case in which both primary and secondary condensers are omitted and there is no leak; that is, when

$$C_1=\infty\,;\quad C_2=\infty\,;\quad M^2=L_1L_2.$$

The theory of the transformer for this case has been synthetically developed by the writers,[1] and may be identified with the results obtained for this case from the general discussion in this paper.

The general relation between primary and secondary currents (81) and (85) is

$$(86)\qquad I_2=\frac{M\omega I_1}{\sqrt{R_2^2+\left(\frac{1}{C_2\omega}-L_2\omega\right)^2}}.$$

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[1] "Theory of the Transformer," Electrical World, Vol. XXXI., beginning No. 12, March, 1893.

This expression is evidently in accordance with the law that the current in a circuit is equal to the impressed electromotive force divided by the impediment. For the particular case under discussion this becomes

$$I_2=\frac{M\omega I_1}{\sqrt{R_2^2+L_2^2\omega^2}}.$$

The general value for the primary current in (81) reduces for the particular case to

$$I_1=\frac{E\sqrt{R_2^2+L_2^2\omega^2}}{R_1R_2\sqrt{1+\left(\frac{L_1\omega}{R_1}+\frac{L_2\omega}{R_2}\right)^2}}. \tag{87}$$

These results are the same as those obtained on page (340) in the series of articles just referred to.

The angular relation between the primary and secondary currents is seen by a comparison of (80) and (84). The secondary current lags behind the primary by an angle of 90° plus an angle whose tangent is $\frac{1}{C_2R_2\omega}-\frac{L_2\omega}{R_2}$. If there is no condenser in the secondary, this lag is $-90^\circ-\tan^{-1}\frac{L_2\omega}{R_2}$, which is in accordance with the well-known transformer diagram. A condenser in the secondary might reduce this angle to 90° or make it even less, as is seen from the equations, and may likewise be shown synthetically. The geometrical construction of transformer diagrams may be thus analytically established.

If we consider that there is no magnetic leakage and put $L_1L_2-M^2=0$, and that there are no condensers in circuit, the expressions (78) and (79) for α and β become

$$\alpha=\omega^3(R_1L_1+R_2L_2),$$

and $$\beta=-\omega^2R_1R_2.$$

Hence $$\tan^{-1}\frac{\beta}{\alpha}=\tan^{-1}-\left(\frac{L_1\omega}{R_1}+\frac{L_2\omega}{R_2}\right),$$

and by (82), $$\Phi=\tan^{-1}-\left(\frac{L_1\omega}{R_1}+\frac{L_2\omega}{R_2}\right)+\tan^{-1}\frac{L_2\omega}{R}.$$

This easily reduces to

$$(88)\qquad \Phi = \tan^{-1} \frac{-\dfrac{L_1\omega}{R_1}}{1 + \dfrac{L_1 L_2 \omega^2}{R_1 R_2} + \dfrac{L_2^2 \omega^2}{R_2^2}},$$

which is identified with the result given on page 340 of the articles in the *Electrical World* referred to above.

Although the analytical expressions might be found by means of the general equations (16) and (19), when there is an harmonic impressed electromotive force, to cover all cases which arise when there are condensers present in the circuits of a transformer, yet it will be found that many problems will lend themselves to readier solution by the graphical methods in which diagrams are built up by synthetic processes. It is considered that the above examples, which identify the results independently obtained from differential equations with the diagrams made from other considerations, are sufficient to show that any diagram has its analytical equations, and any equations, properly derived from the harmonic law, have their corresponding geometrical interpretation.

MAKE AND BREAK WITH HARMONIC ELECTROMOTIVE FORCE.

In the discussion of the current flow in the primary and secondary of a transformer subjected to an harmonic electromotive force, the exponential terms which constitute the complementary function have been omitted from the equations, and the currents are simple harmonic functions of the time. These exponential terms modify the current for a short time after the make, but their effects rapidly diminish and become negligible after a fraction of a second. The exponential terms have been discussed in the earlier part of this paper, and the effects there described are to be superimposed upon the simple harmonic flow of current which would take place if they were not present. Whether these terms are oscillatory or not depends, as before, upon the relation between the various constants of the circuits. When the complementary function is oscillatory, the resultant current for a short time oscillates about its final sinusoidal form, its form depending upon the relation between the period of the impressed electromotive force and the natural period of the circuit, and upon the time of introduction of the electromotive force. The periods may be such that distinct beats are obtained. These oscillations are the same in nature as those which occur after the make of a single circuit containing resistance, self-induction, and capacity.

Current Flow in a Single Circuit.

The equations for a single circuit containing resistance, self-induction, and capacity are directly derivable from those for a transformer by assuming that the secondary is removed. To find the values for α and β for this case, take out from (78) and (79) the factor $R_1R_2\omega^2$, and let $\frac{L_2\omega}{R_2}=0$; $\frac{M\omega}{R_2}=0$; $\frac{1}{C_2R_2\omega}=0$. For the primary circuit alone, we then obtain the values

$$\alpha = R_1R_2\omega^2; \qquad \beta = \left(\frac{L_1\omega}{R_1} - \frac{1}{C_1R_1\omega}\right)R_1R_2\omega^2.$$

Substituting these values in (80), the expression for primary current becomes

$$(89) \qquad i_1 = \frac{E}{\sqrt{R_1^2 + \left(\frac{1}{C_1\omega} - L_1\omega\right)^2}} \sin\left\{\omega t + \tan^{-1}\left(\frac{1}{C_1R_1\omega} - \frac{L_1\omega}{R_1}\right)\right\}.$$

This is the value for the current at any time in a simple circuit containing resistance, self-induction, and capacity when subjected to an harmonic electromotive force, and is fully discussed for general and particular cases in the writers' treatise on *Alternating Currents.*

Conclusion.

We have considered two mutually related circuits with constant coefficients of self-induction, thus not taking into consideration the changes in the coefficient of self-induction, which occur at high magnetization, when iron is present due to the hysteresis loss. The analytical work can only be rigorously correct when there is no iron, and must be looked upon as an approximation, when iron is present, which is justified when a high degree of magnetization is not reached. We have developed the expressions for the current flow in such circuits in general due to any impressed electromotive force, and have illustrated how they may be reduced to simpler forms for particular cases, — as for the case of an ordinary transformer and for a simple circuit. The limits of this paper make it impossible to enter into a full discussion of the analytical results obtained or to take up the many particular cases covered.

ON THE FREEZING-POINTS OF DILUTE SOLUTIONS. I.

By E. H. Loomis.

THE conflicting views in regard to the nature of solution arise largely from the fundamental disagreement in the results which different observers have found for the freezing-points of dilute aqueous solutions. The difficulties in making such determinations are so great that the results have been affected apparently by large errors. The methods which have been employed to overcome these difficulties are so uncertain in practice that different observers, even when using identical methods, have obtained radically different results, and these have furnished the material for the construction of the conflicting theories. While the estimated errors of the principal observers have been placed by themselves as low sometimes as 0°.0005 C., and seldom higher than 0°.005 C., the observed freezing-points of a given solution have differed by many times the latter amount. One may cite, for example, the case of an aqueous sugar solution, in middle concentration, say $\frac{1}{10}$ normal, or about 3.3 per cent. The observed freezing-points, together with the estimated error of the observer, are given in the following table. These results are obtained by interpolation, for the most part within narrow limits. The first determination of Raoult is reduced to numerical value from his curve of results, which is the only published data. To gain a clearer idea of the magnitude of these differences it must be remembered that the total depression of the freezing-point in a $\frac{1}{100}$ normal sugar solution does not exceed 0°.02. One seems warranted in believing that theories which require for their support such a degree of certainty in the freezing-point of a solution, that 0°.001 C. becomes critical, rest so insecurely upon the present experimental data that they should be held with some little reservation, and at least advocated with no display of temper.

Observer.	Year of Publication.	Freezing-point.	Estimated error.
[1] Raoult, I.	1886	0°.24	0°.01 (*a*)
[2] Arrehnius, I.	1888	0°.210	0°.005 (*b*)
[3] Traube	1891	0°.235	0°.005 (*c*)
[4] Eykman	1891	0°.216	
[5] Arrehnius, II.	1891	0°.204	
[6] Tamman	1891	0°.206	
[7] Pickering	1891	0°.202	0°.0005 (*d*)
[8] Raoult, II.	1892	0°.205	0°.0020(*e*)

Some two years ago, at the suggestion of Professor Dr. W. Hallwachs, I began to work upon the freezing-points of dilute solutions. The apparatus employed was the well-known one of Beckmann in its improved form,[9] which commends itself so highly on account of its great ease of manipulation and large range of application. The results which I obtained, however, showed so little agreement among themselves, that it soon became evident that the desired degree of accuracy was not to be expected with this method. Not the least of the many difficulties which were encountered, and one which made me always suspicious of my own results, was the inability to free the observations from a large element of arbitrariness. The thermometer seemed to be incapable of assuming a fixed position, and when at times it did become stationary, the slightest incautious jar or change in the rate of stirring, or even continued stirring at a uniform rate, would disturb the mercury. Thus a slight increase in the vigor

[1] Ann. de Chim. et de Phys. (VI.) 8, 1886, p. 313. (*a*) same, (V.) 1883, p. 135.
[2] Zeit. Phys. Chem. II., 1888, p. 491. (*b*) same, p. 493.
[3] Ber. Chem. Berichte, 1891, p. 1853. (*c*) same, p. 1854.
[4] Ber. Chem. Berichte, 1891, p. 1783.
[5] Ber. Chem. Berichte, 1891, p. 2255.
[6] Ber. Chem. Berichte, 1891, p. 2287.
[7] Ber. Chem. Berichte, 1891, p. 3328. (*d*) same as [7], 1892, p. 1860.
[8] Comptes Rendus, 1892, p. 268. (*e*) same as [8], 1892, p. 271.
[9] Zeit. Phys. Chem. II., 1888, p. 638.

of the stirring or its uniform continuation is attended by a steady *rise* in the thermometer, which becomes less and less as the stirring is gradually decreased, and finally the mercury becomes stationary at almost any prescribed point at which the stirring is allowed cautiously to cease altogether. Thus the danger that the bias of the observer may affect the results was so great that the method was finally abandoned.

The only apparent way of avoiding this danger is to insure the presence of a large quantity of ice by allowing a large degree of overcooling to precede the freezing; but here, again, a still more serious difficulty presents itself, namely, the change in the concentration of the solution due to the freezing of a portion of the solvent. My experiments convince me that the corrections which thus become necessary are extremely uncertain, and when they amount, as some observers find, to more than 4 per cent. of the total depression, they cannot fail to produce utter confusion in the results.

My experience, however, with the Beckmann apparatus furnished the key to the method which finally developed itself, since it was during this time that it first became evident how marked was the influence of the temperature of the freezing mixture upon a mixture of ice and water obtained by exposing water in any suitable vessel to its action. The temperature of this partially frozen water, especially when the initial overcooling which always precedes the freezing does not exceed 1° C., is always lower, within certain limits, the lower the temperature of the freezing mixture is chosen, reaching, however, a maximum depression when the freezing mixture becomes about $-7°$ C. Beyond this point a surprising fact presents itself; for now the *lower* the temperature of the freezing mixture, the *higher* becomes the observed temperature of the ice and water mixture surrounded by it. The highest temperature is observed when the freezing mixture is reduced to about $-15°$ C. All this is easily understood as soon as one observes that below a certain point, by no means, however, fixed, but rather dependent on the shape of the vessel and manner and rate of stirring, an ice sheath is formed on the wall of the vessel, which thus serves as a shield against the extreme cold of the exterior bath. When this sheath is complete and compact in texture, the protection which it affords to the remaining ice and water mixture becomes complete, and the water assumes its true freezing temperature. Further reduction of the outside temperature beyond the point at which a complete ice sheath is formed has no appreciable influence upon the inner temperature. That this final temperature should be higher than it is when the freezing mixture is only $-3°$ C., for example, is plain, since at this much warmer temperature the stirring prevents the formation of an ice sheath, and the freezing water is exposed to the full action of the cold environment. It is equally plain that any circumstances which facilitate or hinder the formation of the ice sheath affect the course of these various phenomena, but in no way their general nature.

For similar reasons the temperature of freezing water is determined by the extent of the initial overcooling. When this overcooling is little, the quantity of ice which forms as a result is small, and the influence of the freezing mixture thus manifests itself strongly, while, on the other hand, an initial overcooling of more than 1° C. is followed by a rapid formation of a large quantity of ice throughout the water and the freezing mixture ceases to influence sensibly the thermometer.

That the temperature of an ice and water mixture is exceedingly sensitive to the temperature of its surroundings appears, however, more directly from a simple fact which every observer in this field must have noticed; namely, while allowing the ice to melt in the tube of a Beckmann apparatus, the usual practice is to hold it in the air until, with uninterrupted stirring, the ice is about to disappear. A glance at the thermometer in the meantime shows that as the ice has been melting, the thermometer has steadily risen, slowly at first while the quantity of ice is still relatively large, and finally with great rapidity as the ice nears the point of disappearing. We may safely say, when little ice is present, the effect of a cold freezing mixture or the warm air upon a mixture of ice and water is the same in kind and hardly less in degree than when no ice whatever is present.

This high sensitiveness of an ice and water mixture to the temperature of its surroundings is surpassed by that of a partly frozen solution. The combined action of these disturbing influences has occasioned large errors in the results, and accounts, I think, in no small measure, for many of the great differences between the results of different observers, — differences which not infrequently amount to 40 per cent. I may perhaps venture to suggest one combination of these influences which accounts for the fact that the observed depressions have been commonly too large. Let us take, for example, a $\frac{1}{100}$ normal solution of any substance. To measure the depression of the freezing-point, we must first find the freezing-point of the solvent, — let us suppose water, — and then that of the solution itself. The difference is the desired depression. The freezing-point of the water is most accurately determined by allowing a large degree of overcooling, and thus insuring the formation of a large quantity of ice. As this in no way affects the validity of the observation, and contributes greatly to the ease and certainty in making it, a common practice has been to allow a large degree of overcooling. On the other hand, in taking the freezing-point of the solution, great care is exercised to confine this overcooling within certain narrow limits, so that the corrections which must be applied on account of the change in the concentration may be as small and uniform as possible. What the effect of this method of procedure is, becomes evident from what has already been said in regard to the influence of the freezing mixture, and its relation to the quantity of ice which is allowed to form; the observed freezing-point of the water, owing to the large quantity of ice present, will be approximately the true freezing-point, while the observed freezing-point of the solution will be too *low*, since the quantity of ice is too small to protect the solution from the action of the freezing mixture. The observed depression is thus too large. This error may easily reach a number of thousandths. As the depression for a $\frac{1}{100}$ normal solution must be multiplied by 100 to obtain the *relative* or so-called *molecular depression*, we observe that an error of a thousandth has the magnitude of a *tenth* of a degree on any chosen scale or curve of results. That some such error has played a prominent part in the various determinations seems evident from a most casual glance at the successive results, from the first observations to the most recent. The observed depressions have steadily decreased, in seeming proportion as these errors have been eliminated, exactly as appears in the case of the $\frac{1}{10}$ normal sugar solution already mentioned.

Before entering upon a detailed description of the present method it needs to be observed that the central idea about which it has grown up lies in the fundamental conception of the freezing-point of a liquid. For the sake of clearness, let us confine ourselves to water as a type.

The freezing-point of water may be defined as *that temperature at which ice and water may exist in contact, and no reaction take place between them.* It is thus the temperature of equilibrium in a mixture of ice and water. However objectionable the term "freezing-point of water" may be when used to designate this temperature, the term "melting-point of ice" is certainly no less so, and I have uniformly used the former as less likely to produce confusion. It is to be observed that this state of equilibrium is exceedingly delicate, and that the mixture of ice and water is at its freezing-point only when this equilibrium is maintained. Thus the temperature of an ice and water mixture is higher when the ice is melting than when it is freezing, and the difference, which may be considerable in spite of thorough stirring, increases in proportion as the forces which tend to disturb the equilibrium become greater, and the extent of surface-contact between the ice and water becomes less. If this surface of contact is small, as is the case when little ice is present and massed in relatively large pieces, the temperature of the mixture may be made to vary many tenths of a degree, according as it is exposed to the temperature of the air or that of a freezing mixture. In the former case, the temperature rises much above the freezing-point, since, owing to the little ice-surface exposed, the melting cannot proceed fast enough to use up the large amount of heat which the mixture receives from the air; in the second case, the temperature is, on the contrary, much below the freezing-point, since, for the same reason, the freezing cannot take place fast enough to prevent the overcooling of the water.

The necessity, then, of protecting the liquid whose freezing-point is to be determined from the disturbing influences of outside temperatures is fundamental; and almost from the beginning, even while still using the Beckmann apparatus, I sought to accomplish this by making the freezing-tube much longer, and so remove the solution as far as possible from the air, and by using a secondary freezing mixture with a temperature only a few tenths below the freezing-point of the liquid to be examined, in which the apparatus was placed at the time of making an observation. This practice still remains a prominent feature of the new method.

The Apparatus.

It consists of a thermometer with enclosed scale divided in hundredths of a degree. Its dimensions are such that a hundredth measures about 0.4 mm. This instrument was made according to special order by Herrn R. Fuess, Berlin, and I may take this opportunity of thanking him for the masterly way in which he met all of my requirements. The scale is divided with the greatest precision, and in all my work with it I have observed no deformation of the mercury meniscus, which has given me so much trouble in other instruments of this class, and which appears to indicate an unclean capillary and impure mercury. This thermometer is held firmly by two paraffined corks in a large test-tube (28 x 3 cm.). One cork is fixed to the thermometer and serves as a stopper to the tube, while the other is placed a little way above the level of the liquid in the tube, and serves to guide the stirrer and maintain the thermometer in a central position. This test-tube has a simple modification which is fundamental to the success of the method. The bottom is bottle-shaped, *i.e.* rounding *inward*, and not outward, as is usually the case with test-tubes. This tube is set in a second test-tube of slightly larger dimensions so as to allow for a millimeter air space between the tubes. They are held together at their upper ends by a short piece of thin-walled rubber tubing of large calibre, while a small rubber band stretched over the inner tube keeps them apart at the bottom. The walls of the tubes should not exceed 1 mm., and the thermometer should reach to within about 2 cm. of the bottom.

The stirrer is a most important part of the apparatus. Of the many forms which I have used, one alone gives complete satisfaction. This is the common "ring" stirrer, with this essential alteration: around the outside of the platinum ring a stripping from a goose feather is bound with platinum wire in such a way that the edge of the feather border just rubs the wall of the tube. This gentle and uniform friction against the wall of the tube, together with the peculiar shape of the bottom of the tube itself, prevents the troublesome massing of the ice between the bulb of the thermometer and the bottom of the tube, and obviates all tendency of the ice to freeze in a sheath. The glass rod of this stirrer runs through openings in the two corks, which act as guides to its motion and so prevent its striking against the bulb of the thermometer. It may be added that in using this stirrer, no tendency has been observed in the ice to mass itself together and float at the surface.

For the sake of clearness and brevity, this double tube with its thermometer and stirrer will be referred to as the *freezing-tube.*

At the time of an observation, this stands up to its neck in a thick mixture of salt water and finely powdered and sifted ice (snow answers the purpose admirably) in such proportions as to have a temperature of 0°.30 C., below the freezing-point of the liquid to be observed. This mixture is contained in a copper vessel 35 cm. high and 6 cm. wide. This is also provided with a ring stirrer and is closed at its mouth with a flat cork through the opening in which the freezing-tube is set. To render the temperature of this mixture as constant as possible, the copper vessel is placed in a thick jacket of felt. This copper cylinder with its weak freezing mixture and felt jacket will be uniformly referred to as the *protection-bath.* This rests upon the base of an ordinary retort-stand, the vertical rod of which supports the framework of a small electric hammer (*H*) similar to those used in common electric call-bells. The coils of this hammer have a combined resistance of about 10 ohms, and are supplied with the current from two accumulators (in series) of about 2 volts each. The blows of this hammer are thus energetic and are delivered directly upon the upper end of the thermometer. Uniform and vigorous jarring of the instrument is thus maintained during an observation. In addition, two other baths are employed,— one a salt and ice freezing mixture at $-10°$ C., contained in a glass battery jar about 30 cm. high, which the mixture fills to a depth of about 14 cm., or, more exactly, to such a depth that it reaches up to the level of the solution in the freezing-tube when this is placed in it.

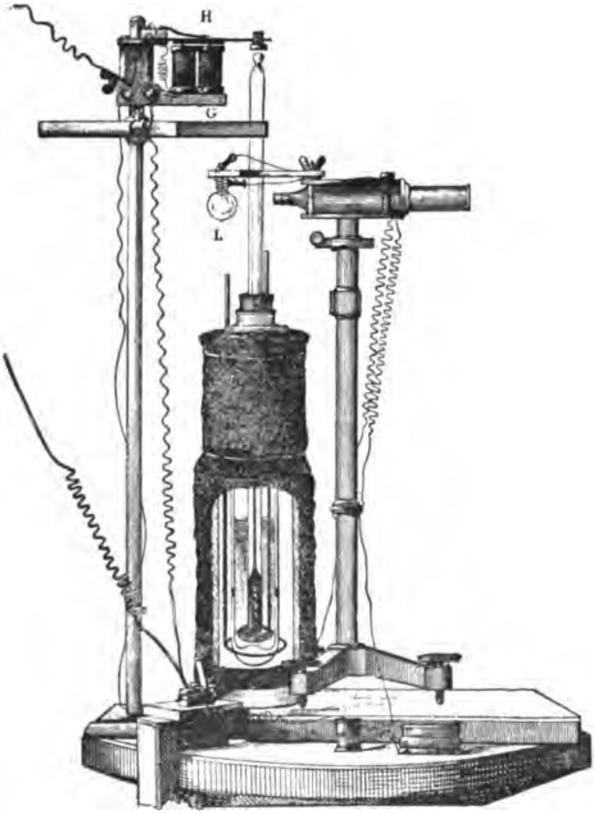

Fig. 1.

This will be known as the *freezing-bath.* The other is merely a mixture of water and coarsely broken ice, in a similar jar, which it fills to such a depth that the freezing-tube may be entirely immersed in it. This is the *melting-bath.* Each of these baths are protected by a heavy wrapping of felt and closed with thick wooden covers, openings in which admit the freezing-tube.

The Reading of the Thermometer.

This is effected with a microscope having a micrometer scale in its eyepiece. The lenses and micrometer are so selected that exactly ten divisions of the micrometer correspond to one scale division of the thermometer. Thus the thousandths of a degree may be directly read and the *ten-thousandths* easily estimated. As the thermometer scale is back of the mercury column, it is obviously impossible to see distinctly the scale and mercury through the microscope at the same time. To read the thermometer, it thus becomes necessary to focus the microscope on the thermometer scale, and bring the micrometer scale into coincidence with it; and then, by a slight sliding backwards of the entire microscope stand, bring the meniscus into focus, in which position the thermometer may be read off on the micrometer scale. To avoid the errors which might possibly arise from this necessary disturbance of the microscope, the stand was made massive and the levelling-screws which supported it rested upon a pane of plate glass which was itself firmly cemented in place. Whatever inconvenience is occasioned by this arrangement is more than compensated for in the fact that, during the interval in which the mercury is coming to its stationary position, no idea may be obtained of what this final position may be, and hence there is no possibility that the observer's bias may affect the result through the unconscious changes in his rate and vigor of stirring, which this bias may induce. To the microscope stand is also attached a small incandescent lamp on the free end of a movable arm, so that it may be readily swung around behind the thermometer, directly in line with the axis of the microscope. The lamp, mercury, and microscope always occupy the same relative position, and errors arising from a variable illumination of the meniscus are avoided. The lamp is only used when the daylight becomes insufficient, and then only during the moment of reading the thermometer. The arrangement of the various parts and the minor details necessary for the few simple adjustments are easily gathered from Fig. 1, taken from a photograph. The figure shows in addition the interior of the protection-bath and freezing-tube.

The Thermometer Corrections.

The thermometer was furnished with a certificate from the German government scientific laboratory at Berlin (Reichsanstalt in Charlottenburg near Berlin), from which the corrections for 0° and −4° — the limits of the scale — were taken. Then by the most careful calibration with a detached mercury column, whose average length in thermometer scale-divisions was 0°.9886, it was found that the interval 0° to 1° was 0°.0015 too great, or instead of being a true degree it was in fact 1°.0015. Owing to the great uniformity in the capillary tube in this region, it was assumed that this error was uniformly distributed over the interval, and thus a correction proportional to the depression had to be added to the observed depression. For the greatest observed depression, namely, 0°.7688 in the case of the $\frac{2}{10}-H_2SO_4$ solution, this correction amounts evidently to 0°.00116. For depressions of less than 0°.1 this correction almost vanishes.

The corrections for variations in the atmospheric pressure are more considerable. The thermometer with its large bulb and minute capillary is a true aneroid barometer, and has a sensitiveness determined by the thickness of the bulb walls and the relations of its other dimensions. In this case, according to the Berlin certificate, a change of 6 mm. in the barometer corresponds to a change of 0°.001 in the reading of the thermometer. Thus this correction may easily exceed 0°.002. To avoid the possible errors which might arise from applying such a large and partly uncertain correction, care was exercised to make all observations on the very *dilute* solutions immediately before or after the determination of the freezing-point of water.

The Method.

The best idea of the practical details of the method may be obtained from a description of the determination of the freezing-point of water, which must be made daily to eliminate the errors due to changes in the thermometer. Further, to render these changes as little as possible, the thermometer is kept during the entire period of the observations at about 0°. The freezing-tube is filled to the depth of 10 cm. (about 70 cm^3.) with distilled water of a high degree of purity, which is then cooled to the freezing-point and the thermometer put in place. The freezing-tube is now placed in the protection-bath, whose temperature is previously reduced to about $-0^\circ.32$ by the addition of proper quantities of ice or salt. Here it is allowed to remain twenty minutes to insure the cooling of the entire apparatus to the vicinity of 0°.[1] During this interval the temperature of the protection-bath has generally risen to the required $-0^\circ.30$. The freezing-tube is now removed to the "freezing-bath" (−10° C.), where it is kept, with uninterrupted stirring, until the freezing begins. It is then placed in the "melting-bath" (0° C.), and there, by the mere mechanical energy of vigorous stirring, the ice is brought almost to the point of disappearing.[2] This may be known by observing the thermometer itself. It rises slowly so long as considerable ice remains, and more rapidly as the melting proceeds. When but little ice remains, the rise becomes very rapid. A little experience enables one to judge with great certainty when the ice has been melted to the required degree. In the case of water it is generally when the thermometer has reached $+0^\circ.17$.[3]

At this moment the freezing-tube is placed in the freezing-bath where the temperature rapidly falls, until, after an overcooling of between 0°.10 and 0°.20, freezing begins, and the thermometer rapidly rises. At this instant the freezing-tube is thrust into the protection-bath, the electric hammer set in motion, and after two minutes, during which the stirring has been vigorously maintained, the thermometer is read. In this interval the mercury uniformly reaches its highest point, and remains stationary so long as the jarring and stirring are maintained. After the stirring and jarring are stopped, the mercury continues perfectly stationary from one to two minutes, and then without exception begins slowly to sink. A jar, however, will cause it to drop suddenly much sooner. To obtain a second reading the freezing-tube is again placed in the melting-bath, and the ice again brought to the point of disappearing, at which moment it is transferred to the freezing-bath, and after freezing has again begun, it is placed as before in the protection-bath, when a second reading is made. This is repeated six to ten times, and the middle of the results taken as the required observation. It must not be forgotten that the stirring is to be maintained continuously, from the time that the freezing-tube comes in the freezing-bath, until the thermometer is to be read. The "strokes" should reach from the bottom of the tube to within a few millimeters of the surface of the liquid. The wooden fork (*G*) may be used as an index to control the height to which the stirrer should be lifted. In case the overcooling exceeds 0.°20, it may be known with certainty that the overwarming in the melting-bath was carried too far, and in the following observations the overwarming should be lessened, say to 0°.15. Should, now, the overcooling fall short of 0°.10, one knows with equal certainty that the overwarming was not carried far enough, and this overwarming must be cautiously increased until the subsequent overcooling is brought within the prescribed limits. This relation between the degree of overwarming in the melting-bath and the overcooling in the freezing-bath is most interesting. The experiments show that the greater the overwarming, the greater will be the following overcooling. Thus, one may effect almost any desired amount of overcooling by properly varying the degree of overwarming. In the case of water, for reasons still unknown to me, the degree of overwarming necessary to confine the overcooling within the prescribed limits (0°.10 – 0°.20) is changeable; sometimes, for example, after having remained for many weeks at 0°.15, it may rise to 0°.22, or perhaps fall as low as 0°.08, and there remain constant for a longer or shorter time. On the contrary, with a solution, this degree of overwarming is very constant, and by varying it the overcooling may be *controlled* to the hundredth of a degree.[1]

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[1] This precaution is absolutely necessary, since otherwise the first determinations are too high, owing to the warm walls of the freezing-tube and part of the thermometer above the surface of the liquid. This error may amount to more than 0°.004, and has led some observers *arbitrarily* to reject the first observations. When the source of error is discovered, its elimination becomes easy, and the first observations no longer show any irregularities.

[2] For obvious reasons, then, the melting of the ice is accomplished in the "melting-bath" instead of in the warm air, since the disturbing influences of the air and hands give rise to very considerable errors.

[3] This warming of a liquid and its "ice" above its freezing-point may be denoted by the term *overwarming;* while, on the other hand, the cooling of such a mixture below its freezing-point may be denoted by the term *overcooling.* The fact that this latter term is commonly used to denote the cooling of a liquid alone, below its freezing-point before freezing is induced, can hardly lead to confusion.

It is not without interest to observe that the overwarming necessary to effect the same degree of overcooling in a solution is less than in the case of water, and decreases uniformly as the concentration is increased and with different rates in solutions of different substances.

Thus in a $\frac{1}{10}$ normal solution it is 0°.06 for sugar, while it is only 0°.04 for sulphuric acid. I have not studied this phenomenon systematically enough to warrant me in proposing any explanation of it. To return to the method, we have still to determine in exactly the same way as already described for water, the freezing-points of the solutions, — always remembering that the temperature of the protection-bath must be successively lowered so that for each solution its temperature is 0°.30 lower than the freezing-point of the solution under observation. The difference between this and the freezing-point of water is the observed depression.

It should be added that the various details of the method were developed during a long series of experiments relative to the freezing-point of water alone. In fact, several thousand determinations of the freezing-point of water were made before the various sources of error were so far discovered and suppressed that such a freezing-point could be determined with absolute certainty to the fraction of a thousandth of a degree; and what was more important, that this extreme accuracy could be attained when only the barest traces of ice were present, — a vital condition with solutions to avoid the increase in the concentration which the presence of much ice involves. To indicate to what extent success was attained in this latter respect, a random selection from my note-book may be made.

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[1] It is to be noticed that in case ice is once allowed to form in "lumps," as is not infrequently the case with water before the required degree of overwarming is found, all that has been said about the relation between the degree of overwarming and overcooling has no longer any application, and the overcooling can no longer be made to exceed about 0°.08, and is independent of the amount of overwarming. The troublesome lump must be entirely melted by persistent and vigorous stirring in the melting-bath, before further progress may be made.

No.	Overwarming.	Overcooling.	Quantity of ice.	Reading for freezing-point.
1	+ (?)	− 0°.45	Much ice	+ 0°.0274
2	0°.21	0°.07	Little ice	0°.0273
3	0°.22	0°.05	Very little ice	0°.0283
4	0°.23	0°.065	Little ice	0°.0273
5	0°.23	0°.065	Little ice	0°.0270
6	0°.24	0°.070	Little ice	0°.0274
7	0°.34	0°.100	Little ice	0°.0274
8	0°.60	1°.0	Very much ice	0°.0278
9	0°.60	0°.6	Much ice	0°.0273
10	0°.60	0°.9	Very much ice	0°.0278

Middle of 6 observations, little ice present, + 0.02745.
Middle of 4 observations, much ice present, 0.02758.

It thus appears that the disturbing influences of the freezing mixture and air have been so far eliminated that the actual freezing-point of water, and so too that of a solution, may be determined with the greatest certainty even when only a trace of ice is present. *Not until this had been accomplished were any observations on solutions made.*

Although the observations are thus so largely independent of the quantity of ice present, for the sake of a last degree of accuracy, I have always confined the overcooling in the case of water, as is the necessity in the case of solutions, to the limits, 0°.10 to 0°.20. The conditions are thus identical with water and the solutions.

(*To be concluded.*)

MINOR CONTRIBUTIONS.

SOME RAPID CHANGES OF POTENTIAL, STUDIED BY MEANS OF A CURVE-WRITING VOLTMETER.

BY G. S. MOLER.

AT the general meeting of the American Institute of Electrical Engineers, held in Chicago, on June 6th, 1892, a paper was read describing an instrument called a Dynamo Indicator or Instantaneous Curve-Writing Voltmeter, and some of the curves obtained from a dynamo were shown and explained. When the instrument was designed, it was intended to serve mainly as a means of tracing, during a single revolution of the armature of a dynamo, a curve whose ordinates should represent the differences of potential between the two ends of a single coil on the armature, or between one end of a coil and one of the brushes. The instrument consists of an enclosed voltmeter of the permanent magnet form, having a needle whose rate of oscillation is slightly over a hundred per second. By means of a lever, the pointer attached to the needle can be made to press lightly against a small smoked cylinder, which is about one and one-fourth inches in diameter, and two and one-half inches long. The movement of the pointer is through a short arc whose chord is parallel to the axis of the cylinder, so that when the cylinder is revolving, a curve will be drawn upon it, whose ordinates are parallel to the axis. By rolling the smoked drum across a dampened sheet of paper, the smoke may afterwards be transferred to the paper, showing the curve as a white line against a black ground.

Since writing the above-mentioned paper, the instrument has been used for investigating some other sudden changes of potential besides those produced in the coils of a dynamo armature. It is the purpose of this paper to give some of these, which may seem to be of interest.

First, I shall describe some of the curves obtained while running a separately excited motor by means of a series-wound dynamo. The motor, not being loaded, speeds up until it produces a counter-electromotive force sufficient to cut down the driving current till the series-wound dynamo loses its magnetism, and then has its polarity reversed by the motor, which it then checks and sets revolving in the opposite direction, when the polarity is again reversed, and so on continuously.

The machine used as a motor (see *a*, Fig. 1) was a small shunt-wound Weston dynamo, which at normal output gives 10 amperes at 60 volts. The field of this, which has a resistance of 58.55 ohms, was put in series with a 120-volt incandescent lamp, and was connected to an Edison dynamo which was running at 124 volts.

The brushes of the motor were connected to the terminals of a Siemens and Halske series-wound dynamo *b*,'which at normal rate gives 40 amperes at 60 volts.

When the field of the motor was excited, and the series-wound dynamo was set running, the armature of the motor began to revolve, first in one direction and then in the other, and so on back and forth. During the experiment it made about eight and one-half revolutions in each direction, which was determined by means of a speed counter. It also, by actual count, reversed direction thirty times in twenty seconds, that is, it ran right-handed fifteen different times, and left-handed just as many more, in twenty seconds. A compass needle, held near one of the poles of the series-dynamo, showed, with each reversal of the motor, a reversal of the polarity of the dynamo, which took place, as near as could be observed, while the motor was at its greatest speed in either direction. An ammeter in the circuit showed a reversal of the current at about the same time. The ammeter was a dead-beat one, and was so quick acting, that readings

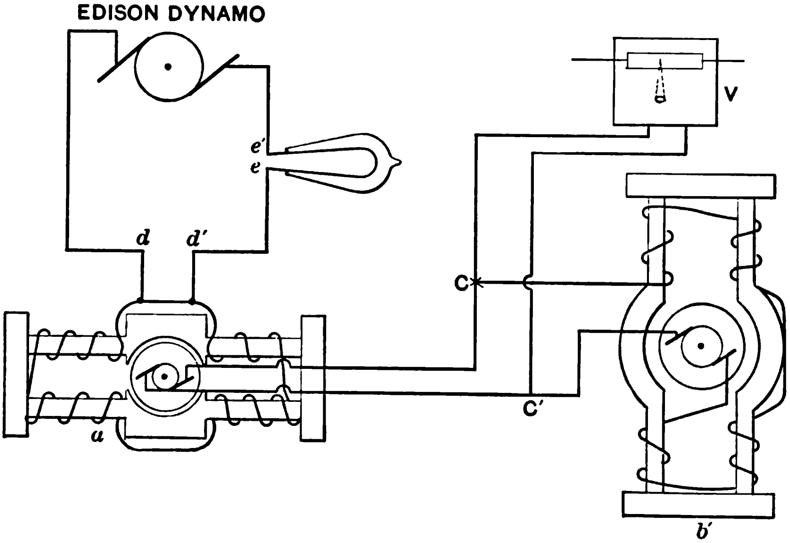

Fig. 1.

could be made with considerable accuracy. The maximum current, which lasted for only a moment each time, was 42 amperes, and the maximum difference of potential of the terminals of the series-dynamo, which also lasted for only a moment, was 41 volts. The incandescent lamp, in circuit with the field, was observed to flash up and be much brighter while the armature of the motor was running in one direction, than while it was running the other. The change in the difference of potential of the terminals of the lamp was found to be from a minimum of 85 to a maximum of 120 volts, while the Edison generator, as said before, was running at a potential of 124 volts. When the motor was not running, the voltmeter around the lamp showed 103 volts, which lies just about half-way between the two readings taken while the motor is in operation. The terminals of the curve-writing voltmeter V were connected at C and C', Fig. 1, and a curve was obtained, the form of which is shown at A, Fig. 2. In drawing the curves shown in Fig. 2, their ordinates were magnified so as to make them more easily studied. The points m, n, and o, on the curve A, correspond very nearly to the times of greatest motion of the motor, and the ordinates of greatest length, very nearly to the times of reversal of motion when no counter-electromotive force is produced. The curve shows that there are no abrupt changes of potential, for it does not differ greatly from a sine curve. The terminals of the instrument were next connected to the points d and d' to show the changes of potential between the terminals of the field of the motor; this gave the curve B: then the curve C was obtained by connecting to the points e and e' to find the fluctuations at the terminals of the lamp. It was found that at the same time when the curve B drops nearly to the base line, the curve C runs up to a point. This occurs while the potential of the dynamo is

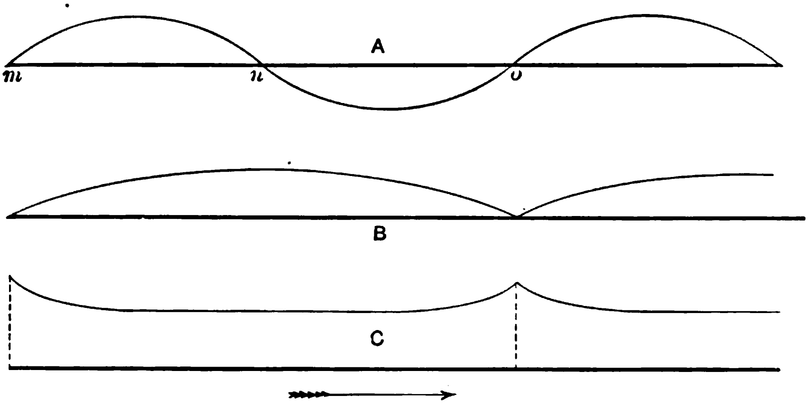

Fig. 2.

changing most rapidly in one direction but not in the other; that is, there is a high point upon the curve *C* for every pair of reversals. The inductive effect in the field circuit, which caused the lamp to change so much in brightness, was produced by the rapidly changing current in the armature, which was also rapidly changing its motion.

Another study which was made, was of the changes of potential which take place between the carbons of an arc light when it is suddenly started up and while the arc is forming. A Weston lamp, which runs at 20 amperes and 30 volts, was taken, and the instrument was connected to the upper and lower carbons, then the smoked drum was made to revolve at a uniform rate, and the circuit was closed through the lamp. The curve produced is represented at *a*, Fig. 3. At the point *t* the switch is closed, and at *u* the arc begins to form: the line from *u* to *v* covers the period during which the carbons are being separated.

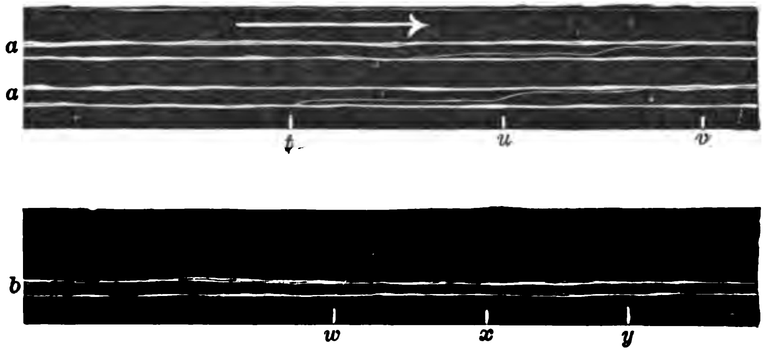

Fig. 3.

Differing greatly from the above curve is the curve *b* obtained from a Thomson-Rice lamp, which operates at 6.5 amperes and 50 volts. In the Weston lamp the carbons touch so long as there is no current flowing, but in the one last mentioned the carbons are held apart by a spring until they are made to touch by the current through the shunt coil of the lamp; then they are separated again, and the arc is formed. There is a cold air gap between the carbons when the circuit is first closed, and the only flow of current that can take place is that through the shunt coil, so the difference of potential between the carbons runs up to some high value, and remains at that till they are made to touch. This is shown by the great height of that part of the curve from *w* to *x*.

This lamp was in series with a rheostat and was connected to an Edison dynamo running at about 125 volts, so the longest ordinates of this part of the curve must represent that voltage. When the carbons first come together, the contact seems to be rather poor, but as the tips of the carbons heat up, it becomes so good that the ordinates of the curve are practically reduced to zero, as shown at y. Immediately after this the carbons are separated, and the curve rises till its ordinates represent about 50 volts, the amount at which this lamp operates.

All of the above-described curves were made over and over again a number of times. They were always of the same form, proving that they were not accidental, but that they represent the actual changes taking place. These are changes which do not manifest themselves in instruments as ordinarily used. Still more delicate and more quickly operating instruments will no doubt develop new facts which will contribute their share toward broadening the field of electrical science.

PHYSICAL LABORATORY OF CORNELL UNIVERSITY,
July, 1893.

IRREGULARITIES IN ALTERNATE-CURRENT CURVES.[1]

BY FREDERICK BEDELL, K. B. MILLER, AND G. F. WAGNER.

CURVES of a more or less sinusoidal nature are commonly plotted to represent the instantaneous values of an alternating current, and various conclusions are drawn from their forms and relative positions. For any perfectly symmetrical generator the curves for successive periods are the same, both in shape and size, but decided differences may exist between the successive portions of the curves for machines which are not symmetrical. In the larger machines this difference is not likely to exceed one or two per cent., not introducing serious error into the mean results, although occasionally a greater difference may be found. In the smaller machines, however, the instantaneous curves show marked irregularities, so that to take one period as typical is not always justifiable.

To investigate this point experimentally, curves 1, 2, and 3 were taken from three small eight-pole Westinghouse alternators, showing the instantaneous values of the electromotive force. The electromotive force was measured by means of a Thomson multicellular voltmeter connected around one of two incandescent lamps connected in series across the terminals of the machine, the whole electromotive force being readily found by a proper calibration. The instantaneous reading was obtained by means of a revolving contact-maker on the shaft which made the contact at a definite point in the revolution of the armature, as indicated by a graduated disk. The three machines from which curves 1, 2, and 3 were taken were apparently alike in all respects. An inspection of the accompanying figures shows a decided difference in the curves from different machines, and also differences between successive periods of the same machine. This is further seen by comparing the areas of consecutive loops of the curves, as marked upon them. The resistance used was non-inductive, and whether the curves represent current or electromotive force is merely a question of scale. In any case, the ordinates of the current curve represent the instantaneous values of the current, $i = \frac{dQ}{dt}$, whence the area inclosed by the curve is $\int idt = Q$; that is, the area inclosed by the current curve is proportioned to the quantity of electricity which flows, and for a complete cycle or revolution of the armature must be zero, inasmuch as there is no continual flow one way or the other. Likewise, the ordinates of the electromotive force curve represent the instantaneous values of the electromotive force, $e = \frac{dN}{dt}$, and the area inclosed is proportional to the change in the number of lines of magnetization through the armature. Evidently the algebraic sum of these areas must be zero for a complete cycle, since the magnetization is the same at the beginning and end of a complete revolution of the armature. To establish this point experimentally, curves 2 and 3 were taken, each for a complete revolution of the armature. In these curves, ordinates represent electromotive force, and abscissæ the

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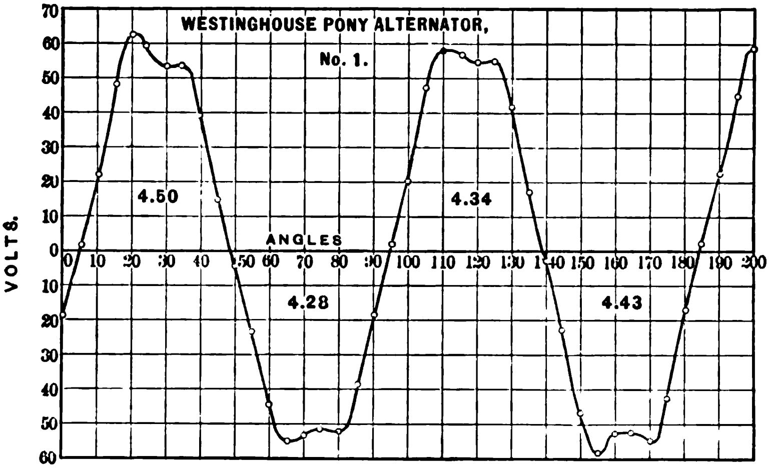
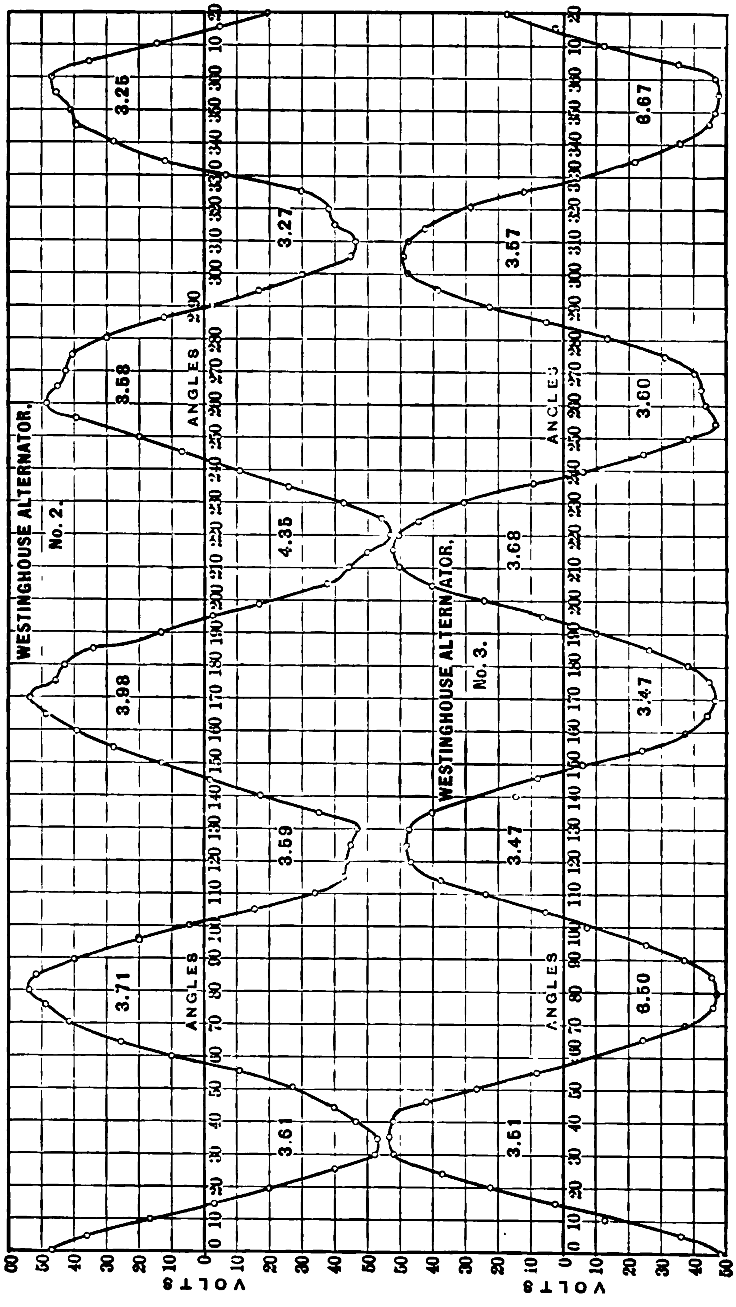
[1] A paper read before the Madison meeting of the American Association for the Advancement of Science, August, 1893.




WESTINGHOUSE ALTERNATOR,
No. 2.
WESTINGHOUSE ALTERNATOR,
No. 3.
ANGLES
ANGLES
ANGLES
ANGLES
VOLTS
VOLTS
3.25
3.27
3.58
4.35
3.98
3.59
3.71
3.61
6.67
3.57
3.60
3.68
3.47
3.47
6.50
3.51
0
10
20
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30
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50

portion of the cycle, with reference to the position of the armature at which readings were taken. The areas of these curves were obtained by a planimeter, and in arbitrary units are: —

Areas, Curve No. 2.		**Areas, Curve No. 3.**	
Positive.	**Negative.**	**Positive.**	**Negative.**
3.71	3.61	3.51	3.50
3.98	3.59	3.47	3.47
3.58	4.35	3.68	3.60
3.25	3.27	3.57	3.67
14.52	14.82	14.23	14.24
Total, – .30		Total, – .01	

Although, separately, the positive and negative areas widely differ, the sum of the positive and negative areas are very nearly equal, thus experimentally establishing the conclusion reached above.

Physical Laboratory of Cornell University,
July 21, 1893.

Note on the Infra-Red Spectra of the Alkalies.

B. W. Snow.

In the first number of the 48th volume of *Wiedemann's Annalen*, Messrs. Kayser and Runge point out certain errors in my article "Ueber das Ultrarothe Emissionsspectrum der Alkalien," a translation of which appeared in the first and second numbers of the *Physical Review*. These authors then show further that, when certain errors are corrected, my experimental values for the wave-lengths of the infra-red cæsium lines coincide as closely as could be expected with the values predicted by means of their empirical formula. I desire, through the columns of the *Review*, to acknowledge this criticism, and, at the same time, to express to these gentlemen my sincere thanks for their kindness in calling especial attention to these errors, which, however, in the form in which the work was conducted, were unavoidable.

That my values for the wave-lengths of the lines in the infra-red spectra of all the metals studied were not without error I was perfectly aware, and it was on this account that I gave in detail on page 220 of the original paper, and on page 40 of the translation, the original data from which was compiled the curve of calibration of the prism. Even assuming no constant error to be present in these determinations, these three series of calibrations differ among themselves by amounts of the same magnitude as the errors which Kayser and Runge point out. From the very beginning I saw at once that any method for wave-length determination by means of a calibrated prism is inferior to the beautiful grating method employed by Kayser and Runge. For three reasons, however, under the conditions under which I worked, the use of the prism seemed preferable to that of the grating, notwithstanding the difficulties to be encountered in obtaining a correct curve of calibration: first, because of the greater intensity of the prismatic spectrum; secondly, because of the freedom in this spectrum from all overlapping spectra; and in the third place, because of the failure of the grating in certain spectra, or in certain portions of spectra, to represent the true distribution of energy. The method then employed in the calibration of the prism was the one first used by Becquerel,[1] and afterwards rediscovered by Rubens.[2] While this method is capable of yielding most excellent results, it is quite impossible to obtain by means of it a determination of wave-length with the accuracy attainable by the use of the grating. This failure in the utmost accuracy of this method lies chiefly in determining the exact point of tangency of the curve representing the energy spectrum, crossed by interference-bands, and the outer and inner envelopes of this curve. In my determination of wave-length, therefore, I used this method simply as the best means at hand, and trusted, with the care employed in the calibrations, that the remaining errors might be small.

The other and no less serious criticism made upon this investigation by Messrs. Kayser and Runge refers to the contamination of the various spectra by the presence of impurities in the salts used. This, too, was a difficulty which I recognized throughout the entire series of determinations, and in the original paper I not only made mention of these impurities, but asked that due allowance be made for their presence. With the exception of the sodium chloride, the various metallic salts were obtained from one of the most reliable chemical manufactories in Germany, and a special request was made that they be sent in the purest possible condition. As there was no means at hand for further purification, these salts were used in the condition in which they were received from the factory, although it needed but a glance at the visible spectrum of any of them to detect therein the presence of easily recognized foreign lines.

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[1] H. Becquerel, Comptes Rendus, XCVII., p. 71, 1883.

[2] Rubens, Wied. Ann., XLV., p. 238, 1892.

Notwithstanding the fact that each infra-red spectrum undoubtedly contained lines more or less intense which did not belong to the metal there studied, I felt in no way justified in making any corrections or alterations in the distribution of energy as I found it in the infra-red. After passing into the unknown region beyond the limits of the visible spectrum, I did not consider myself capable of saying which lines were true and which were false. Kayser and Runge, however, after a careful analysis of my results, have succeeded in detecting in my cæsium spectrum the spurious lines, and have then shown that when these were eliminated, even with no correction applied to the wave-lengths of those remaining, the latter coincide remarkably well with the values they had previously calculated. Kindly permit me to say in conclusion that this result not only gives me the greatest pleasure, but furnishes one of the strongest proofs of the correctness of their formulæ.

Physical Laboratory, University of Wisconsin,
October 13, 1893.

NOTES.

The American Association for the Advancement of Science.—The forty-second meeting was held at Madison, Wisconsin, August 17–22, 1893. The work of the local committees had been unusually well performed. The various sections found themselves luxuriously housed under a single roof, that of the Science Hall, of the University of Wisconsin, while the state house afforded ample accommodation for large evening gatherings. The Saturday excursion to the "Dells" of the Wisconsin River and the Monday evening lawn party and water fête were features of unusual novelty and beauty.

In Physics, aside from the vice-presidential address "On the Phenomena of the Time-infinitesimal," nineteen papers were read. The list was as follows: —

"Application of Interferential Methods to Measurements of Expansion of Long Bars," Edward W. Morley and Wm. A. Rogers. — "A Preliminary Study of the Constants of the Morley Interferential Comparator," Wm. A. Rogers. — "On Physical Addition or Composition," Alexander Macfarlane. — "Some Rapid Changes of Potential Studied by Means of a Curve-Writing Voltmeter," G. S. Moler. — "Some Applications of Electric Heating in Physical Laboratory Practice," Edward L. Nichols. — "Note on the Use of a Rotating Disk in Photometry," Ervin S. Ferry. — "Irregularities in Alternate Current Curves," F. Bedell, K. B. Miller, and W. F. Wagner. — "Experiments with an Alternate Current Condenser," Frederick Bedell, N. F. Ballantyne, and R. B. Williamson. — "Fatigue in the Elasticity of Stretching," Joseph O. Thompson. — "The Electric Strength of Solid, Liquid, and Gaseous Dielectrics," A. Macfarlane and G. W. Pierce. — "Elastic Properties of Glass," W. S. Franklin and L. B. Spinney. — "An Automatic Toepler Pump," Edward W. Morley. — "An Apparatus for the Generation of Oxygen and Hydrogen by Electrolysis," E. L. Nichols and G. S. Moler. — "On So-Called Negative Lightning," W. LeConte Stevens. — "Note on the Surface Tension of Liquids," Ernest F. Nichols. — "Electrolytic Polarization," John Daniel. — "On the Continuous Spectrum of the Alkalies," B. W. Snow. — "On the Effect of Evaporation upon the Relative Dimensions of Bars of Metal Partially Submerged in Water," Wm. A.

Rogers. — "The Effect of Temperature and of Electric Driving on the Period of Tuning Forks," John S. Shearer.

In the case of the first two papers, increased interest was given to an important topic by the exhibition of the interferential comparator in actual operation. The instrument was mounted in the basement of the physical laboratory of the University and a variety of experiments were performed with it during the meeting.

A number of subjects of interest to physicists were presented in other sections of the Association; notably Vice-President Doolittle's address on "Variations of Latitude," and the papers on "The Atomic Weight of Oxygen," by E. W. Morley; on "An Accurate Method of Measuring Heavy Liquid Pressure," by D. S. Jacobus; "On the Changes in the Dimensions of Metals which may be Due to Changes in Molecular Structure Depending on their Age," by Wm. A. Rogers, and on "The Electro-deposition of Iridium," by W. L. Dudley.

The number of papers on physics was as large as at Rochester (1892) or at Washington (1891), although the total attendance in 1893 was not more than two-thirds of that of the previous year, and was less than half of that of 1891.

The Madison meeting was indeed much the smallest of recent years, the registration falling considerably below that of any meeting of the Association since the Saratoga meeting in 1879. There were several local and temporary causes for the diminished attendance, such as the World's Fair in Chicago, and the great financial depression existing throughout the country, but the failure of the Association to increase with the growth of science in the United States is not to be ascribed to any such influences. There is undoubtedly wide-spread apathy among the men who should be leaders in the cause for the promotion of which the Association was organized. There are, for example, eighteen living past-presidents. Of these one only, the retiring president, who had a definite and important function to perform, was present at Madison. Section B, since its organization in 1882, had been presided over successively by eleven vice-presidents, all of whom are still alive. They were all absent this year. That, under these circumstances, the work in physics (to say nothing of other departments) has steadily improved and that the quasi-science, which it is *not* the purpose of the Association to promote and which in earlier years had been so troublesome an element, has so largely disappeared, is due to the exertions of younger physicists who have entered the Association and whose enthusiasm has not yet had time to grow cold.

In what striking contrast to all this is the condition of the British Association which gathers annually nearly all the foremost men of science of the United Kingdom, together with numerous foreigners of distinction. Its discussions constitute one of the events of the scientific year, not for England alone, but for the world. Founded, like the younger association on this side of the Atlantic, for the dissemination of science among the people at large, it has yet had time to inaugurate and carry through to completion scientific undertakings of the highest importance. With us, the task which such an association is to perform, must be done under conditions more difficult than those which have existed in Europe; shall the masters stand aside on that account and leave the work to apprentices?

The Electrical Congress. — The International Congress of Electricians (Chicago, August 21–25, 1893) was not the complete success that it might have been had the arrangements been placed in the hands of a permanent, well-organized, and representative body like the American Institute of Electrical Engineers. To compare the meeting with the latest previous event of the kind, we find the registered attendance to be about half as great as at the Electro-technical Congress at Frankfort (1891) and the number of papers read to be in about the same ratio 31 (at Chicago) and 53 (at Frankfort).

The congress suffered, like the numerous other gatherings organized under the official auspices of the World's Fair Auxiliary, in being obliged to meet in the Bedlam of the Chicago Art Institute. The division into sections of pure theory, theory and practice, and pure practice, while entirely logical, did not result in anything approaching a uniform division of the papers to be presented or of hearers. The first and last sections were, relatively speaking, sparsely attended. In spite of these and various other drawbacks, which, under the untoward circumstances, could not be removed even by the untiring efforts of those upon whom the burdens of organization were laid, the Electrical Congress was an important event. Valuable papers were read, and many interesting discussions occurred. Above all, a gathering of electricians and physicists was brought about, the like of which has never before taken place upon the American continent. Professor von Helmholtz was made Honorary President; Dr. Elisha Gray, Chairman; Professor F. B. Crocker, Permanent Secretary; and Professors Rowland, Cross, and Houston, Chairmen of sections A, B, and C. The following is the programme: —

"On the Analytical Treatment of Alternating Currents," Prof. A. Macfarlane. — "Complex Quantities and their Application in Electrical Engineering," Charles P. Steinmetz. — "General Discussion of the Current Flow in Two Mutually Related Circuits containing Capacity," Dr. Frederick Bedell and Dr. Albert C. Crehore. — "Explanation of the Ferranti Phenomenon," Dr. J. Sahulka. — "Measuring the Power of Polyphase Currents," A. Blondel. — "Extended Use of the Name 'Resistance' in Alternate Current Problems," Prof. W. E. Ayrton, F.R.S. — "Signaling through Space by Means of Electro-Magnetic Vibrations," W. H. Preece, F.R.S. — "Materials for Standards of Resistance and their Construction," Dr. St. Lindeck. — "Variation of P. D. of the Electric Arc with Current, Size of Carbons and Distance Apart," Prof. W. E. Ayrton, F.R.S. — "Ocean Telephony," Dr. Sylvanus P. Thompson, F.R.S. — "Iron for Transformers from the Magnetic Point of View," Prof. J. A. Ewing, F. R. S. — "Note on Photometric Measurement," Prof. B. F. Thomas. — "Some Measurements of the Temperature Variation in the Electrical Resistance of a Sample of Copper," A. E. Kennelly and R. A. Fessenden. — "Various Uses of the Electrostatic Voltmeter," Dr. J. Sahulka. — "On a Method of Governing an Electric Motor for Chronographic Purposes," Prof. A. G. Webster. — "On the Construction of Cables for Subterranean High Tension Circuits," Dr. A. Palaz. — "Periodic Variation of the Candle Power of Alternating Arc Lights," Prof. B. F. Thomas. — "Transformer Diagram Experimentally Determined," Dr. Frederick Bedell. — "London Electrical Engineering Laboratories," Prof. Andrew Jamieson. — "On the Source and Effects of Harmonics in Alternating Circuits," Prof. H. A. Rowland. — "A Pair of Electrostatic Voltmeters," Prof. H. S. Carhart. — "On the Maximum Efficiency of Arc Lamps with Constant Number of Watts," Prof. H. S. Carhart. — "On Direct Current Dynamos of Very High Potential," Prof. F. B. Crocker. — "On an Improved Instrument for Measuring Magnetic Reluctance," A. E. Kennelly. — "The Swinburne-Thompson Unit of Light," Dr. Silvanus P. Thompson, F.R.S. — "Rotary Mercurial Air-Pumps," F. Shulze-Berge. — "The Conversion of Alternating into Continuous Currents," Dr. C. Pollak. — "The Use of Accumulators in Central Stations," Dr. C. Pollak. — "Underground Electric Construction in the United States," Prof. D. C. Jackson. — "A New Incandescent Arc Light," L. B. Marks.

The *chamber of delegates*, which constituted the official portion of the congress, consisted of the following delegates all duly appointed by their respective governments. United States — H. A. Rowland, T. C. Mendenhall, H. S. Carhart, Elihu Thomson, E. L. Nichols. Great Britain — W. H. Preece, W. E. Ayrton, S. P. Thompson, Alexander Siemens. France — E. Mascart, J. Violle, de la Touanne, Hospitalier, Luduc. Germany — H. von Helmholtz, Schraeder, E. Budde, E. Voit, O. Lummer. Italy — Galileo Ferraris. Austria — J. Salhulka. Switzerland — A. Palaz, Thury. Sweden — Wennman. Canada — O. Higman. Mexico — A. M. Chavez.

Professor Rowland was elected president, and Professor Nichols secretary of the chamber.

The principal business transacted by the chamber of delegates consisted in defining the following units, viz.: —

"As a unit of resistance, the international ohm, which is based upon the ohm, equal to 10^9 units of resistance of the C.G.S. system of electro-magnetic units, and is represented by the resistance offered to an unvarying electric current by a column of mercury at the temperature of melting ice, 14.4521 grammes in mass of a constant cross-sectional area and of a length of 106.3 centimeters.

"As a unit of current, the international ampere, which is one-tenth of the unit of current of the C.G.S. system of electro-magnetic units, and which is represented sufficiently well for practical use by the unvarying current, which, when passed through a solution of nitrate of silver in water in accordance with accompanying specifications,[1] deposits silver at the rate of 0.001118 of a gramme per second.

"As a unit of electromotive force, the international volt, which is the electromotive force that steadily applied to a conductor whose resistance is one international ohm, will produce a current of one international ampere, and which is represented sufficiently well for practical use by $\frac{1000}{1434}$ of electromotive force between the poles of electrodes of the voltaic cell, known as Clark's cell; at a temperature of 15 degrees C. and prepared in the manner described in the accompanying specification.[1]

"As a unit of quantity, the international coulomb, which is the quantity of electricity transferred by a current of one international ampere in one second.

"As a unit of capacity, the international farad, which is the capacity of a condenser charged to a potential of one international volt by one international coulomb of electricity.

"As a unit of work the joule, which is equal to 10^7 units of work in the C.G.S. system, and which is represented sufficiently well for practical use by the energy expended in one second by an international ampere in an international ohm.

"As a unit of power the watt, which is equal to 10^7 units of power in the C.G.S. system, and which is represented sufficiently well for practical use by work done at the rate of one joule per second. As a unit of induction the henry is recommended, which is the induction in a circuit when the electromotive force induced in this circuit is one international volt, while the inducing current varies at the rate of one international ampere per second."

The motion to accept the name of Henry for the unit of induction was made by Professor Mascart and seconded by Professor Ayrton. It was unanimously carried.

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[1] The specifications referred to are those of the English board of trade, excepting in the case of the Clark cell, the specifications for which are to be modified by a committee consisting of Professors von Helmholtz, Ayrton, and Carhart.

The chamber made no recommendations concerning the rating of arc lamps, and it formulated no definition of a practical standard of light. The very complete system of notation presented by M. Hospitalier was ordered printed, but without official adoption.

Professor von Helmholtz's Visit to America.—The arrival of Baron von Helmholtz in August aroused much enthusiasm among the younger physicists of the country, many of whom have been his pupils. He was accompanied by the Baroness von Helmholtz and by Drs. Lummer, Lindeck, Feussner, Pringsheim, and Leman of the *Electrotechnische Reichsanstalt* in Charlottenburg.

Some Recent Appointments in Physics.—The following are some recent collegiate appointments: Daniel Shea, Ph.D., to the chair of physics in the Illinois State University; Benjamin W. Snow, Ph.D., to the chair of experimental physics in the University of Wisconsin; Frederick Bedell, Ph.D., as assistant professor of physics in Cornell University; John C. Shedd, M.S., to the chair of physics in Marietta College; Arthur L. Foley, A.M., to the chair of physics in Indiana University; Albert C. Crehore, Ph.D., as assistant professor in physics at Dartmouth College.

NEW BOOKS.

Transactions of the Frankfort Congress.[1]

It was remarked by foreign visitors at the electrical exhibition held in Frankfort in 1891, that it was a German rather than an international exhibition, England, France, and the United States being inadequately represented. The same remark might be made, but with less force, with reference to the International Congress for Electro-technics, held during the exhibition. The report of the proceedings of the Congress contains some forty-eight papers, of which all but seven were presented by electricians from the greater Germany, including Austria and German Switzerland. Of the remainder, six were read by English participants in the Congress, while one was from the hand of Professor Hospitalier of Paris.

The proceedings of the Congress are divided into two parts, the first consisting of seven papers read in general session, the other containing the proceedings of the various sections. For the purpose of considering special topics the Congress was divided into sections, to which were assigned: —

First: Theory and Methods of Measurement.
Second: The Technics of Heavy Current.
Third: Signalling, Telegraphy, and Telephony.
Fourth: Electro-chemistry, and Special Applications of the Electric Current.
The Fifth section, for the discussion of legal questions relating to electrotechnics, listened to no papers, but resolved itself into a commission for the consideration of the subject assigned to it.

In giving the first place upon the programme of the general session to Professor Wilhelm Kohlrausch's paper on the proper training of the electrical engineer, "Welches ist der geeigneste Bildungsgang für Elektrotechniker?" the committee of arrangements made a fortunate selection. The published report (pp. 16–39), by the occasional insertion of bracketted remarks, gives some faint indication of the interest with which the paper was received. To appreciate the situation, however, one must have been present in the auditorium at Frankfort and have listened to the discussion which followed.

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[1] **Bericht ueber die Verhandlungen des internationalen Elektrotechniker Congresses zu Frankfurt am Main. 2 Parts; pp. xxxix. 141 and 393. Johannes Alt, Frankfurt a/M., 1892.**

Kohlrausch, speaking from his experience in the polytechnicum at Hannover, appealed for more mathematics and physics, and chemistry, in the school-training of the electrician. He was met in the discussion by the opinion, already familiar in England and America through the dictum of Kelvin, that electrical engineering is nine-tenths mechanical engineering, and on the part of Slaby of the Charlottenburg technical school, with the statement that all necessary special training in electro-technics might be embraced in a single semèster: " . . . ein meist nur einsemestriger Unterricht im Laboratorium," p. 24.

Physicists in America will read these pages with great interest. Few directors of physical laboratories are there in these days who have not been confronted with this question. Many men who are at present entirely occupied in the training of electrical engineers, began like Kohlrausch, "by being physicists," and these at least will sympathize with him in his plea for more thorough study of the fundamental sciences in the schools.

The most striking paper in the first part of these transactions is that of Zipernowsky on interurban electric railways of high speed, in which are outlined with considerable detail the plans for an electric road between Budapesth and Vienna. That a project which contemplates 250 kilometers an hour as an attainable speed should, after two years, still await fulfilment is not surprising. The reader of Zipernowsky's article, however, will certainly be impressed with the practical character of his plans, and will probably admit that he indicates many of the conditions necessary to the solution of the problem of travel at high speeds.

As of special interests to physicists, two subjects in the transactions of the general sessions may be mentioned. Firstly, the discussion of Hospitalier's system of symbols and nomenclature; which is especially useful in that it brings out the fact, since newly emphasized by the treatment which the same excellent scheme received in the chamber of delegates of the Chicago Congress, that the adoption of a uniform system of notation is not a matter of the immediate future; secondly, the discussion following Epstein's paper on *experiment stations for electro-technical investigation* (pp. 116–133).

It is chiefly to the second part of this report, however, that readers interested in physics will turn, and especially to the section devoted to *theory and systems of measurement.*

Here among other matters, chiefly of minor interest, are three contributions from the "Reichsanstalt" in Charlottenburg, viz. "A Study of the Clark and Fleming Normal Elements" by Dr. Lindeck; "Determinations of the Electrical Properties of Copper-Nickel and Copper-Manganese Alloys" by Dr. Feussner, whose results with the latter class (Weston alloys) confirm in the main those previously obtained by B. H. Blood,[1] and of "The Magnetic Behavior of the Alloys of Iron" by Dr. Holborn.

Of much deeper interest than these, from the point of view of the physicist, indeed by far the most important memoir in the volume, is Professor H. F. Weber's paper on "The General Theory of the Glow-Lamp" ("Ueber eine allgemeine Theorie des elektrischen Glühlichtes").

This paper embodies the researches of many years. In it Weber seeks to establish the possibility of computing accurately the absolute temperature of any incandescent carbon filament by means of an equation of the form

$$C \cdot F(T \cdot e^{aT} - T_0 e^{aT_0}) = \frac{I \Delta P}{J},$$

where T is the temperature of the filament, T_0 of the surrounding globe; I is the current, ΔP the difference of potential between the terminals of the lamp, and J the mechanical equivalent of the calorie. F is the radiating surface, a a constant, called the temperature factor, the numerical value of which (0.0043) is the same for all solids. C is termed the constant of total radiation.

The determination of C for thirty-three kinds of glow-lamps showed that filaments are to be classified in two groups, black ($C = 0.000017$) and gray ($C = 0.000013$).

Making use of these constants in the formula, it was found that the "normal" temperature of all filaments is nearly the same; lying between $T = 1565$ (1292° C.) and $T = 1580$ (1307° C.); also that the entire range of temperatures with which one has to do in the case of the glow-lamp is very small; lying between $T = 1400$ and $T = 1650$ as extreme limits.

The value of Weber's results, if they be found capable of direct experimental verification, is not confined to photometry. They offer a foundation upon which to base quantitative study of radiation at temperatures lying between the red heat and the melting-point of platinum such as has not existed hitherto.

The technical electrician will find in the remainder of this volume many matters of the highest interest. Particularly noteworthy is the treatment of the departments of applied electricity in which the Frankfort exhibition excelled; viz. the application of motors and dynamos of the rotary field system to long-distance transmission.

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[1] See American Journal of Science, Vol. 39, p. 431.

When we compare the Frankfort Congress, as its results appear in this report, with the Paris Congress of 1881 or that of 1889, we find that it has less claim to be considered a world's congress than these. It was more strictly an *electro-technical* gathering than either, or than the Chicago Congress of 1893, the university element even of Germany, being almost entirely absent. Professors in the technical schools were almost universally present, but the entire group of universities of the German empire furnished but seven participants, out of a total of 472 Germans in attendance, which list did not contain the names of von Helmholtz nor G. Wiedemann nor Heinrich Hertz. But one of the seven university professors (Quincke of Heidelberg) read a paper.

On the other hand, the Congress met at an important period in the development of industrial electricity, and while it burdened itself with no attempts at legislation and offered no official recommendations, it afforded occasion for the presentation of many valuable papers and for much fruitful discussion. The manner in which the work of the compilers has been done is worthy of much praise. The detailed reports of the discussions are faithful to a degree of minuteness not often attained, outside of Germany.

E. L. Nichols.

Lehrbuch der allgemeinen Chemie. W. Ostwald. IIte Auflage;
Band II., erster Teil, 1104 pp., Mark 34. Leipzig, W. Engelmann, 1893.

The great advances made in recent years in the theory of chemical processes have made it necessary to rewrite and enlarge Ostwald's famous *Lehrbuch*, which now appears in three volumes. The work is a very clear and complete account of the development of chemical theory, especially from a physical point of view, and includes the quantitative relations constituting the thermodynamical theory of solutions, thermochemistry, electrochemistry and the subject of chemical equilibrium. The historical method of treatment, which was the charm of the earlier edition, is preserved in the new.

An unusual measure of interest is excited by the present volume — on Chemical Energy — for the reason that here for the first time in the history of theoretical chemistry the phenomena of thermochemistry, electrochemistry, etc., are regarded as transformations of the energy forms involved. In the Introduction are emphasized: the substantiality of energy; its forms; its capacity and intensity factors, as

Capacity.		Intensity.		Energy.
distance	×	force	=	distance energy,
surface	×	surface tension	=	surface energy,
volume	×	pressure	=	volume energy,
quantity	×	potential	=	electricity;

further, the intensity law of Helm (1887), that for equilibrium the intensity of an energy form must be uniform throughout the system; and the ***principle of virtual energy changes***, that two energy forms are in equilibrium in a system when their virtual changes are equal. To illustrate this last, suppose a vaporization or similar change, equilibrium being possible between heat and volume energy. The principle requires

$$\frac{\rho}{T}dT = vdp,$$

ρ representing heat of vaporization, etc. This gives

$$\frac{dp}{dT} = \frac{\rho}{Tv},$$

the vaporization formula of Clausius. Again, in a reversible voltaic cell equilibrium may subsist between heat and electrical energy,

$$\frac{Q}{T}dT = edE,$$

$$\frac{Q}{e} = T\frac{dE}{dT},$$

the Gibbs-Helmholtz relation between heat absorption $\frac{Q}{e}$ per unit electrical quantity, and the temperature coefficient of electromotive force. In general, representing capacities and intensities by C and I, energy equilibria are presented by

$$\Sigma CdI = 0.$$

An interesting history of Energetics is appended.

In the chapters on Thermochemistry, Thermodynamics is regarded as that special phase of the energy theory treating the equilibria between heat and work. Here, in addition to the theory, appears a remarkable collection of all reliable thermochemical data.

The conservation of energy, impossibility of creating energy out of nothing ("perpetual motion of the first kind"), and the impossibility of transforming energy without intensity differences, such as gaining work from the constant temperature heat of a mass of water ("perpetual motion of the second kind"), are expressed as the two fundamental

Laws of Energetics.

I. A perpetual motion of the first kind is impossible.

II. A perpetual motion of the second kind is impossible.

The historical account of electrochemistry, from Grotthus to the free ion theory, from Hittorf and Kohlrausch to Arrhenius and Ostwald, from

Volta to Helmholtz and Nernst, is most interesting. The conductivities of electrolytic solutions and the successful application of Ostwald's formula $\alpha^2/(1-\alpha)v = \text{const.}$ are discussed in great detail. With P the electrolytic-solution-pressure of a metal, and p the osmotic pressure of its free ions in a solution of its salt, the potential difference e between metal and solution (*i.e.* the work it does in dissolving) is

$$e = -\int_P^p v\,dP = -RT\int_P^p \frac{dP}{P} = RT\log\frac{P}{p}.$$

The electromotive force of a reversible cell is then

$$E = RT\left(\log\frac{P_1}{p_1} - \log\frac{P_2}{p_2}\right),$$

where R is the gas-constant, and T absolute temperature. These formulæ are used by Ostwald in the entire construction of an electrochemistry; he draws many new conclusions in doing this and confirms them by experiment; the whole reads like an original paper, and is inspiring in the extreme.

The mutual transformations of radiant energy and chemical energy (photochemistry) complete the volume.

J. E. Trevor.

The Dynamo: Its Theory, Design, and Manufacture. By C. C. Hawkins and F. Wallis. 8vo., pp. xiv + 520. London, Whittaker & Co.

The author's endeavor has been "to furnish to students a simple, yet accurate account of the theory and design of the modern dynamo." "Such a book," they explain in the preface, " . . . must combine both the theoretical and the practical sides; . . . must be free from excessive technicality; and should not be too mathematical; . . . its statements should be entirely accurate."

With this object in view, the writers start with an introductory chapter in which they acquaint the reader with the nature of current flow, which is explained at length by the familiar hydraulic analogies, and they state in a general way the functions of the several members of a dynamo. The reader then becomes familiarized, in Chap. II., with the magnetic field, by the use of the exploring magnet and iron fillings, and obtains a rather clear idea of a line of force from a physical standpoint.

The discussion of the magnetic circuit in the following chapter is, for the most part, clear and sufficient, yet it lacks in certain essential points. A dynamo is fundamentally a magnetic circuit, and no treatise on the dynamo is adequate which is lacking in its treatment of the magnetic circuit. Now the relation which exists between the magnetizing force and the current in the magnetizing coils producing it, is the foundation upon which the principles of the magnetic circuit are based, and from which every calculation of magnetic field is made. This theorem, the key to all electromagnetic problems, is thus summarily disposed of: "It will be found," (empirically) "that the strength of the field is . . . proportional to the ampere-turns of the solenoid; and . . . inversely proportional to the mean circumference of the ring. If, therefore, the strength of field within the solenoid or number of lines per unit area of cross-section of the ring is symbolized by H, the general law is summed up by the equation

$$\mathrm{H} \propto \frac{\mathrm{AT}}{l},$$

where AT are the ampere-turns, and $l=$ mean circumference of the ring; or if l is in centimeters, and H in C.G.S. lines per square centimeter,

$$\mathrm{H} = k \cdot \frac{\mathrm{AT}}{l},$$

where $k = 0.4\,\pi$, or 1.256.

The total number of lines, z, passing through the solenoid is equal to the density of the lines per unit area multiplied by the area, or $z = \mathrm{H}a$, where $a =$ area of cross-section in square centimeters encircled by each loop of the solenoid." The constant "$k = 0.4\,\pi$, or 1.256," is thrown in without explanation, and the reader is allowed to rack his brains to discover whether it is empirical or found by analysis. The complete proof of the theorem from which this expression is obtained may, perhaps, be beyond the grasp of the average reader for whom the book is intended, but its statement in fine print or a reference to some proper source, whereby the reader may know that the formula has some rational basis, certainly seems called for.

In the following two chapters, on "The Production of an E.M.F.," and "The Magnetic Pull," the two fundamental equations of the dynamo are developed at length. These are

$$(1)\ \mathrm{E} = \mathrm{BLV};$$
$$(2)\ \mathrm{F} = \mathrm{BL}c,$$

where B denotes induction per square centimeter; L, length of conductor; V, its velocity; and c, the current traversing it.

The main portion of the book is devoted to the full description of the various classes of dynamos, typical forms being discussed at length, with one chapter devoted to the detailed description of actual machines, chiefly of English manufacture. Armatures and field-magnets are fully treated and the various methods of compound winding. The idea of self-induction is well brought out in a separate chapter, the development being "simple, yet accurate."

In the discussion of the heating of dynamos, the authors introduce an empirical formula of their own for the maximum rise of temperature of the outside of the armature in degrees Fahrenheit; viz.:

$$t^\circ \text{F.} = \frac{85 \times \text{w}}{\text{s}\left(1 + \frac{v}{2000}\right)},$$

where w = total watts expended in armature, s = cooling surface of the armature in square inches, and v = peripheral velocity in feet per minute.

The book closes with a detailed problem in dynamo design, and a chapter on the management and working of dynamos.

The authors have confined themselves closely to their subject, the dynamo, and have well covered the ground which they have assigned themselves. To digress is a temptation, but the writer who starts a chapter with a discussion of 'dry-back and water-tube' boilers and in the same chapter develops the differential equations for alternating-current flow through concentric cables (as seen in a recent technical work), is yielding too much to temptation and presuming too much upon the reader's mental elasticity. Messrs. Hawkins and Wallis keep close to their text and present in a logical way that which directly pertains to the dynamo. They make little claim to originality, — with the exception of their treatment of magnetic leakage, the heating of dynamos, and the E.M.F. of an alternator, — and have simply restated the work of Kapp, Hopkinson, S. P. Thompson, and Ewing; also, of Swinburne and Esson. In presenting this material in one volume and systematically developing the subject in a way intelligible to the novice in electrical engineering, the writers have performed a service. Their notation is poor and at times inconsistent. In using the small capitals H and B, they scarcely ascribe the dignity usually accorded these quantities. The use of larger type throughout for symbols would be advantageous. The product of current and electromotive force is variously expressed as *c*E, EC, *ce*, with *ei* and EI at times to avoid monotony. These are minor points, excusable perhaps in a work presenting new material; but in a book of this sort, the merit of which depends entirely upon the style of setting forth, such matters cannot be given too careful attention.

The book is not strong in clear and precise statement; good facts are often concealed amid lengthy explanation. Although the treatment is based upon the simple ideas of current and magnetic field, in no place is the relation given between the volt, the ampere, and the ohm.

The typography is good, the illustrations poor. The book is carefully indexed.

F. Bedell.

Traité de Photométrie Industrielle. A. PALAZ. pp. vii. + 280. Paris, Georges Carré, 1892.

Professor Palaz's Treatise on Industrial Photometry brings together into convenient form a large amount of useful material which, owing to the industrial prominence of electric lighting, is of especial interest at the present day. In the introductory chapter upon the basis of photometry, in which the law of Bouguer and Lambert is developed, together with brief discussion of the sensitiveness of the eye to various colors, the question of the composition of the light, of artificial illuminants, etc., is taken up. The treatment of the subject of luminosity and of the composition of the light of artificial sources is incomplete and not altogether satisfactory. This is one of the most difficult as it is one of the most important portions of the science of photometry. Upon a knowledge of the relative effects of the various wave lengths of the visible spectrum upon the retina, and of the relative distribution of energy in the spectra of sources to be compared, all exact photometry of lights varying in composition must be based. To give any sufficient account of the subject in the four pages allotted to it by the author was of course impossible. The table representing the visual sensation produced by equal amounts of energy of various wave lengths, according to Langley, is of course of the greatest importance, but to be of use in photometry, it must be supplemented by tables giving the absolute distribution of energy in the spectra of various sources of light. Reliable data upon this latter point are unfortunately not easily obtained. There are, however, tables of relative luminosity (*vide* Vierordt, *Annalen der Physik*, Bd. 137, p. 200; Schumann, *Elektrotechnische Zeitschrift*, Bd. 5, p. 224, etc.), which would have been serviceable to the readers of Professor Palaz's treatise.

The introduction of the results obtained by O. E. Meyer (p. 15) is of doubtful value, since they are not consistent with the measurements of Crova, Pickering, Vogel, and Nichols and Franklin, all of which are in all essential particulars in agreement with one another. In the form in which Meyer's data are presented, moreover, they are misleading. The numerical values given in the table are in point of fact the ratios $\frac{\text{gas-light}}{\text{petroleum-light}}$, $\frac{\text{gas-light}}{\text{sun-light}}$, etc., instead of being, as the context would make it appear, the reciprocals of those ratios. The tables from Violle's measurements (p. 14) are a much more valuable feature of this portion of the book, and the development of methods of determining mean horizontal and mean spherical candle-power, with which the first chapter closes, will be found especially useful. When we come to the description of photometers in the second chapter, we find an excellent summary of the well-known types, beginning with the original photometer of Lambert, 1760, and coming down to the forms introduced by Lummer and Brodhun, Crova, Weber, and other contemporaries. The portion of this chapter devoted to the spectro-photometer is unfortunately much less complete than that given to the other forms of instrument. Indeed, spectro-photometry in Palaz's work seems scarcely to have received recognition as a legitimate branch of the subject with which his treatise has to do. The third chapter deals with photometric standards, and forms by far the best summary of data concerning them available at the present time. The British and German candles, the Munich candle, and the bougie decimale are here brought side by side for comparison, together with such light sources as the carcel, the petroleum lamp, the acetate of amyl lamp, the Methven slit, and the pentane standard of Vernon-Harcourt. Numerous tables give the results of the many fruitless attempts made by Monie, Dibden, Uppenborn, and others to fix the proper relationship between these various arbitrary standards. The remainder of the book is devoted to a description of auxiliary apparatus, such as the mirror, the revolving wheel, the various forms of holder for getting the distribution of candle power in the case of incandescent and arc lamps, and to the photometry of the incandescent lamp and of the arc lamp. The treatment of these subjects leaves comparatively little to be desired, and it will serve a useful purpose in bringing into juxtaposition material hitherto scattered and difficult of access. Taken all in all, Professor Palaz's treatise is a great advance upon those which have preceded it. It is much more complete than the works of Dibden and of Kruess, and it is written in a broader and more scientific spirit. It will be found a convenient and fairly accurate work of reference for those who have to do with practical photometry, and beyond that will give the reader a glimpse into the field of investigation which this branch of optics presents.

E. L. NICHOLS.

A Guide to Stereochemistry. ARNOLD EILOART. 96 pp. Price $1.00. New York, Alexander Wilson, 1893.

The theory of geometrical isomerism, more briefly termed Stereochemistry, strives to account for the fact that some organic compounds of identical composition and strikingly similar chemical properties yet differ slightly in certain properties, by ascribing to the molecules of these substances different arrangement in space of their constituent atom groups. There has been of late much speculation regarding the stereochemistry of carbon and nitrogen compounds, and it is summarized in the book by Dr. Eiloart, which is designed to serve as a text-book. The author devotes very little attention to the established features of the theory, but discusses the more recent speculations in considerable detail. For this reason the classical little *Stéréochimie* of van't Hoff (new edition, 1892) is likely to be of more service in giving a clear idea of results actually reached. The latter half of Dr. Eiloart's book is an alphabetical list of literature citations, and to it is appended an illustrated description of wooden models (tetrahedral) designed to represent the "structural formulæ" of organic compounds.

J. E. T.

Experimental Science. By GEORGE M. HOPKINS. 8vo, pp. xiii. + 840. New York, Munn & Co., 1893. 17th edition.

The appearance of the seventeenth edition of this book is evidence of the very large number of persons who find pleasure in the phenomena of physics, particularly when presented in simple guise. The extraordinary popularity of Mr. Hopkins' book is likewise an indication of the prevalence of innate mechanical skill in this country; for a very large proportion of his readers doubtless are not readers merely, but repeat some of the experiments with apparatus of their own constructing.

A work of this kind defies classification. While the traditional sequence of our text-books is followed, it is in no sense a text-book; much less can it lay claim to be a logical well-proportioned treatise. It is entirely devoid of theory and innocent of even the most elementary mathematics; it deals with the most familiar topics of experimental physics, topics many of them already used and abused in a multitude of text-books. Nevertheless, to put this work down as a member of that family, already too large, the "Science made easy," the "A B C of natural philosophy," the "Science of familiar things" family, whose physics is not physics, whose chemistry is not even alchemy, would be a gross injustice. With all its simplicity of statement it is seldom grossly inaccurate; in spite of the plethora of material, which in a book bound to logical continuity of treatment would lead to embarrassment and confusion, the complete independence of each paragraph from all that precedes or follows, makes it as easy to use, thanks to an excellent index, as a dictionary. To the novice, without in any degree supplanting a good text-book, it will be an attractive work, while on the other hand few professional demonstrators of physics will glance over its pages without gleaning therefrom suggestions of new and ingenious methods of illustration.

E. L. NICHOLS.

JOHN TYNDALL.

Volume I. *January–February, 1894.* *Number 4.*

THE

PHYSICAL REVIEW.

A STUDY OF THE POLARIZATION UPON A THIN METAL PARTITION IN A VOLTAMETER. I.

By John Daniel.

Introduction.

THE curious polarization phenomena of very small electrodes in a sulphuric acid voltameter bearing a strong current, accounts of which appeared in the *Annalen* during the winter of 1892, suggested to Dr. L. Arons, of the University of Berlin, to try a very thin metal partition in a voltameter — expecting, as he said, that there would be a development of heat at this partition, resulting in the destruction of the partition. Dr. Arons used gold-leaf as a partition. The gold-leaf was pasted with Canada balsam over a hole about 15 mm. in diameter bored in a glass plate, which was slipped into the groove of a wooden or vulcanite frame in the middle of the voltameter. This did not give a tight partition, but sufficed for the observation that there was not even visible development of gas on the gold-leaf; whereas platinum 0.02 mm. thick substituted for the gold-leaf showed profuse development of gas with the current strength used, even when punctured with a hole 2 mm. square. I think Dr. Arons also tried the gold-leaf partition in a solution of a salt of copper or of silver, and observed that the metal was deposited on the partition.

At the suggestion and under the kind direction of Professor A. Kundt and Dr. L. Arons, I undertook a more thorough investigation of the subject.

Several problems presented themselves:—

(1) By quantitative measurement to determine whether there be a critical thickness below which there will be no polarization at the partition, and if so, to determine this thickness.

(2) To determine the other critical thickness for which the polarization is as great as for very thick plates.

(3) The quantitative measurement of this polarization with the same current strength for plates of various thicknesses between these limits.

(4) By varying the current strength in these different cases, to determine what function the polarization is of the current strength.

(5) To learn how the polarization in these cases varies with the time during which a given current strength flows, readings being made periodically until the polarization becomes constant, or nearly so.

These are the principal direct problems that have been attacked. Numerous minor observations have been made and will be discussed in their place. The experiments have been made in a room of fairly constant temperature, and no attempt has been made to account for slight variations of temperature as affecting resistance, etc., as the quantities to be measured are interesting mainly as regards their relative values; but especially because so slight a correction would be cloaked by such unavoidable variations as irregular escape of gas from the electrodes or partition, in case of its development there.

Apparatus.

The instruments employed, as well as the plan of the experiments, are very simple. As a galvanoscope, as well as current measurer, a well-damped ring galvanometer was used. The galvanometer has four coils of about 22 ohms each, which may be connected as desired. Deflections were read with a telescope from a scale divided into millimeters, fixed at a distance of 1.5 m. The instruments are supported by an independent stone pier. As nearly as possible the same deflection of the galvanometer was always used, the resistance being so adjusted as to effect this, and the "figure of merit" of the galvanometer for this position was determined by connecting a Daniel cell with the galvanometer in series with a known resistance of such value as to give the deflection to be used, and found to be, by Ohm's law,

$$\gamma = 0.000000265 \text{ ampere per scale division.}$$

For another position used later, $\gamma = 0.000000224$.

The connections for the galvanometer circuit are shown in Fig. 1, in which *B* is a battery of storage cells, *V* the voltameter (with partition *a*), *C* a commutator, *G* the galvanometer (with key *k*). *R* is an auxiliary resistance in the main circuit, r_1 a resistance with which the galvanometer is in shunt, ρ an auxiliary resistance of 1010 ohms. The total resistance in the galvanometer circuit is 1100 ohms. *I* is the main current in amperes; i_1 and i_2 are the currents in the branch circuits.

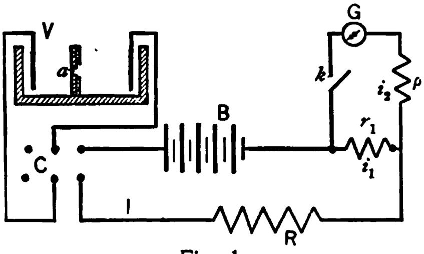

Fig. 1.

To measure the strength of current flowing through the voltameter at any time, we observe the scale deflection, and apply the following simple relations of current and resistance: —

$$(1) \quad I = i_1 + i_2,$$

$$(2) \quad i_1 r_1 = i_2 r_2.$$

But $\quad i_2 = \gamma a$, and $r_2 = 1100$, always.

$$\therefore\ 1100 \times 0.000000265 \times a = i_1 r_1 = i_2 r_2.$$

Here, r_1 being known, i_1 is determined, and from (1), *I* is found. The shunt resistance, r_1, was changed as the main resistance *R* was altered, so as to maintain the same deflection of the galvanometer.

For measuring the ohmic resistance of the voltameter, Kohlrausch's method with induction coil and electrodynamometer was used.

The connections for this measurement are indicated in Fig. 2, in which *B* is the battery, *C* the commutator, *I* an induction coil, *E* an electrodynamometer (with key *k*), *R* a resistance box of the P.O. pattern, and *V* the voltameter.

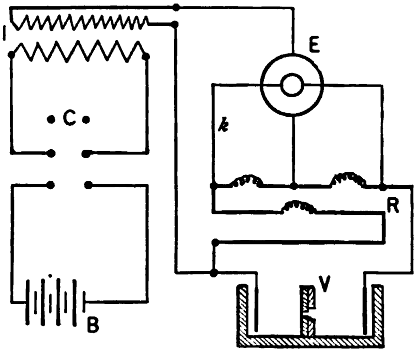

Fig. 2.

A solid iron core was used in the induction coil to avoid the great difference in the direct and inverse currents which exists when a wire core is used, and which results in a polarization of the voltameter. The P.O. box was used as a wheatstone bridge, of which the voltameter was one arm, and the movable coil of the electrodynamometer was connected across the branches of the bridge, the fixed coils being in series with the main circuit. Being thus used only as a galvanoscope, its constant need not be discussed; suffice it to say that the arrangement was sensitive to several decimals, though resistance was measured only to hundredths of an ohm.

The voltameter is simple, though its construction involves the only novelty in the apparatus. The voltameter first used was a glass vessel 18 cm. long, 6 cm. wide, 8 cm. deep, with a hard rubber partition sealed in with sealing-wax, in the middle. The opening in the vulcanite was 3 cm. wide and extended to within 2 cm. of the bottom. This frame for the partition was made by sealing two pieces of vulcanite together with a rim between of the same thickness as the glass plates bearing the metal partitions, which were slipped down into the groove thus formed. This was convenient, as the plates could be easily and quickly changed; but a quantitative test showed leakage around the edge of the glass plate of nearly 20 per cent of the conductivity when the plate was open, using solid and bored glass respectively; hence the results obtained with this voltameter are only relative, — not quantitatively correct.

This voltameter and its results would not be mentioned here but for the fact that I was so careless as to think that this tight-fitting glass would not allow sensible leakage under the circumstances, and to make a long series of measurements before testing the leakage quantitatively, and that, too, with some solutions which I have not had time to repeat.

The voltameter as finally adopted consists of a glass vessel, 20 cm. long, 12 cm. wide, 11 cm. deep. In the middle is fixed, by sealing-wax, a plate-glass partition, 3 mm. thick, with a hole in the center, 2 cm. in diameter. A smaller glass plate, with hole 1.5 cm. in diameter, is sealed over this opening, the metal partition being first sealed over the hole in the small glass. This gives two water-tight compartments, with no electrical communication except through the metal under experiment, which covers the hole in the glass. The end electrodes are of platinized platinum, 4 by 6 cm., and 0.1 mm. thick. With $CuSO_4$ solution, copper electrodes were used. Very thin plates of metal, even ordinary gold-leaf, were easily sealed over the hole in the small glass plate by laying the foil on smooth paper, carefully melting sealing-wax placed around the hole in the form of powder, and then cautiously picking the thin metal up with the melted wax. This accomplished, it is easy to seal the small glass plate over the opening in the partition. Each side of the voltameter could then be filled up to the lower edge of the opening; and, by carefully pouring the solution alternately in the two sides, little at a time, using a glass rod, the vessel could be filled above the gold-leaf, which showed surprising strength in being able to stand a difference of level of several millimeters.

For the voltameter in Fig. 1. we have the following relations: —

The voltameter included in the circuit and the partition open,

$$I = \frac{E - e}{R + r_3 + \frac{1}{\frac{1}{r_1} + \frac{1}{r_2}} + v}$$

We are concerned only with the *change* in the auxiliary resistance, and not its absolute value, in order to make the current the same when the partition is changed, and hence may simplify the case by representing the sum in the denominator by W; therefore, —

$$\left.\begin{array}{ll}(1) & I=\dfrac{E-e}{W}, \text{ with partition open.}\\[2ex] (2) & I=\dfrac{E-e-x}{W-w}, \text{ with metal partition.}\end{array}\right\} \therefore \frac{E-e}{W}=\frac{E-e-x}{W-w}$$

$$\therefore\ x=w\left(\frac{E-e}{W}\right)=Iw.$$

(A) $x=Iw=$ polarization on the partition.

(3) $I=\dfrac{E}{W+w_1-v}$, with voltameter cut out of the circuit.

$\therefore$ from (1) and (3),

(B) $e=I(w_1-v)=$ polarization on the end electrodes.

In every case the auxiliary resistance was so adjusted as to make the current as nearly as possible the same with partition in and partition out. This could not be done exactly, but was corrected by interpolation. We shall be mainly concerned with the polarization on the partition, and we see from (A) that this polarization expressed in volts is the simple product of current strength in amperes and the *change* of resistance necessary to keep the current constant when the partition is added that it was with partition open.

The same box of resistance was used for all purposes (in determining the "figure of merit" of the galvanometer, the resistance of the voltameter, and as rheostat to regulate the main and branch currents); hence we are not concerned with the question as to whether the units are true ohms.

Solutions used as Electrolytes.

With the old voltameter, relative values have been obtained for 30% H_2SO_4, 5% H_2SO_4, and very dilute H_2SO_4, such as to give voltameter resistance of about 20, 60, and 100 ohms. With the new voltameter, quantitative measurements have been made with

$$30\%\ H_2SO_4;\ \text{Sp. Gr.} = 1.22;\ v = 1.1\ \text{ohms.}$$
$$CuSO_4;\ \text{Sp. Gr.} = 1.12;\ v = 24.5\ \text{ohms.}$$
$$NaCl;\ \text{Sp. Gr.} = 1.12;\ v = 5.5\ \text{ohms.}$$

Also some observations with

$$3\%\ H_2SO_4;\ \text{Sp. Gr.} = 1.02,$$
$$KOH;\ \text{Sp. Gr.} = 1.11,$$

and *very* dilute sulphuric acid.

As partitions in the voltameter, platinum, gold, silver, and aluminum have been used, such as are described below. The thickness of the thick plates was measured with a micrometer gauge; that of the thin plates was calculated from their size, weight, and specific gravity. The plates marked S. & W. were from Sy and Wagner, of Berlin, and are supposed to be quite pure. Those marked M. are from F. Mueller, of Dresden, and are some pure, as indicated, others of the quality of the finest gold-leaf, *i.e.* 0.925 pure. It will be convenient in the tables to refer to these partitions by the *numbers*, and the following description will suffice once for all: —

THICKNESS OF PARTITIONS, EXPRESSED IN MILLIMETERS.

No.	Gold.		Platinum (pure).		Silver (S. & W., pure).	Aluminum (commercial).
0	S. & W., pure	0.25				
1	M., pure	0.0472	S. & W.	0.1	0.0023	0.4
2	M., $\frac{925}{1000}$	0.0241	S. & W.	0.002		0.00051
3	M., $\frac{925}{1000}$	0.00433	M.	0.000152		
4	M., $\frac{925}{1000}$	0.00183	S. & W.	0.0173		
5	M., $\frac{925}{1000}$	0.000586	S. & W.	0.00407		
6	M., $\frac{925}{1000}$	0.000382				
7	M., $\frac{925}{1000}$	0.000087				

The series of gold plates is fairly complete. They were beaten to order by Ferdinand Mueller in Dresden. Effort was made to get a similar series of platinum, but as platinum foil for "silvering" seems to have fallen into disuse, I was unable to find any one prepared to make it for me. This is unfortunate, as platinum is

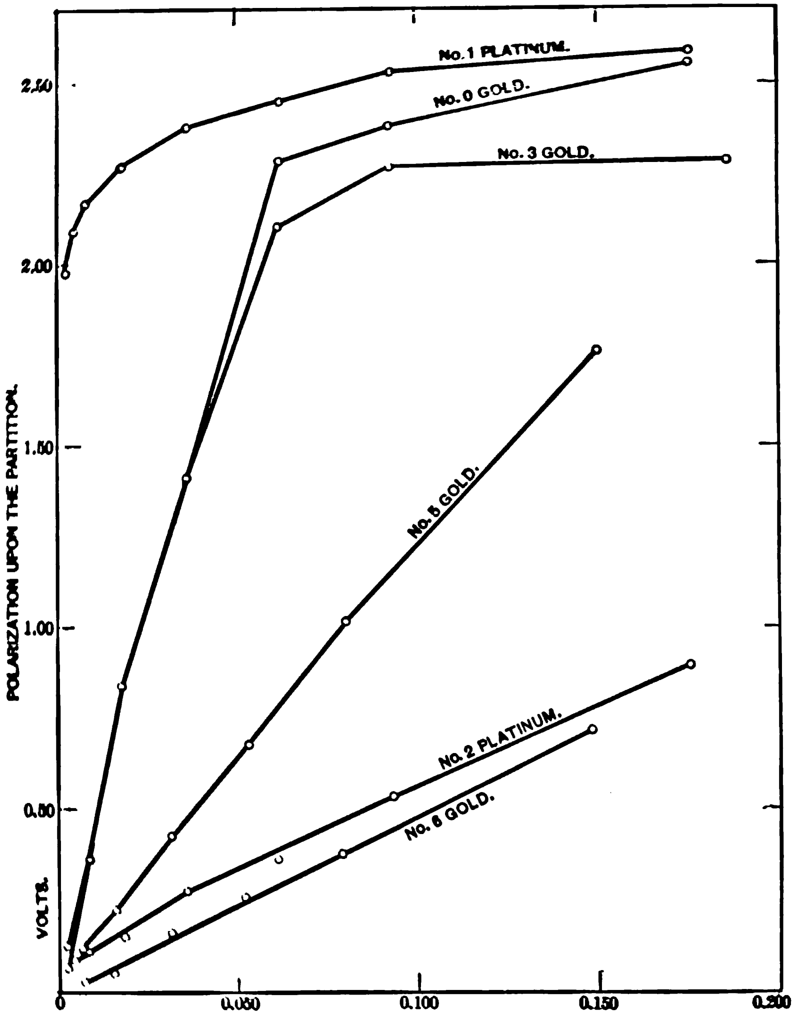

Fig. 3.

the only metal which seems to remain free from chemical action under a strong and long-continued electric current. However, as No. 2 and No. 3 of the platinum happen to fall below and between the critical limits respectively, they serve as a very important check upon the results obtained with the more complete gold series, and show that these results are at least qualitatively correct. Below will be found reasons for believing them to be quantitatively correct.

In the tables which follow, $I =$ current in amperes passing through the voltameter; $x =$ polarization, in volts, on the metal partition; $v =$ ohmic resistance of the voltameter, partition open.

Table I. was obtained with the old voltameter.

TABLE I.

H_2SO_4	V	I	Gold Plates.					
			No. 1.	No. 3.	No. 4.	No. 5.	No. 6.	No. 7.
30 %	1.6	0.1345	0.67	0.78	0.50	0.52	0.35	0.0
30 %	1.6	0.0665	0.33	0.35	0.23	0.24	0.16	0.0
30 %	1.6	0.0443	0.22	0.26	0.17	0.17	0.12	0.0
30 %	1.6	0.0222	0.12	0.14	0.10	0.10	0.05	0.0
5 %	5.7	0.78	1.42	1.65	1.19	0.98	0.70	0.04
5 %	5.7	0.0513	0.93	1.11	0.79	0.67	0.46	0.02
5 %	5.7	0.031	0.56	0.68	0.48	0.41	0.28	0.01
Very	101.7	0.0047	1.67	1.72	1.21	1.06	0.70	0.04
Dilute	101.7	0.0029	1.02	1.07	0.74	0.67	0.42	0.03

The results in Table II. were obtained with the new voltameter without leakage. They are shown graphically in Fig. 3. The readings were taken after waiting two minutes for the current to become steady when a new plate was inserted, or the resistance changed for a new current strength; and in case of the thicker plates where the time change in the polarization was more pronounced and continued longer, five to ten minutes was allowed between readings, or in each case until the galvanometer indicated that the current had become about constant. Later tests given in another table show that this time change of polarization, especially for thick plates, though at first rapid and then very slow, really continues some time.

TABLE II.

CURRENT ALTERNATELY DECREASING AND INCREASING.

30% H_2SO_4; $v = 1.1$ ohms.

I	0.175	0.093	0.061	0.037	0.018	0.009	0.004	0.002
No. 0 Au. (25 mm.)	[1]2.83 2.66 [2]2.66 [3]2.66 2.38	2.37 2.34 2.39 2.56 2.25	2.03 2.15 2.19 2.33 2.15	1.28 1.45 1.42 1.53 1.15	0.67 0.71 0.73 0.79 0.55	0.33 0.35 0.38 0.39 0.33	0.17 0.16 0.18 0.20 0.19	0.09 0.09 0.11 0.12 0.12
No. 1 Au.	[4]2.36	2.39	2.29	2.27	2.18	2.10	1.90	1.02
No. 1 Pt.[5]	2.52 2.59	2.48 2.52	2.42 2.44	2.39 2.39	2.28 2.28	2.17 2.18	2.08 2.09	1.98 1.98
No. 2 Pt.[6]	0.89 0.89	0.53 0.53	0.36 0.35	0.26 0.25	0.15 0.15	0.108 0.108	0.085 0.085	0.07 0.07
No. 1 Ag.[7]	0.87 [8]0.53	0.46 0.28	0.42 0.31	0.26 0.15	0.20 0.15	0.12 0.12	0.09 0.096	0.08 0.08

REMARKS.—[1] Profuse gas. [2] Without opening circuit. [3] Decreasing after circuit was closed 2.5 hours. [4] Very long between readings; gas. [5] Gas; negative side of partition gilded, the solution having been often used with gold plates. [6] No visible gas. Plate gilded. [7] No gas. [8] Oxidized and very irregular.

No. 1 Al. too much oxidized and too irregular to measure.

	No. 2 Pt.					No. 4 Pt. (very constant).		
	H_2SO_4 30%.	$v = 1.1.$	NaCl.	Sp. Gr. = 1.09.	$v = 6.24.$	NaCl.	Sp. Gr. = 1.12.	$v = 5.5.$
I	0.176	0.092	0.177	0.036	0.089	0.176	0.089	0.035
X	1.13	0.58	3.34	1.55	3.00	3.71	3.58	3.31

REMARKS.—With $CuSO_4$, Sp. Gr. = 1.12; $v = 24.5$; Cu. electrodes. No. 1 Pt. very constant and, for $I = 0.18$, $x = 2.19$. No. 7 Au., No. 3 Pt., and No. 2 Al., no polarization. No. 1 Ag. dissolved.

TABLE II. (*continued*).

30% H_2SO_4; v = 1.1 ohms.									
No. 2 Au.	I	0.152	0.081	0.053	0.032	0.016	0.008	Long time readings; gas.	
	X	3.05	2.96	2.98	2.86	2.79	2.72		
No. 3 Au.[1]	I	0.184	0.093	0.061	0.037	0.019	0.009	0.004	0.002
	X	2.57 2.29	2.42 2.28	2.00 2.11	1.41 1.41	0.83 0.83	0.37 0.37	0.15 0.15	0.07 0.07
No. 5 Au.	I	0.150	0.080	0.053	0.032	0.016	0.008	No visible gas.	
	X	1.76 1.73	1.02 0.99	0.68 0.65	0.42 0.41	0.22 0.22	0.105 0.105		
No. 1 Pt.[2]	I	0.168	0.091	0.062	0.036	0.018	0.009	0.004	0.002
	X	2.87	2.72	2.65	2.55	2.39	2.23	2.07	1.99

REMARKS. — No. 7 gold, no gas: circuit closed 2.5 hours. $I = 0.15$; no polarization. No. Al. and No. 3 Pt., no gas; no polarization. [1] Gas developed. [2] Fresh solution, circuit closed 4 hours.

	No. 6 Au.				No. 1 Au.[1]		No. 2 Au.[3]	
	I	X	X		I	X	I	X
30% H_2SO_4; $v = 1.1$.	0.149	0.71	0.71	3% H_2SO_4; $r = 6.3$.[3]	0.174	2.78	0.156	2.67
	0.079	0.38	0.38		0.089	2.66	0.091	2.59
	0.052	0.25	0.25		0.060	2.65	0.061	2.61
	0.032	0.15	0.15		0.036	2.61	0.037	2.56
	0.016	0.06	0.06		0.018	2.55	0.018	2.49
	0.008	0.03	0.03		0.009	2.49	0.009	2.43
	No visible gas.				0.004	2.38	0.004	2.43
					0.002	2.30	0.002	2.28

REMARKS. — [1] Long-time readings; gas; gold oxidized. [2] Long-time readings; gas. [3] A trial made with No. 7 gold; circuit closed 2.5 hours; no gas; showed X too small to measure.

TABLE III.

CHANGE OF X WITH TIME. VALUES FOR DIRECT (*a*) AND REVERSED (*b*) CURRENT.

I = 0.175; H_2SO_4, 30%; v = 1.1 ohms.

Time.	End electrodes.		No. 1 Pt.[1]		No. 2 Pt.[2]		No. 0 Au.[3]		No. 4 Au.[4]
	a	*b*	*a*	*b*	*a*	*b*	*a*	*b*	*a*
1′	1.73	1.44	1.91	—	0.97	0.96	2.14	1.94	2.37
3′	—	—	2.08	2.01	1.02	1.02	2.25	—	2.43
5′	1.77	1.76	2.14	2.13	—	1.06	—	2.06	2.49
10′	1.78	1.77	2.21	2.15	1.07	—	2.34	2.14	2.55
20′	1.79	1.78	2.24	2.17	1.09	1.11	2.45	2.17	2.63
30′	1.80	—	2.30	—	1.10	1.12	2.48	2.19	2.66
45′	1.81	1.81	2.33	—	1.11	1.13	—	2.25	2.70
60′	—	—	2.35	—	—	—	—	2.27	2.54
75′	1.84	—	2.36	2.35	—	—	—	2.28	—
100′	1.84	1.84	—	—	—	—	2.53	—	2.41
150′	—	—	—	—	—	—	—	—	2.27
220′	—	—	—	2.46	—	—	2.55	—	0.00

REMARKS. — [1] Slightly gilded. [2] No visible gas. [3] Oxidized; gas. [4] Gas, till oxidation had reduced the plate to a thin film. The oxide then fell off, and polarization ceased.

Time.	No. 1 Pt.[1]		No. 5 Pt.[2]		No. 2 Pt.[3]	No. 4 Pt.
	a	*b*	*a*	*b*	*a*	*b*
1′	2.17	1.98	2.37	1.99	2.97	3.51
3′	2.34	2.10	2.45	2.08	—	3.56
5′	2.43	2.16	2.53	2.12	3.34	3.67
10′	2.53	2.30	2.65	2.24	3.34	3.71
20′	2.68	2.42	2.74	—	3.34	3.72
30′	—	2.53	2.83	—		
45′	2.85	2.73	—	2.59		
60′	—	—	—	2.63	Gas.	Gas.
75′	—	—	—	2.67		
100′	2.92	—	2.90	—		
150′	—	2.87	—	—		
220′	—	—	—	2.68		

REMARKS. — [1] Fresh solution; 30 % H_2SO_4. [2] NaCl. (Sp. Gr. 1.09), $v = 6.24$. [3] NaCl. (Sp. Gr. 1.12), $v = 5.5$.

No. 3 Pt., No. 7 Au., and No. 2 Al. showed no polarization.

The following is a sample page of my note-book, showing how the observations were recorded and the tables derived: —

NO. 7 GOLD IN 30% H_2SO_4.

Zero of scale.	Scale reading.	Deflection, a.	Shunt, r_1.	Auxiliary resistance, R.	Branch resistance, r_2.	Sensibility, for 1 ohm, σ.	Change of resistance, w.	Polarization, $Iw = x$.
337.5	940.0	—	—	—	—	—	—	—
336.0	939.0	603.0	1	28	1100	20.0	0.1	0.014
338.0	981.0	—	—	—	—	—	—	—
336.0	981.0	644.0	2	55	1100	12.0	0.02	0.016
338.0	974.0	—	—	—	—	—	—	—
337.0	973.0	636.0	3	85	1100	7.5	0.0	0.0
338.0	976.0	—	—	—	—	—	—	—
337.0	977.0	639.0	5	143	1100	5.0	0.0	0.0
338.0	984.0	—	—	—	—	—	—	—
337.0	983.5	646.0	10	287	1100	2.5	0.2	0.003
337.0	958.5	—	—	—	—	—	—	—
337.0	958.5	621.0	20	597	1100	1.0	1.0	0.008

The No. 7 gold removed with a glass rod, not opening circuit.

						Current I.
336.0	940.5	—	—	—	—	
336.0	942.0	605.0	1	28	1100	0.1492
336.0	979.3	—	—	—	—	—
336.0	981.3	644.3	2	55	1100	0.0795
336.0	972.0	—	—	—	—	—
336.0	972.0	636.0	3	85	1100	0.0524
336.5	975.0	—	—	—	—	—
336.0	976.0	639.0	5	143	1100	0.0316
336.5	981.5	—	—	—	—	—
336.0	982.0	645.0	10	287	1100	0.0161
336.0	956.5	—	—	—	—	—
336.0	956.5	620.0	20	597	1100	0.0078

The two readings in each case are for decreasing and increasing current respectively.

Then, $i_2 = \gamma a \quad = 0.000000224 \times 605 = 0.0001355.$

$i_1 r_1 = i_2 r_2 \quad = 0.0001355 \times 1100 = 0.14905 = i_1$, since $r_1 = 1.$

$I = i_1 + i_2 = 0.1492$ amperes.

$w = R - R' =$ two scale divisions $= \frac{2}{20} = 0.1$

$(20 = \sigma =$ scale divisions for changing R one ohm).

$I w = x = 0.014$ volt.

In this manner, from such a table in the note-book, one line in Table II. was obtained. The sample page of my notes for No. 7 gold was selected to illustrate the method of observation and reduction, because it also serves as a fair sample of many efforts to measure quantitatively the polarization on gold-leaf in good-conducting (30 %) H_2SO_4. One has only to look at the time variation of the polarization on the end electrodes to be able to account for such slight variations from zero as occur here (see Table III.). Having become satisfied from many such series of observations that there was no measurable polarization on gold-, aluminum-, and platinum-foil for the currents used, I then adopted an efficient method of testing them for polarization in various solutions, viz. to allow the strongest current to flow through the plate long enough to become constant, and then, without opening the circuit, to remove the metal partition with a glass rod and simply observe for a change of deflection of the galvanometer. On this method of observation is based the statement under the tables, that for these thinnest partitions there is no polarization for the currents used. The qualitative results of Table I., with the old voltameter, seem to indicate pretty clearly a small polarization on No. 7 gold in very dilute (bad-conducting) acid. A preliminary test with the new voltameter of water containing only a few drops of acid, using gold-leaf as a partition, showed sufficient polarization to develop visible gas.

TABLE IV.

RELATION OF POLARIZATION TO THICKNESS; 30 % H_2SO_4; GOLD PLATES.

Current (I).	No. 6, 0.00038 mm.	No. 5, 0.00059 mm.	No. 3, 0.00433 mm.	Ratio, $d^2 : x$.
	x	x	x	
0.01	0.04 volts	0.9 volts	0.40 volts	441 : 484
0.02	0.09	0.26	0.77	1274 : 1081
0.03	0.13	0.39	1.13	1911 : 1694
0.04	0.19	0.50	1.48	2450 : 2299
0.05	0.28	0.76	2.02	3724 : 3388
0.10	0.47	1.23	2.35	6027 : 5681
0.15	0.71	1.76	2.35	8624 : 8591

From this table it seems that for a given current strength the polarization is fairly proportional to the square of the thickness of the partition for plates near the lower critical thickness (see Fig. 4). The last column of Table IV. shows this ratio as worked out for No. 5 and No. 6 gold.

We may now briefly summarize the experimental observations.

(1) The polarization on a gold-leaf partition in good-conducting H_2SO_4 is zero, or too small to detect with our apparatus, for the range of current used.

(2) The "critical thickness" in good-conducting solutions of H_2SO_4, $CuSO_4$, and NaCl is *greater* than 0.00009 mm. for gold; 0.00015 mm. for platinum; and 0.0005 mm. for aluminum, under the above conditions. It is *less* than 0.0004 mm. for gold; 0.002 mm. for platinum; and 0.002 for silver.

(3) The "upper critical limit" of thickness under these conditions seems to be about 0.004 mm., rather less than for No. 3 gold.

(4) Tables I., II., and III. all point to the conclusion that between the "critical limits" of thickness the polarization for a given current increases with the thickness.

(5) Table II., showing relation of polarization to current, expresses two interesting facts: (*a*) that the polarization on "thick" plates is about the same, in this voltameter, for all currents be-

tween 0.2 ampere and, say, 0.01 ampere, provided time enough be allowed in each case for the current to become constant; *i.e.* between the upper limit of current at which the development of gas is so profuse as by mechanical obstruction and irregular escape to interfere, and the lower limit, at which the formation of gas is no faster than it can be dissipated. (*b*) Quite different is the case for "thin" plates, where, within the limits of current and thickness prescribed, the polarization is dependent upon the current and gives for each thickness a different curve, or rather straight line, for they are all straight lines converging to the origin, and differ-

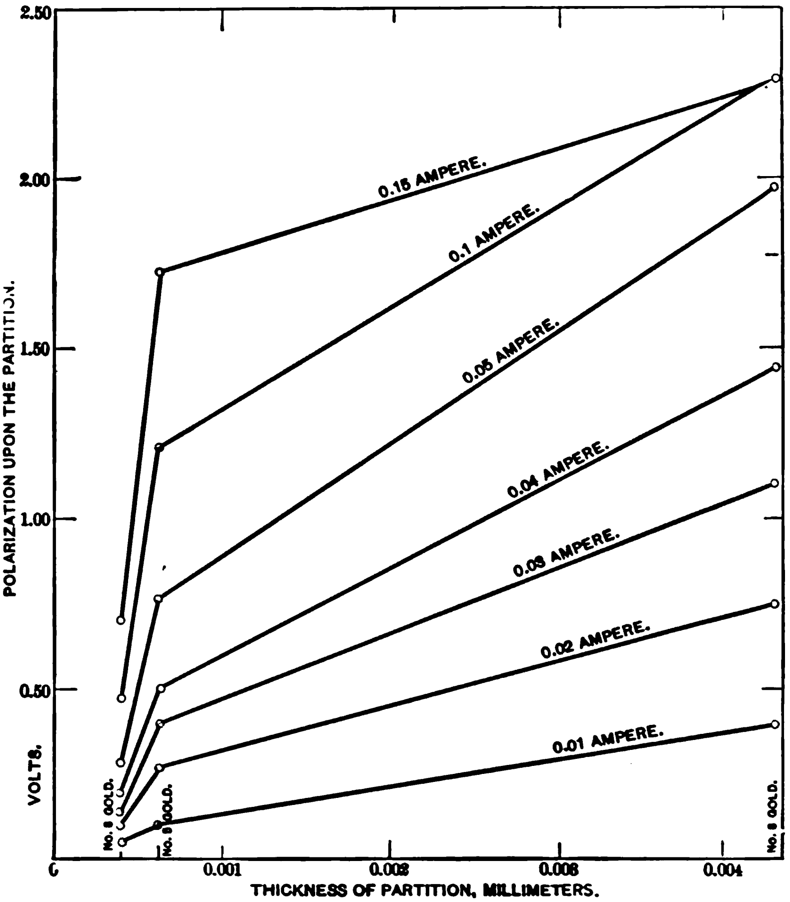

Fig. 4.

ing only in *slope*. The current strength at which the polarization on very thin plates would reach a maximum is far above that used, being, perhaps, expressed in amperes instead of tenths and hundredths.

By *thick* plates are defined those above the "upper critical limit"; by *thin* plates, those below this limit of thickness.

(6) Inspection of Table III., which gives the time change of the polarization, will show a similar distinction between "thick" plates and "thin" plates, as was noted in the last paragraph; viz. that for thick plates the change is considerable and continues slowly for hours; for thin plates, the change of polarization with time is both less pronounced and extends over much less time.

(7) It was noted, especially in the case of $CuSO_4$ as electrolyte, that there was polarization on gold-leaf if the gold exposed contact with the solution some distance beyond the edge of the hole in the glass plate to which it was sealed; thus in $CuSO_4$, for the stronger currents used, there was a symmetrical deposit of copper decreasing in thickness from the outside toward the center, and vanishing at a small distance from the edge of the hole, this distance being less the stronger the current. If only one corner was left exposed, the copper was deposited there. This phenomenon was further tested by bending a thick strip of aluminum 4 cm. long into the shape of a narrow U and simply hanging this U in the *open* hole of the glass partition in $CuSO_4$ and closing the circuit on the voltameter; the two ends of the metal strip being thus in contact with the $CuSO_4$ on opposite sides of the glass two cm. from the edge of the opening, there was decided deposit of copper on one end and escape of oxygen from the other end.

(8) In $CuSO_4$, all the plates except those below the critical thickness were destroyed by oxidation. No. 1 silver was destroyed in less than one minute. Of course gold and silver above the critical thickness could not be used in NaCl because of chemical action, though the thinnest plates were quite unaffected. Only the No. 7 gold was tested in KOH, as it dissolved the sealing-wax.

(9) Thick plates of gold were strongly oxidized in H_2SO_4, especially with strong currents. Thin gold plates were appar-

ently only oxidized under action of strong or long-continued currents. Compare Tables II. and III. Silver was even more easily oxidized than gold. Aluminum was so intensely oxidized by the current that no satisfactory measurements could be made for this metal, though the thin foil was unaffected.

(10) With H_2SO_4 as electrolyte, after a thick plate of pure gold had been used as partition for the time change of Table III., the end cathode was found to be gilded. A thick platinum plate being then substituted for the gold in the same solution for the results of No. 1 platinum in Table III., the platinum partition was found, on removal, to be gilded. The polarization for No. 1 platinum in this case was somewhat less than for the same platinum after both it and the end electrodes were thoroughly cleansed, the electrodes re-platinized, and fresh solution made.

(11) The polarization in $CuSO_4$, using copper electrodes, reached a maximum almost immediately, and remained very constant. The maximum polarization for thick platinum in $CuSO_4$ was hardly 75 per cent that for the same in H_2SO_4. In NaCl the polarization became constant very quickly also, but its value was decidedly greater, especially on thin plates, than in H_2SO_4; though the same distinctive behavior of thick and thin plates was observed.

(12) In H_2SO_4 of different concentrations, the maximum polarization for a partition was of the same order of magnitude; but its value for very weak currents was decidedly greater in weak solutions than for the same current in stronger solutions up to 30 per cent. This shows itself especially with thin plates, and also in the shorter time required for thick plates to reach a maximum polarization with weak currents. The greater change in temperature and the greater change in concentration of weak solutions may account for this.

For currents between 0.1 and 0.2 ampere, the polarization on the end electrodes was 1.84 for H_2SO_4; 1.98 for NaCl; and 0.00 for $CuSO_4$, with copper electrodes; though if the current density was too great, or the time too long, the anode would oxidize and become irregular.

C. Fromme, in a paper, "Ueber das Maximum der galvanischen

Polarisation von Platinelektroden in Schwefelsaeure" (*Annalen d. Physik u. Chemie*, Band XXXIII., S. 80–126), states that the maximum polarization varies both with the concentration and the relative size of the electrodes, the extreme limits being given as 1.45 to 4.31 volts—the minimum polarization coinciding with maximum conductivity. His method for measuring polarization was somewhat similar to that used in this work. As bearing upon "the change of polarization with time," I would refer especially to the investigation of Dr. E. Root upon this subject, discussed by Professor von Helmholtz.[1] These experiments by Dr. Root seem to prove clearly that the liberated ions penetrate deeply into the electrode, even when liberated upon but one side of it, as in his experiment. I take great pleasure in expressing here my thanks and deep obligation to Professor A. Kundt and Dr. L. Arons for their kind sympathy and direction in this work.

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[1] Wissenschaftliche Abhandlungen, Bd. I., p. 835.

PHYSICAL LABORATORY, UNIVERSITY OF BERLIN,
August, 1892.

SOME MEASUREMENTS OF THE TEMPERATURE VARIATION IN THE ELECTRICAL RESISTANCE OF A SAMPLE OF COPPER.[1]

BY A. E. KENNELLY AND REGINALD A. FESSENDEN.

I.

PRECISION in the determination of the temperature variation of resistance in copper is important not only to electrical science, but also to its applications. Our estimate of the temperature of remote or inaccessible positions, as, for example, the ocean bed on which a submarine cable lies stretched, or the interior layers of a dynamo armature winding, are often directly dependent for their accuracy upon the completeness of our knowledge of this temperature coefficient.

Electrical text-books, in stating the temperature coefficients of copper, usually quote the results of Dr. Matthiessen, or of Dr. Siemens, or both. The results of these two authorities are discordant.

Within the range of chamber temperatures, say from 0° C. to 35° C., the difference between these results is practically of little importance. Taking, however, the resistivity of copper at zero centigrade as unity, its resistivity at 100° C. is 1.422 by Matthiessen's observations, and 1.388 by those of Siemens, a variation of nearly 2½ per cent, while above 100° C., or below 0° C., this discrepancy increases rapidly.

First in order of date are the elaborate researches of Matthiessen (and of his collaborator Von Bose), appearing in the *Philosophical Magazine* for February, 1857, and February, 1861, also in the *Philosophical Transactions* for 1858, 1860, 1862, and 1864, the most important series from our present standpoint being those for 1862. The wires tested were varnished with shellac, and immersed in a bath of oil whose temperature was raised by the application of Bunsen burners, and read off by an immersed thermometer. Readings were taken both in ascending and descending series of temperature, and observations are adduced in support of the statements that the application of varnish did not affect the results, and that the observations on a wire heated in a bath of oil were sensibly the same as when the heating took place in air. Matthiessen took six copper wires, all from the same electrolytic source, three annealed and three hard-drawn. Six observations are given of the resistances of each between 0° and 100° C. Having ascertained that all 36 observations accorded very fairly with a parabolic relation between conductivity and temperature, the parabola of closest conformity computed by the method of least squares was $\lambda = 1 - at + bt^2$, or numerically $\lambda = 1 - 0.0038701\,t + 0.000009009\,t^2$, λ being the conductivity at temperature t° C. From this equation, the resistivity (the reciprocal of λ), retaining the terms necessary for accuracy in the fifth digit, at the limit of 100° C. becomes,

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[1] Read at the International Electrical Congress, Chicago.

$$\rho = 1 + 3.8701\,t \times 10^{-3} + 5.968\,t^2 \times 10^{-6} - 1.177\,t^3 \times 10^{-8} - 9.93\,t^4 \times 10^{-11} - 2.769\,t^5 \times 10^{-13}.$$

The graph of this equation, taking values of ρ as vertical ordinates from a horizontal axis of temperatures as abscissæ, yields a curve bending upwards, so that the temperature variation increases with the temperature, the increase in resistance per degree centigrade being at 0° C. 0.387 per cent, and at 100° C. 0.50 per cent of the resistivity at zero C. Matthiessen points out that this bending upwards of the curve is distinctly indicated by his results, and that no straight line can represent them. Not only the 36 observations on copper wires, but more than 200 quoted observations on other metals, all point to a temperature coefficient augmenting with temperature, and negative the supposition of a simple linear relationship between the variables.

Dr. Siemens' researches formed the subject of his Bakerian lecture in 1871. They were undertaken with the object of obtaining a practically reliable scale for the electropyrometer which bears his name, rather than for the direct purposes of scientific research. After pointing out that Matthiessen's formula can on its own evidence be only fairly applicable between the limits of 0° and 100° C., Dr. Siemens proceeds to advance some interesting, although arbitrary, hypotheses for the law of temperature variation in metallic resistances, and then shows that his experimental observations on copper and four other metals are capable of close representation by the empirical formula so obtained. These observations were made on wires heated in air, and also in oil, up to 350° C. with mercury thermometers, and in one series up to 850° C. with a platinum ball pyrometer, an instrument whose indications assumed a constant specific heat in the platinum ball throughout the range of temperature employed.

Dr. Siemens' formula for copper is $\rho = 0.026577\sqrt{T} + 0.0031443\,T - 0.29751$, where ρ is the ratio of the resistivity at any absolute temperature, T, to that at zero centigrade, or $T = 273°$. For 100° C., or $T = 383°$, $\rho = 1.3885$, and the rate of increase of resistivity is at 0° C. 0.394 per cent, and at 100° C. 0.383 per cent of the resistivity at zero C. The graph of the equation is a curve bending slowly downwards towards the axis of temperatures, and the temperature variation diminishes as the temperature rises. All the 170 observations recorded in the paper indicate that the curve bends downward, while all the 250 observations in Matthiessen's 1862 paper make the curve bend upwards. The discrepancy between these two series of results becomes very noticeable between 70° and 100° C.

Professors Dewar and Fleming have published in the *Philosophical Magazine* for October, 1892, a number of observations on the resistivity of metals and alloys at temperatures between −200° C. and +100° C., one series for copper being included. The resistivity of the copper wire is stated to have been 1353 C. G. S. U. at 0.7° C., equivalent to 1349 at 0° C.; and since Matthiessen's standard is 1594 at zero C., this represents a conductivity 18 per cent higher than Matthiessen's standard. Aside, however, from this remarkable and perhaps debatable statement, the graph of the observed values of resistance with respect to temperature is very nearly a straight line throughout the whole range, the resistivity at 100° C. being 1.424 times greater than that at 0° C., and the temperature coefficient being approximately 0.424 per cent per degree centigrade for all temperatures between $-200°$ and $+100°$ C.

Messrs. Cailletet and Bouty, in the *Comptes Rendus* for 1885, give an observed temperature coefficient for copper of 0.418 per cent per degree C., expressed by $\rho_t = \rho_0 (1 + 0.00418\,t)$ between zero and $-58°$ C., and 0.425 per cent from $-69°$ to $123°$ C., $\rho_t = \rho_0 (1 + 0.00425\,t)$.

In Poggendorf's *Annalen* for June, 1858, Herr Arndtsen quotes a uniform temperature coefficient of 0.369 per cent per degree from zero to 200° C. He gives, however, one series of observed resistance with a copper wire (containing 0.15 per cent of iron) between 0° and 100° C., showing a linear relation, or a temperature coefficient of 0.394 per cent per degree C., $\rho_t = \rho_0 (1 + 0.00394\,t)$, and he points out that the divergences from the straight line are within the limits of the observation error.

In view of the discrepancies existing between these best known measurements of the temperature coefficient of copper, Arndtsen, Cailletet, and Bouty giving results practically represented by straight lines, Siemens' results with the line bending distinctly downward and Matthiessen's results with the line bending as distinctly upward, the writers of this paper made a number of measurements in the spring of 1890 upon a sample of copper wire. These measurements were made with great care, and repeated until similar results were obtained in successive series. The wire tested was sealed within the bulb of an air thermometer, so that there could be no appreciable variation between the temperature of the wire itself and the temperature indicated by the pressure of the air in the bulb it occupied. The final results, after full corrections for expansion of the bulb, etc., indicate a linear relation between the resistance and temperature of the wire between the limits of 20° C. and 250° C., represented by the equation $\rho_t = \rho_0 (1 + 0.00406\,t)$, indicating a uniform temperature coefficient of 0.406 per cent per degree C. throughout the range, the maximum observed being 0.4097 per cent, and the minimum 0.399 per cent at any point. The details of these measurements are here submitted in the form of an appendix, not only in support of the statements made concerning them, but also because the experimental arrangements of apparatus finally successful were the outcome of a series of experimental failures, and it is believed that the details of construction may be of service to those who desire to adopt the same method of measurement.

Concerning the conclusions that may be drawn, we feel only justified in saying that good commercial copper can be found in which a linear relationship holds between resistance and temperature between 20° C. and 250° C., and within the range of small observation errors. It is, of course, possible that in different samples of wire the temperature coefficient may increase or diminish with the temperature. In other words, the second differential coefficient of resistance with respect to temperature may perhaps in some samples be either positive or negative, but it seems desirable that fresh measurements should be made, and evidence collected to settle this point, and we submit the view that the best experimental means of measuring the temperature, and to ensure the coincidence between the measured temperature and that of the tested wire, is to enclose the latter in the bulb of an air thermometer in the manner here described.

II.

General Outline of Apparatus.

Within the cylindrical glass tube of an air thermometer was enclosed about 240 cm. of fine copper wire. Short platinum wires, sealed into the glass bulb, brought this copper wire into communication with apparatus for measuring its resistance. The bulb rested in an oil bath heated electrically, and the height of a mercury column required to balance the pressure of the internal air was measured by cathetometer, at the moment that the resistance of the copper wire was noted. The bulb was thus operated as an air thermometer at constant volume, and corrections were applied to the expansion of the glass walls of the bulb, and for the variation of barometric pressure during the period of observations; also electrically for the resistance of the leads up to and including the platinum seals.

Thermometer Bulbs and Contents.

A vertical section of the thermometer bulb and its accessories is shown to approximately one-sixth true scale in Fig. 1. *AB* is the cylindrical glass bulb 15 cm. long, and 3.15 cm. external, 2.85 cm. internal diameter. The capacity of the chamber so enclosed was approximately 75 c.c. Three separate platinum wires, 2 cm. long and 0.048 cm. in diameter, were sealed in at *D*, and welded with three exterior copper wires, *E*, *F*, and *G*, forming the leads to the apparatus. The central wire was also welded within the bulb to a copper wire, 10 cm. long and 0.08 cm. in diameter, and the seal supported this wire axially along the bulb to its termination at *H*.

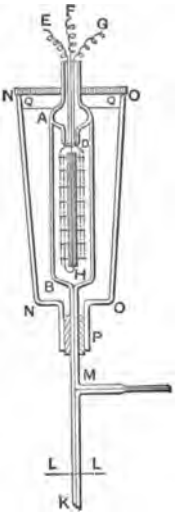

Fig. 1.

This copper wire *DH* was entirely covered by a glass tube, 10 cm. long and 0.25 cm. in external diameter, closely threaded, on which were beads or short cylinders cut from glass tubing, each bead being 1 cm. long, 0.28 cm. internal and 0.4 cm. external diameter. The beads served as sleeves or washers to clamp between them at their junctions circular disks of mica, 2.15 cm. in diameter and 0.02 cm. thick, with a hole in the centre, 0.25 cm. in diameter. Two circles of holes were drilled concentrically around this disk, each circle having fifteen holes. The diameters of these circles were 1.0 cm. and 1.65 cm. respectively. A plan view of a mica disk is shown in Fig. 2, to full scale.

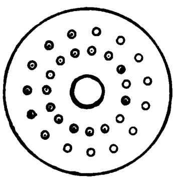
Fig. 2.

The copper wire tested was of good commercial quality, 0.002″ (0.0051 cm.) in diameter, and double silk covered. The silken covering was dissolved off with hot caustic soda, diluted so as not to oxidize the wire, and the bare wire threaded up and down through the mica disk, and parallel to the axis of the bulb; first filling up the inner cylinder of holes, and then the outer. The inside end was soldered to platinum wire, No. 1, connected with the lead *E*; the final exterior end soldered to platinum wire No. 2, from lead *G*; and the junction or midway point connecting the inner and outer cylinders was connected to the copper wire at *H*, which communicated with lead *F*. By this means two separate loops of wire of nearly equal length were provided within the bulb, arranged in two cylinders, one within the other. The outer cylinder between leads *F* and *G* had a diameter of 1.65 cm.; the inner, between *E* and *F*, the diameter of 1.0 cm. The object of this arrangement was to furnish a check by duplicate, upon the observed resistance of the tested wire, and also to ascertain what difference in temperature, if any, existed between the air at radius 0.5 cm. and the air at radius 0.8 cm. from the axis of the bulb, the source of heat being entirely external. Had the mean temperature of the inner and outer cylinders differed at any time in virtue of gradient of temperature within the bulb, by one-tenth of one degree centigrade, it would have been within the range of observation; and had it amounted to one-fifth of a degree, it could not have escaped detection. No appreciable difference was at any time discovered, the increase of resistance in the two cylinders being always proportionally coincident, so that finally the two loops were combined into one for facility in observing, and only divided for an occasional check. The diffusion and convection of the air within the bulb must have been sufficiently rapid to practically equalize the temperature within the air space. The course of the wire within the bulb is indicated by the dotted lines in Fig. 1.

The bulb communicated with the mercury through a glass tube *DK*, round which a hair, secured 15.2 cm. below the bulb at *LL*, formed the fiducial mark. The diameter of this tube was 0.6 cm. externally and 0.1 cm. internally, and the volume of its bore from bulb to fiducial mark, including the offset tube at *M*, was 0.3 c.c. or $\frac{1}{250}$ of the volume of the bulb.

The bulb was held vertically upon the axis of a glass percolator, *NNOO*, the lower tube passing through a rubber stopper at *P*. The space between the bulb and walls of the percolator was filled with boiled linseed oil up to the level *QQ*, and a cylindrical grid of platinoid wire, not shown in the figure, was immersed in the oil. A steady current of from 2 to 3 amperes through this platinoid wire of 16 ohms resistance served to raise the temperature of the oil and immersed bulb at a convenient rate. A disk of asbestos cloth, *NO*, rested as a cover upon the percolator, and more of the same material was wrapped around the exterior surface *NNOO*, in order to impede the escape of heat from the chamber.

When the apparatus was in position, a vertical wooden board, rising above a wooden trough set to catch any mercury that might escape, supported a long vertical tube in front of the cathetometer. A bottle of mercury and an equilibrating bottle of sand were supported by a cord running over pulleys fixed into the ceiling. Lowering the sand bottle raised the bottle of mercury and the attached rubber tube, bringing the level of the mercury in the index tube (allowing for capillarity) up to the same elevation.

Electrical Measuring Instruments. — The resistance of each cylindrical loop of wire within the bulb was about 10 ohms at the normal or initial temperature, in the vicinity of 20° C., making 20 ohms in all, and at the highest temperature reached in the measurements, these resistances doubled, so that the total range of observed change in resistance amounted to 10 ohms in each loop, or 20 when the loops were in series.

In the first trial, the resistances were measured by Wheatstone's bridge. This method was found to be unsatisfactory both in respect to swiftness and precision. In swiftness because the readjustment of the balance required some seconds to effect, and during that time the temperature of the wire might have altered; and in precision, since the resistance of the leads had to be subtracted, and these were likely to vary appreciably with the temperature of the room.

Later, two differential galvanometers were employed. One balanced the two loops within the bulb against each other to detect variations of temperature within the bulb, and the other compared the resistance of both loops in series against a fixed standard in platinoid wire (30 ohms).

Finally, the first galvanometer fell into disuse, since no variation could be detected between the resistances of the two loops, and measurements were confined to the second differential galvanometer, with one wheatstone bridge reading as a check at the outset, and another at the culminating temperature of the series.

The differential galvanometer method is very convenient for such measurement. It enables the variations of the tested resistance to be constantly watched, and balance can be quickly readjusted with a dead-beat instrument by rheostat thrown into the circuit of preponderating influence.

The main circuit consisted of the bulb with its two loops in series, a standard resistance of 30 ohms in platinoid, and an additional resistance of 50 ohms in a rheostat. A single Edison-Lalande cell delivered a steady current of from 6 to 7 milliamperes through this circuit. The two coils of the differential galvanometer had about 2700 ohms each; one was connected to the terminals of the standard 30 ohms, and the other to the terminals of the bulb. A change of resistance in circuit of either coil, amounting to 1 ohm, could be plainly observed on the scale.

Modus Operandi. — The bulb was repeatedly exhausted through its attached tube, then heated to 200° C. under a vacuum for an hour, to expel all moisture, and finally filled with dried air. It was then tightly connected with the mercury apparatus and index tube by double rubber pipes well jointed. The offset tube was opened through a chamber containing calcium chloride so as to acquire internally the pressure of the air, which was noted by mercurial barometer. The level of the mercury was raised at the same time to the fiducial mark, the temperature of the oil bath around the bulb observed, and the resistance of the enclosed copper wire measured. The temperature of the air close to the index tube was also taken. The offset tube was then sealed off with a blow-pipe, and the circuit of the platinoid grid in the oil bath closed. The level of the mercury was then raised in the index tube about 3.5 cm. to cathetometer observation, by lifting the mercury bottle through that distance, driving mercury into the bulb tube above the fiducial mark. The increasing temperature and pressure of the air within the bulb slowly forced the mercury in this column back to the fiducial mark, leaving the index elevation practically unaltered. At the moment that the mercury crossed the fiducial mark, the resistance for balance at the differential galvanometer was noted. The level of the mercury in bottle and index tube would then be raised again another 3.5 cm., the mercury column elevated above the fiducial mark, the cathetometer reading taken, and the resistance balance when the mercury again crossed the line. This process was repeated step by step until the maximum temperature was reached. Meanwhile readings were kept of the barometer pressure in the room, and the temperature of the air by the side of the index tube.

No appreciable time lag existed in the bulb, owing to its small thermal capacity. After the highest desired temperature had been attained in a series, the differential galvanometer would indicate a lowering in the resistance of the copper wire, within ten seconds of the interruption of the heating current. The reduction in temperature could be detected electrically ten or fifteen seconds before the mercury could be observed to retreat. This lag in the mercury was traced, principally, at least, to the influence of fluid friction in the narrow tube. Tapping this tube with the finger was found to accelerate the mercurial movement. Later, the bulb tube whose internal diameter was 1 mm., in order to have as little volume of unheated air-space as possible, was welded into one of larger caliber (0.2 cm.) just above the fiducial mark, increasing volume of air-space outside the bulb to 0.4 c.c., but materially diminishing the fluid friction, so that tapping the tube was scarcely necessary.

A similar series of cathetometer and resistance readings was obtained as the oil bath and bulb cooled down. An entire set of observations generally lasted four hours.

The mercury employed was filtered and kept clean. The internal diameter of the index tube was 0.6 cm., but as its capillarity error entered equally into all the readings, no correction was required on this account.

The coefficient of expansion of the glass forming the bulb was measured by taking two globes blown from the same tubing as the bulb, drawing out their necks to a fine bore, filling them with a measured mass of mercury, at normal temperature, heating them up to their necks in mercury over a sand bath to 157° C. till they ceased to overflow, and the mass of mercury remaining was observed after cooling down.

The mean cubical expansion of the glass was

$$3\beta = (2800 \pm 113) \times 10^{-8},$$

and this value was assumed in all voluminal corrections.

Results. — The first four series of observations were rejected. When computed and plotted, with ordinates of resistance to abscissas of temperature, they showed curves bending slightly upwards, after the manner of Matthiessen's, but the descending curve was distinctly below the ascending branch, so that the diagram appeared to form a loop as though the wire had a lower resistance for a given temperature when cooling than when heating. The cause of this error is not known, but may have been due to fluid friction of the mercury in the bulb tube. With each successive series the curvature of the line diminished, the ascending and descending branches approaching one another. The fifth series was considered to be satisfactory. Its graph is practically a straight line with a slight divergence between upward and downward branches. The reduced observations are given in the accompanying

TABLE I.

	Temperature °C. by air thermometer.	Linear coefficient per °C.	Weighted mean.	Discrepancy.
	29.8	0.004037	0.004059	0.000022
	67.0	0.004076		0.000017
	79.8	0.004066		0.000007
	92.9	0.004085		0.000026
	104.7	0.004063		0.000004
	113.0	0.004055		– 0.000004
	131.4	0.004062		0.000003
	143.0	0.004058		– 0.000001
	155.0	0.004031		0.000022
	168.6	0.004037		0.000023
	181.7	0.004110		0.000051
Maximum	196.2	0.004084		0.000025
	169.0	0.004062		0.000003
	135.3	0.003999		– 0.000060
	123.6	0.003995		– 0.000064
	108.0	0.004010		– 0.000049
	92.0	0.004017		– 0.000042

The sixth series appeared to be the most reliable. Its graph is also practically a straight line up to the maximum temperature of 255 C. with no appreciable deviation between the ascending and descending branches, the latter being carried in this instance no lower than 207° C. The reduced observations referred to linear coefficients are given in the following

TABLE II.

	Temperature of copper wire in ° C.	Linear coefficient of increase per degree.	Weighted mean.	Discrepancy.
	27.8	0.004007	0.004065	– 0.000058
	42.64	0.003984		– 0.000081
	56.55	0.004038		– 0.000027
	72.25	0.004027		– 0.000038
	87.11	0.004063		– 0.000002
	105.07	0.004172		0.000107
	123.99	0.004080		0.000015
	139.48	0.004141		0.000076
	154.17	0.004143		0.000078
	169.94	0.004082		0.000017
	183.71	0.004028		– 0.000037
Accidental fall in temperature	181.68	0.003968		– 0.000097
	197.03	0.003990		– 0.000075
	215.53	0.004022		– 0.000043
	230.59	0.004022		– 0.000043
	244.87	0.004049		– 0.000016
	255.26	0.004070		0.000005
Maximum	255.26	0.004074		0.000009
	255.26	0.004088		0.000023
	235.44	0.004097		0.000032
	207.52	0.004088		0.00023

Between the fifth and sixth series the apparatus was taken apart, the mercury refiltered, the bulb re-exhausted, and then replaced. The method of measuring the temperature of the copper wire here described and advocated involves considerable more arithmetic labor in computing the results, owing to corrections for barometric pressure, temperature of the mercury column in the index tube, and expansion in the bulb, but it eliminates all doubt as to the coincidence between the temperature of the wire and the temperature indicated by thermometer, and avoids all differences between the true thermometric scale by air thermometer based upon Boyle's law, and the slightly divergent scale of the ordinary thermometer based upon the expansion of mercury.

Tests of the Copper Wire used. — The resistivity of the wire in the bulb was observed to be 1637 C.G.S. units at 0° C. from its mass and resistance, allowing a specific gravity of 8.90. Several different observations did not agree very closely, however, owing probably to the small diameter of the wire, and its liability to become stretched and variable in diameter.

An analysis of the copper wire used was made by Mr. McCoy of the Purdue Chemical Laboratory and one of the writers. The results are as follows: —

Antimony Arsenic	less than 0.01 per cent;	more than 0.0025 per cent.
Iron	less than 0.025 per cent;	more than 0.0025 per cent.
Zinc	less than 0.03 per cent;	more than 0.0025 per cent.

III.

Note on the Temperature of Lowest Visible Red Heat.

A few measurements were made of the resistance of copper wires enclosed in exhausted glass tubes and gradually raised to just visible red heat, by gradually increasing the current strength through them. The tubes were 30 cm. long, and 2 cm. external, and 1.8 cm. internal diameter. The platinum wires were sealed into the glass at each end, and connected with the copper wire, — one to carry the heating current, and the other to act as "pressure wire," in order to eliminate the resistance of the first, or platinum electrode. The copper wire stretched along the axis of the exhausted tube was 30.4 cm. long, and 0.0015' (0.0038 cm.) in diameter. Measuring the resistance of these wires at normal temperature with a very feeble current, they were then raised to redness, and their resistance observed under that condition in a darkened room.

Reckoning back with the linear temperature coefficient of 0.00406 from the normal temperature to zero centigrade, the resistance of the wire was found to be three times that at zero when visible luminosity was just attained, the mean calculated ratio being, in fact, 3.001. If the same linear temperature coefficient be assumed throughout that whole range, the corresponding temperature of lowest visible luminosity becomes 493° C. in this instance.

The method is very sensitive in application, and repeated trials with the same wire and same observer would usually fall within two degrees centigrade by resistance valuation. There was, however, a systematic variation between the observations when the observers were exchanged, amounting to about three degrees of centigrade; and since the criterion of appreciable visibility is merely physiological, it is perhaps impossible to accurately define it. From the sensations experienced in observing, it might be supposed that habit or physical condition would appreciably influence the range of visual appreciation, after the manner of a personal equation.

In conclusion, we desire to express our acknowledgments to Mr. Thomas A. Edison, in whose laboratory the above research was conducted.

ON THE FREEZING-POINTS OF DILUTE SOLUTIONS. II.

By E. H. Loomis.

The Accuracy of the Method.

TWO prominent questions arise in deciding upon the accuracy of the method described in the first part of this paper:[1] —

(1) How much reliance can be put upon the constancy of these fine thermometers?

(2) How much agreement appears in the individual observations of a series out of which the middle is taken?

The former may be best answered from the following table of the freezing-points of water, or determined zero points of the thermometer during the period Oct. 29 to Dec. 4, 1892.

Date.	Observed zero point.	Barometer.	Room Temperature.	Result corrected to 760 mm.
Oct. 29	+0.02900	748.0 mm.	9°.5	+0.03100
31	0.02916	745.0	10°.5	0.03166
Nov. 1	0.02927	743.5	10°.5	0.03202
2	0.02895	748.0	11°.0	0.03095
4	0.03012	755.5	11°.0	0.03087
5	0.03102	756.5	11°.0	0.03160
9	0.03030	757.0	11°.0	0.03080
11	0.03049	755.0	10°.8	0.03112
12	0.03084	752.0	10°.5	0.03217
18	0.03018	750.0	10°.3	0.03185
26	0.03080	759.5	8°.0	0.03088
29	0.03073	759.0	7°.5	0.03089
30	0.03081	756.0	7°.5	0.03148
Dec. 1	0.03000	756.0	7°.0	0.03067
2	0.03096	756.0	7°.0	0.03163
4	0.02893	744.0	7°.0	0.03160

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[1] See *Physical Review*, Vol. I., p. 199.

It appears then that the maximum variation in the zero point of the thermometer during the five weeks was 0°.0014. The highest value, +0°.03217, was on Nov. 12; the lowest, +0°.03067, on Dec. 1. A glance at these results forces one of two conclusions — either the thermometer is subject to sudden changes, or the determinations themselves are correspondingly erroneous. Neither of these assumptions is agreeable. The latter is, however, inadmissible, since in such a case the depressions based upon the erroneous zero points would show corresponding errors. This is by no means the fact, and we are forced to admit that the thermometer is subject to slight and dangerously sudden changes which may affect seriously the accuracy of an observation.

The second question is most satisfactorily answered by inspecting a series of observations on any particular solution. For this purpose, as an example of a series where the agreement of the individual observations is least satisfactory, let us choose the $\frac{2}{10}$ normal magnesium sulphate solution. The detailed observations are as follows: —

1 Temperature of protection bath.	2 Overwarming.	3 Overcooling.	4 Reading of thermometer.
– 0°.72	– 0°.330	– 0°.53	– 0°.3867
	0°.335	0°.51	0°.3858
	0°.335	0°.50	0°.3863
	0°.332	0°.49	0°.3858
	0°.330	0°.50	0°.3870
	0°.330	0°.52	0°.3860
	0°.330	0°.52	0°.3859

Barometer = 744.5. Room temperature = 7° C.

As an example of the better series, let us take the one from which the freezing-point of water was obtained on Nov. 12. This selection is made not because the series is especially good, since it is in this particular much surpassed by many others, but because it was in this particular series that the maximum height of the zero point occurred, and the excellence of the series seems to indicate that this sudden change could not arise from an error in the observations, but more likely was due to actual changes in the thermometer itself. Further, this particular series contains an observation, the first, which was purposely made before allowing the apparatus to cool off to 0° in the "protection bath." I have repeated this experiment many times and always with the result that the observation so obtained is about 0°.004 too high.

1 Temperature of protection bath.	2 Overwarming.	3 Overcooling.	4 Reading.
– 0°.31	+ (?)	– 0°.220	+ 0°.0340
0°.31	0°.160	0°.227	0°.0308
	0°.150	0°.073	0°.0308
	0°.155	0°.065	0°.0308
	0°.165	0°.060[1]	—
0°.29	0°.175	0°.070	0°.0310
0°.31	0°.185	0°.060[1]	—
	0°.175	0°.070	0°.0308
0°.30	0°.175	0°.065	0°.0310
	0°.180	0°.100	0°.0308
	0°.180	0°.085	0°.0305
0°.30	0°.180	0°.085	0°.0310

Barometer = 752.0. Room temperature = 10°.5.

In the first series it appears the maximum variation is 0°.0012, while in the second it is 0°.0005. In neither case can the middle value be far from the true freezing-point desired.

The Solutions.

These are made first in such concentration that each liter of the solution contains as many grams of the dissolved substance as there are units in its molecular weight. Such a solution is conveniently referred to as a "gram-molecule solution," and its degree of concentration is unity, which we denote by $m = 1$. The temperature to which these standard solutions were referred was 18°, except in the cases of sugar and urea. For the former it was 4°, and for the latter 6°. These standard or gram-molecule solutions, in the case of sugar, sodium chloride, and urea, were made by direct weighing of the properly recrystallized and dried substances. In the remaining instances they were obtained from stronger solutions of determined strength by dilution. The strength of these stronger solutions was calculated from their specific gravity by using the tables in Kohlrausch's *Leitfaden der Praktischen Physik* (Leipzig, 1892).

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[1] No observation was made, as the overcooling was too little to fall within the prescribed limits.

From these data the proper amount of dilution to obtain the required gram-molecule solution is easily calculated. The accuracy of these solutions was ensured by determining their specific gravities, and in the case of the electrolytes by determining also their electrical conductivities. The values are given at the head of each table of results. From these standard solutions the various fractional dilutions were obtained by use of a 200 c.c. flask and suitable pipettes, all of which were most carefully graduated to actual volume. To insure the introduction of the entire contents of the pipettes into the flask, they are rinsed three or more times directly into the flask, and to avoid error in filling, the pipettes are wetted before final filling with equal thoroughness in a special portion of the standard solution. This method is much more expeditious than that of weighing, and is no less satisfactory. The water used in making the solutions is the same as that in which the zero point of the thermometer is determined.

It must be added that organic solutions are made immediately before they are to be used. This was found to be especially necessary in the case of sugar solutions, which showed unmistakable signs of dissolution in less than a week, even at 4° C. It is further necessary to mention that the dilution of the gram-molecule solution in preparing the various fractional concentrations was uniformly done at the temperature of the room, care being observed that the solution and water used in its dilution had the same temperature. Slight errors arising from the different coefficients of expansion of water and the solution are thus possible, but in no way can they affect the character of the results, since the temperature of the room varied only within narrow limits during long periods.

In making the selection of the electrolytes, preference was given to such as showed great variation in their molecular constitution[1] in the particular region of concentration to which I confine my investigations.

The Results.

In the following table m denotes the strength of the solution in gram-molecules per liter; Δ denotes the depression of the freezing-point, which is *observed* to the fourth place of decimals. The fifth place arises from the reduction of the observations, and has no value except to control the fourth place. The quotient $\frac{\Delta}{m}$ is the molecular depression computed according to Arrhenius, and is furnished directly from the observed data. The quotient $\frac{\Delta}{P/M}$ is the molecular depression as computed according to Raoult, where P denotes the number of grams of the substance dissolved in 1000 g. of water, and M the molecular weight of the substance. To obtain this quotient only the specific gravity of the given solution needs to be known. This was determined for the solution $m = \frac{2}{10}$ in each of the series, and the corresponding value of $\frac{\Delta}{P/M}$ thus directly computed. The values for the more dilute solutions may then be determined exactly enough by lineal interpolation. It is obvious that the two methods of computing the molecular depression lead to essentially different values only in the stronger solutions, and even here the difference is slight except when the molecular volume is large as in the case of sugar.

The results of direct observation are given in ordinary faced numerals in the tables, while the indirectly derived values are given in small-faced type.

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[1] F. Kohlrausch, Wiedemann's Annalen, 1885, 26, p. 161.

Sodium chloride, NaCl. $M = 58.51$.
Gram-molecule solution ($m = 1$) at 18° contains 58.51 g. per liter.
Observed Sp. Gr. $_{(18°)}$ 1.0393. Conductivity $_{(18°)}$ $596.4 \times 10.^{-8}$ S.E.
Room temperature, 18°. Time, August, 1892.

Gram molecule in a liter (m).	Depression of freezing-point, Δ.	Molecular depression, $\frac{\Delta}{m}$.	Grams per 1000 grams of water, (P).	Molecular depression reckoned according to Raoult, $\frac{\Delta}{P/M}$
0.01	0°.03674	3.674	0.585	3.673
0.02	0°.07193	3.597	1.171	3.595
0.03	0°.10680	3.560	1.757	3.557
0.04	0°.14165	3.541	2.343	3.538
0.05	0°.17653	3.531	2.929	3.527
0.06	0°.21172	3.529	3.515	3.524
0.07	0°.24571	3.510	4.103	3.504
0.08	0°.28008	3.501	4.690	3.494
0.09	0°.31448	3.494	5.278	3.486
0.10	0°.34835	3.484	5.865	3.475
0.20 Sp. Gr. $_{(18°)}$ 1.0069	0°.68779	3.439	11.752	3.422

Cane sugar, $C_{12}H_{22}O_{11}$. $M = 342.07$.
Solution ($m = 1$) at 4° contains 342.07 g. per liter.
Sp. Gr. $_{(17°.5)}$ 1.1286.
Room temperature (4° to 5°.5). Time, February, 1893.

Gram molecule in a liter (m).	Depression of freezing-point, Δ.	Molecular depression. $\frac{\Delta}{m}$.	Grams per 1000 grams of water, (P).	Molecular depression reckoned according to Raoult, $\frac{\Delta}{P/M}$.
0.01	0°.01705	1.705	3.429	1.701
0.02	0°.03545	1.773	6.871	1.765
0.03	0°.05455	1.818	10.334	1.806
0.04	0°.07289	1.822	13.809	1.806
0.05	0°.09210	1.842	17.296	1.822
0.06	0°.11147	1.858	20.798	1.833
0.07	0°.13055	1.865	24.320	1.836
0.10	0°.18971	1.897	34.966	1.856
0.15	0°.28846	1.923	53.008	1.862
0.20 Sp. Gr. $_{(4°)}$ 1.0266	0°.39169	1.959	71.380	1.877

Magnesium sulphate, $MgSO_4$. $M = 120.46$.
Solution ($m = \frac{1}{2}$) at 18° contains 60.23 g. per liter.
Sp. Gr. $_{(18°)}$ 1.0575. Electrical conductivity $_{(18°)}$ 271.5 × 10^{-8}.
Room temperature, 7° to 8°. Time, November-December, 1892.

m	Δ	$\frac{\Delta}{m}$	P	$\frac{\Delta}{P/M}$
0.01	0°.02662	2.662	1.205	2.662
0.02	0°.05108	2.554	2.410	2.554
0.03	0°.07420	2.473	3.615	2.473
0.04	0°.09769	2.442	4.820	2.441
0.05	0°.11994	2.399	6.025	2.398
0.06	0°.14171	2.362	7.230	2.361
0.07	0°.16234	2.319	8.436	2.318
0.08	0°.18330	2.291	9.642	2.290
0.09	0°.20348	2.261	10.848	2.260
0.10	0°.22305	2.231	12.055	2.229
0.20 Sp. Gr. $_{(18°)}$ 1.0228	0°.41576	2.079	24.123	2.076

Ethylalcohol, C_2H_6O. $M = 46.02$.
Solution ($m = 1$)$_{(18°)}$ contains 46.02 g. per liter.
Sp. Gr. $_{(20)}$ 0.99015.
Room temperature, 6° to 7°. Time, December, 1892.

m	Δ	$\frac{\Delta}{m}$	P	$\frac{\Delta}{P/M}$
0.01	0°.01720	1.720	0.461	1.719
0.02	0°.03510	1.755	0.922	1.753
0.03	0°.05306	1.769	1.769	1.766
0.04	0°.07288	1.822	1.845	1.818
0.05	0°.08936	1.787	2.307	1.782
0.06	0°.10792	1.799	2.770	1.793
0.08	0°.14524	1.816	3.697	1.808
0.10	0°.18253	1.825	4.628	1.815
0.12	0°.21919	1.826	5.561	1.814
0.14	0°.25646	1.832	6.496	1.817
0.16	0°.29269	1.829	7.436	1.811
0.20 Sp. Gr. $_{(18°)}$ 0.9969				

Sulphuric acid, H_2SO_4. $M = 98.07$.
Solution ($m = 1$)$_{(18^\circ)}$ contains 98.07 g. per liter.
Sp. Gr. $_{(18^\circ)}$ 1.0615. Conductivity $_{(18^\circ)}$, 3396×10^{-8}.
Room temperature, (0° to -1°). Time, January, 1893.

m	Δ	$\frac{\Delta}{m}$	P	$\frac{\Delta}{P/M}$
0.01	$0^\circ.04493$	4.493	0.981	4.492
0.02	$0^\circ.08619$	4.310	1.963	4.307
0.05	$0^\circ.20652$	4.130	4.912	4.123
0.10	$0^\circ.39679$	3.968	9.841	3.954
0.20	$0^\circ.76996$	3.850	19.762	3.822
Sp. Gr. $_{(18^\circ)}$ 1.0121				

Phosphoric acid, H_3PO_4. $M = 98.01$.
Solution ($m = 1$)$_{18^\circ}$ contained 98.01 g.
Its Sp. Gr. $_{(15^\circ)}$ 1.0508. Conductivity $_{(18^\circ)}$, 469.7×10^{-8}.
Room temperature, $3^\circ.5$. Time, January, 1863.

m	Δ	$\frac{\Delta}{m}$	P	$\frac{\Delta}{P/M}$
0.01	$0^\circ.02310$	2.310	0.981	2.309
0.02	$0^\circ.04344$	2.172	1.962	2.170
0.05	$0^\circ.10033$	2.007	4.912	2.002
0.10	$0^\circ.18860$	1.886	9.854	1.877
0.20	$0^\circ.35828$	1.791	19.808	1.773
Sp. Gr. $_{(18^\circ)}$ 1.0091				

Urea, $CO(NH_2)_2$. $M = 60.09$.
Solution ($m = 1$)$_{6^\circ}$ contained 60.09 g. per liter.
Sp. Gr. $_{(18^\circ)}$ 1.0148.
Room temperature, 6° to $6^\circ.5$. Time, February, 1893.

m	Δ	$\frac{\Delta}{m}$	P	$\frac{\Delta}{P/M}$
0.01	$0^\circ.01722$	1.722	0.601	1.721
0.03	$0^\circ.05279$	1.760	1.805	1.757
0.05	$0^\circ.09067$	1.813	3.011	1.809
0.10	$0^\circ.18235$	1.824	6.037	1.816
0.20	$0^\circ.36691$	1.835	12.129	1.818
Sp. Gr. $_{(6^\circ)}$ 1.0029				

Remarks on the Solutions.

1. *Sugar.* — It is to be observed that this standard sugar solution is considerably lighter than the usual normal solution based on a liter of solution at 18° C., since in the present case 4° C. was the temperature employed.

2. *Alcohol.* — The solution from which the standard or gram-molecule solution was prepared had Sp. Gr. $_{(20^\circ)}$, 0.9048; and thus, according to the Mendelejeff's tables, was 54.1 per cent alcohol. 85.06 g. should then contain 46.02 g. pure alcohol, and after dilution to a liter volume should give the required gram-molecule solution. Such a solution, however, had Sp. Gr. $_{(20^\circ)}$, 0.99015, and according to the same tables thus appears to contain but 45.65 g. per liter, and appears therefore to be about 1 per cent too light.

It is to be further remarked that the alcohol solutions stronger than about $m=0.10$ *lose alcohol by evaporation*, which occasions a marked and constant rise in the freezing-point during the progress of the observations. These determinations are thus affected by an error. For this reason no observations on the stronger solutions were made.

3. *Phosphoric Acid.* — This solution was made from an acid furnished by Tromsdorff. After an initial dilution its Sp. Gr. $_{(15^\circ)}$ was found to be 1.2490, and thus, according to the tables of F. Kohlrausch, its percentage composition was 39.46 per cent. Accordingly 248.378 g. should contain the 98.01 g. of pure acid required for the standard solution. This amount was diluted to a liter volume, and the Sp. Gr. $_{(15^\circ)}$ was then found to be 1.0508, which, however, corresponds, according to the same tables, to 9.3 per cent, and the solution thus appears to contain only 97.72 g. per liter.

The *conductivity*, on the other hand, is but 469.2×10^{-8}, which corresponds to but 8.7 per cent. I am unable at present to explain these irregularities. It is to be remarked in addition that the solution $m=0.20$, as appears in the table on page 281, has a conductivity 142×10^{-8}, which is almost exactly what the Kohlrausch tables give for this dilution. In this case, as in the case of alcohol, I have applied no correction for this apparent inaccuracy in the gram-molecule solution.

The Correction for Concentration.

It has been mentioned already in the description of the method that the solution is allowed to overcool from 0°.10 to 0°.20 before freezing takes place. In general, this is about 0°.15. The quantity of ice thus formed is small, but is sufficient to occasion a slight increase in the concentration of the solution. Assuming for the present that the latent heat of melting ice (80 calories) is unchanged by the presence of the dissolved substance, and that no heat is given off by the solution save during the actual time of overcooling, theoretical conclusions point to a very simple way of determining the amount of ice which results from any particular degree of overcooling. Thus, an overcooling of 80° C. would result in the complete freezing of the water, while an overcooling of 1° would cause $\frac{1}{80}$ of the water to solidify. In the present instance, when the average overcooling is 0°.15, then $\frac{0.15}{80}$ of the water is frozen, or the solution itself is thus concentrated about 0.2 per cent. The observed depression of the freezing-point is thus about 0.2 per cent too much, and a correction of this extent should be applied. That this, however, is not the true correction — that the change in concentration is considerably *greater*, in fact, than here appears, — is evident; first, because the solution continues to lose heat after the freezing begins (in the present method for two minutes), and consequently the heat removed from the solution is greater than that measured by the degree of overcooling alone; and secondly, the latent heat of melting is undoubtedly less for a solution than for pure water, and consequently a certain degree of overcooling results in a *greater* formation of ice than would be the case for pure water. In the hope of obtaining a more exact correction, I proceeded in various ways, the most satisfactory of which are the following: —

Immediately after making the last observation on the $\frac{2}{10}$ solutions ($m=0.2$), I remove the unfrozen portion of the solution, and compare its specific gravity and conductivity in this slightly concentrated state with those of the original solution. The change in concentration is thus determined. The separation of the unfrozen portion from the small quantity of fine ice crystals floating in it is easily effected, without any disturbance of the proportion of ice and water, by using a small glass syphon, care being observed to cool this, before insertion, to the exact freezing-point of the solution in a separate portion of the partly frozen solution itself. This was done with the $\frac{2}{10}$ NaCl and the $\frac{2}{10}$ H_8PO_4 solutions, with the following results: —

	Specific Gravity $\left(\frac{18^\circ}{4^\circ}\right)$.		Electrical Conductivity, $k \times 10^{-8}$.	
	Before.	After.	k. Before.	k. After.
$\frac{2}{10}$ NaCl.	1.00692	1.00695	164.40	164.80
$\frac{2}{10}$ H_3PO_4.	1.00967	1.00971	141.96	142.47

The change in conductivity for NaCl was thus 0.24 per cent, while for H_8PO_4 it was 0.35 per cent, which is in close agreement with the corresponding changes in the specific gravities. These changes in specific gravity and conductivity measure the increase in the concentration of the solution, and indicate a correction of about 0.3 per cent to the observed depression. Although this indirect method is quite exact, I was led to make the direct determination in the following very satisfactory manner: the unfrozen portion is drawn off, and treated identically as the original solution. The unfrozen portion is then again drawn off and again treated in the same manner. This is done three times, and the freezing-point of the third portion is then determined. We have thus magnified the concentration threefold; and it is evident that the desired correction is one-third the difference between the final freezing-point and that of the original solution. Still greater accuracy could be obtained by extending the process of successive refreezings to five or six. However, with three refreezings the results confirm the conclusions drawn from the other methods: that the correction is about 0.3 per cent, and thus considerably *larger* than that which is deduced from the mere extent of overcooling; even when, as in the present case, the solution is exposed but a short time to the action of a very weak freezing-mixture. It seems certain, when the solution is thus exposed many minutes to a very cold freezing-mixture, as is usually the practice in such investigations, that this theoretical correction is wholly unwarranted and inaccurate. The correction, as thus experimentally determined, is still somewhat uncertain; and as its application or omission in no way affects the general character of the results, I prefer to leave them exactly as they were observed. It must, however, be kept in mind that the observed depressions are about 0.3 per cent too great.

The Graphic Representation of the Results.

The best general survey of the results may be obtained when they are graphically represented. For this purpose, instead of the actual depression Δ of a given solution, the quotient $\frac{\Delta}{m}$ is introduced. This is a fictitious depression of this freezing-point, and evidently represents what would be the corresponding depression in a solution of units concentration ($m = 1$), if Δ remained proportional to m. This quotient is conveniently known as the molecular depression. The scale of molecular depression, $\frac{\Delta}{m}$, is so chosen that, for $m = 0.01$, one scale division represents 0°.001 in actual depression. Thus, an error of 0°.0002 displaces the curve in this region a whole millimeter. This great sensitiveness becomes gradually less in the direction of greater concentration, so that at $m = 0.10$, for example, an error of 0°.002 in the observed depression would be required to displace the curve this amount. The course of the curve is thus more certain in proportion as the solutions are more concentrated. The observed points are in general indicated by a cross +, except in urea, where a small circle ⊙ is employed. Since three of the results for urea are identical with those of alcohol, no attempt is made to draw a separate curve.

The curves are drawn by using a flexible ruler, so applied as to best represent the course of the results. The full lines represent the results directly in terms of $\frac{\Delta}{m}$ and m, according to the method of Arrhenius; while the broken and pointed lines, — · — · —·, lying directly below the full lines, represent the same results in terms of $\frac{\Delta}{P/M}$ and m, according to the method of Raoult. The two curves for sugar are widely separated from each other. In the other cases, the two curves are much less widely separated, and in the case of $MgSO_4$ they are so nearly identical that no attempt is here made to draw the second curve.

For the convenience of those especially interested in the question of "disassociation" the theoretical values computed in accordance with the theories of van't Hoff[1] and Arrhenius[2] on the basis of the Kohlrausch[3] determinations of the electrical conductivity of the various solutions are given in dotted lines, and marked "theoretical." As is well known the computation of these theoretical values rests upon the assumption of a certain limiting value for the molecular conductivity in so-called infinitely dilute solutions. This value is in some degree uncertain, and it may be remarked that by choosing a smaller value for this limit in the case of H_2SO_4, say 3600 instead of 3800, the theoretical curve for the molecular depression of the freezing-point becomes much more nearly coincident with actual results of the observations. It needs to be added further that the theoretical values are calculated on the assumption that each H_2SO_4 molecule splits into three parts in the disassociation, and that NaCl and $MgSO_4$ each split into two. No theoretical values are given for H_3PO_4.

Discussion of Results.

A glance at the results as exhibited in Fig. II. reveals an essential difference between the substances which allows them to be grouped in two widely distinct classes. Thus the curves for the four electrolytes, H_2SO_4, NaCl, $MgSO_4$, and H_3PO_4, are convex downward, while the others, the non-electrolytes, sugar,

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[1] van't Hoff., Zeit. Phys. Chem., Leipzig, I. p. 496. See also Ostwald's Allg. Chem., 2d. edition, p. 141. The former is translated in the Phil. Mag., 26, p. 95. The second is also published in English.

[2] Zeit. Phys. Chem., I. p. 631. See also Walker, Phil. Mag., 32.

[3] F. Kohlrausch, Wiedemann's Annalen, 26, p. 161, 1885.

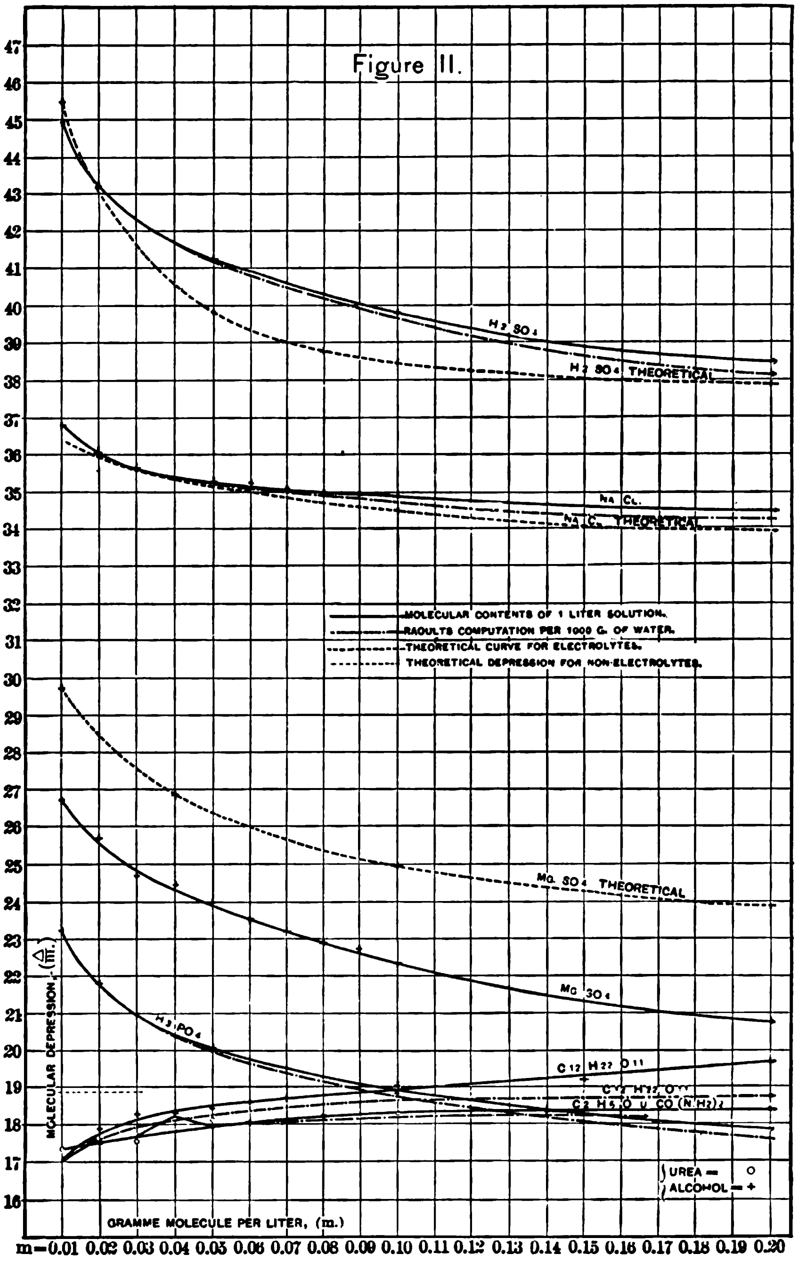

Figure II.

H 2 SO 4
H 2 SO 4 THEORETICAL
NA CL.
NA CL THEORETICAL
MOLECULAR CONTENTS OF 1 LITER SOLUTION.
RAOULTS COMPUTATION PER 1000 G. OF WATER.
THEORETICAL CURVE FOR ELECTROLYTES.
THEORETICAL DEPRESSION FOR NON-ELECTROLYTES.
MG SO 4 THEORETICAL
MG SO 4
H 3 PO 4
C 12 H 22 O 11
C 12 H 22 O 11
C 2 H 5 O H CO (N H2)2
UREA — ○
ALCOHOL — +
MOLECULAR DEPRESSION, (Δ/m.)
GRAMME MOLECULE PER LITER, (m.)
47
46
45
44
43
42
41
40
39
38
37
36
35
34
33
32
31
30
29
28
27
26
25
24
23
22
21
20
19
18
17
16
m=0.01 0.02 0.03 0.04 0.05 0.06 0.07 0.08 0.09 0.10 0.11 0.12 0.13 0.14 0.15 0.16 0.17 0.18 0.19 0.20

alcohol, and urea, are convex upward. Further, the former, with the exception of NaCl, are thus convex downward throughout their entire length, while in the latter this convexity is only exhibited in the region of extreme dilution and soon gives place to a *straight line.* It is also to be noticed that in extreme dilution the members of the second class have almost identical freezing-points for the same degree of molecular concentration, and that two entirely dissimilar organic bodies, alcohol and urea, have identical freezing-points throughout the region in which they have been studied. Obviously not enough bodies have been studied to lead to any general conclusions, but thus far the two classes are sharply defined.

A comparison of the results with those of other observers in general shows little agreement. However, it is to be observed that the results of Pickering's[1] observations, apart from the irregularities which he found, are very nearly parallel to the present values, and in general lie about 5 per cent higher, which suggests some constant source of error in one or the other series of observations. The striking exception must, however, be noticed in the case of NaCl. Here Pickering's[2] results, when graphically represented as in Plate I., give a curve which crosses the NaCl curve in Plate I. no less than ten times in the region there represented. A scarcely less striking agreement presents itself with H_2SO_4 (*Berlin Berichte,* 192, p. 1106) in the general course of the results, an agreement which becomes a practical identity in the more concentrated solutions. Thus far I have found none of the irregularities which appear in the results of Pickering and which he refers to sudden changes in the properties of the solutions at certain degrees of concentration. I thus no longer make the observations so painfully close together, and the later determinations embrace but five points of observation, which, however, fully fix the general course of the results. I have also found nothing which would indicate that the molecular depression in the case of electrolytes attains a maximum beyond which, in extreme dilution, the depression becomes "abnormally small." The present observations, however, do not reach far enough into this region to make the matter anything but conjecture. However, in the case of NaCl, this maximum appears in Pickering's observations between $m = 0.01$ and $m = 0.02$, where I find no indication of such a turning-point in the values of the molecular depression.

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[1] Pickering, various contributions, Berlin Chem. Berichte, 1891–92.

[2] Pickering, Berlin Chem. Berichte, 1892, p. 1314.

So, too, on the other hand, I have found no "extraordinary increase" in the molecular depression of the organic "indifferent" bodies as the dilution is increased—a fact which would make them resemble the electrolytes, as Traube,[1] Raoult,[2] and Pickering[3] have found. The present results show exactly the opposite tendency on the part of the organic bodies thus far examined.

It is gratifying to call attention to the very remarkable agreement between the present results for NaCl and those recently published by Jones[4]—an agreement reaching to the thousandth degree, even in the more concentrated solutions, which becomes the more remarkable when it is remembered that methods are wholly different. Even had the methods been the same, the use of different thermometers would more than account for all the differences in the two series of observations.

Concluding Remark in Regard to the Accuracy of the Method.

The graphic representation throws much new light upon the accuracy of the method, and indicates that it is not so great as the study of the separate series of observations has led us to believe. To make this evident, attention may be called to the depression as found for the $\frac{4}{100}$ alcohol solution. The value given in the tables and in the curve of results is the middle of seven observations which show a maximum variation of $0^\circ.0005$, due mostly to two observations in the series, one of which was as much above the general average as the other was below it. On the assumption that there is no sudden break in the curve at this point, we must assume that this observation is more than 0°.0015 too high.

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[1] Raoult, Berlin Chem. Berichte, 1891, pp. 1321 and 1853.

[2] Ann. Chim. et de Phys., 1886, p. 313. Curves for Alcohol and Sugar. Raoult's second determination shows the same tendency, though in a much less degree. See Compts Rendus, 1892, p. 268.

[3] See quotation from Pickering, Berlin Chem. Berichte, 1891, p. 1327.

[4] Zeit. Phys. Chem., XI., p. 110.

It is interesting to note the history of this particular observation, as it is in some measure a characteristic one. The first observations on alcohol embraced every other hundredth, — thus, $m=0.02$, 0.04, 0.06, etc.

These results, when taken alone, showed a change in the *curvature* of molecular depressions beginning at $m=0.06$. This was so marked that I hesitated to accept the results as final before carefully repeating them with fresh solutions. The three new observations for these points were identical with the original ones, and it became certain that the solution suffered a marked change in properties in this region. To still further confirm the matter, it was determined to find the values at closer intervals; thus, for $m=0.01$, 0.03, and 0.05. There could be no doubt that these would lie so as to still further fix the course of results already so clearly indicated by the observations at $m=0.02$, 0.04, and 0.06.

To my great surprise, the three new observations were not at all in harmony with the observations for $m=0.04$, but indicated another course for the curve as appears in Fig. 2. The conclusion was forced upon me that the method, for an unknown reason, had furnished a result, twice confirmed, fatally inconsistent with all my previous notions of its accuracy. Fortunately, this appears only as an exception, and does not affect one's confidence in the method. It is most probable that such an error is due to some slight impurity on the wall of the thermometer's capillary, which affects the tension of the mercury miniscus at this point, so as to cause it to increase or decrease its curvature slightly at this point, and thus lead to an error in the observation. In all other respects the method commends itself for its great accuracy and simplicity of manipulation.

I wish here to express my deep appreciation of the great help which Professor Kohlrausch rendered me throughout this entire investigation.

PHYSICAL LABORATORY, STRASSBURG,
July, 1893.

MINOR CONTRIBUTIONS.

THE EFFECT OF TEMPERATURE AND OF ELECTRIC DRIVING ON THE PERIOD OF TUNING-FORKS.

By JOHN S. SHEARER.

THE following experiments were undertaken in order to determine: (1) The correction required when an electrically driven fork, whose free period at a certain temperature is known, is used as a standard for the measurement of short intervals of time; (2) The influence of temperature upon the rate of such a fork. The convenience of the driven fork in practical work makes the effect of the various methods of driving of some interest.

The temperature effect for a range from 0° C. to about 30° has been investigated by H. Kayser,[1] and also by A. M. Mayer.[2] Both of these observers used the free fork, however, and made no comparison of the free and forced periods.

In the experiments to be described the method of Lissajous' figures was used to compare the forks, the one vibrating freely under constant conditions, the other driven under the conditions to be studied. The first experiments were made, using a Helmholtz fork-interrupter and a Lissajous vibration microscope. A Morse instrument in circuit with a clock and a signal key was used as a chronograph. The current driving the fork was from a storage battery, and by varying the resistance in the circuit the variations in pitch due to changes of current through the magnets could be determined. The fork used as a standard was bowed, and the other alternately bowed and driven.

It was found that the change in period was in all cases small. In order to keep the apparatus running steadily, a current was required which somewhat increased the amplitude over that observed in the free vibrations, so that the increase of period may have been due to the increased amplitude. The mean of a large number of observations gave 6.43 seconds as the time required for one fork to gain one vibration on the other when both were bowed. When one was driven, the corresponding time was 2.43

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[1] Kayser, Annalen der Physik und Chemie, N. F. 8, p. 444.

[2] Philosophical Magazine (5), Vol. 21, p. 286 (1886).

seconds. By means of small pieces of wax, it was found that the change was due to an increase of period of the driven fork. Assuming that the standard made, as marked, 256 vibrations per second, the other when free, made 255.8445 vibrations, and when driven 255.5935. In other words, the free period was .00390863, the forced period .00391246, so that the effect of driving was to increase the period in the ratio of 1 : 1.00098 nearly. Another set the following day, with a slight variation in the tuning, gave for the time of complete change when free 5.92 seconds, when driven 2.78 seconds. The change of rate due to driving was therefore of about the same magnitude as before.

A series of seven resistances giving a range of current suitable for the instrument were then measured, and the forced period was determined for each. This series was repeated several times with consistent results. Tables I. and II. and curve *a* of Fig. 1, give some of the observations and the results derived from them.

Table I.

INFLUENCE OF CURRENT UPON RATE OF AN ELECTRICALLY DRIVEN FORK WITH MERCURY CONTACT. [Four Sets.]

Set 1.

R	T	N	P	D
11.00	5.18	256.1931	.00391230	00
16.50	5.50	256.1818	.00391248	18
21.66	5.68	256.1761	.00391256	26
29.33	6.04	256.1656	.00391272	42
31.75	6.06	256.1650	.00391273	43
37.75	6.30	256.1587	.00391282	52
41.30	6.94	256.1441	.00391305	75

Set 2.

11.00	5.68	256.1761	.00391256	00
16.50	5.90	256.1695	.00391266	10
21.66	6.06	256.1650	.00391273	17
29.33	6.52	256.1534	.00391291	35
31.75	6.80	256.1471	.00391301	45
37.75	7.40	256.1351	.00391319	63

SET 3.

11.00	4.68	256.2137	.00391199	00
16.50	4.70	256.2128	.00391200	1
21.66	4.84	256.2066	.00391210	11
29.33	5.22	256.1916	.00391233	34
31.75	5.30	256.1887	.00391237	38
37.75	5.44	256.1838	.00391245	46
41.30	5.46	256.1832	.00391246	47

SET 4.

11.00	6.88	256.1453	.00391304	00
16.50	7.20	256.1389	.00391313	9
21.66	7.40	256.1351	.00391319	15
29.33	7.86	256.1272	.00391331	27
31.75	8.38	256.1193	.00391343	39
37.75	9.58	256.1044	.00391366	62
41.30	10.86	256.0921	.00391385	81

Free period for Set 4, .00390387.

Averaging the values of D for each value of R, we get the following results, from which curve *a* in Fig. 1 is plotted.

TABLE II.

R = 16.50	21.66	29.33	31.75	37.75	41.30
D (av.) = 9.50	17.25	34.50	41.25	55.75	74.33

In Tables I. and II., R is the resistance in ohms around which the driving magnets were shunted, therefore R is proportional to the current through the magnets; T is the time required for the fork to gain one vibration; N is the number of vibrations per second, and P is the period. D is the difference of period in hundred-millionths of a second.

A small piece of platinum wire touching a piece of metal was next used instead of the mercury contact of the Helmholtz fork, and the position of the magnets was also varied. In each case the change of period was small so long as the amplitude was not greatly increased above that observed for the free vibrations.

For the more extended study of this method of driving, another pair of forks were used. These were mounted on heavy frames, and the actuating magnets were between the prongs. The microscope was dispensed with, and a ray of light twice reflected was used instead. The circuit was made and broken by means of a platinum spring, attached to the fork, which touched a plate of the same material at each excursion of the prongs outward. This method of driving was found to be more satisfactory than the mercury contact. The fork ran more steadily and could be maintained with a smaller amplitude. The results were the same when the magnets were placed outside the prongs. The observations are given in Table III. with the calculated periods. It will be noticed that for small currents the period is slightly diminished. This was to be expected, since the force of the magnet was added to the recovering power of the fork when the prongs were most widely separated. So long as the amplitude was not considerably increased a slight rise in pitch resulted. With increasing vibration this effect was counterbalanced, and finally the period increased sharply. (See Fig. 1 *b*.)

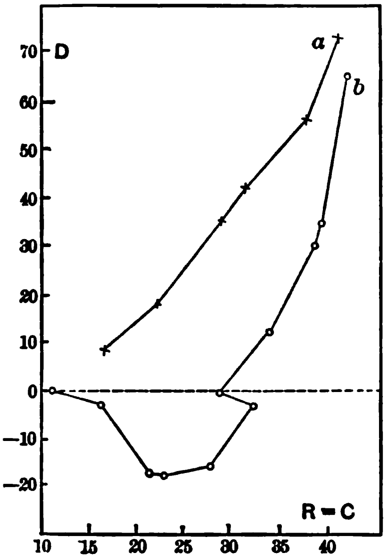

Fig. 1.

Table III.

Set 1.

R	T	N	P	D
11.66	3.79	256.2639	.00390223	00
16.90	3.75	256.2667	.00390218	− 5
22.00	3.62	256.2762	.00390204	−19
23.40	3.62	256.2762	.00390204	−19
27.80	3.64	256.2747	.00390206	−17
29.60	3.80	256.2632	.00390223	0
32.80	3.76	256.2660	.00390220	− 3
34.60	3.90	256.2564	.00390234	+11
38.10	4.10	256.2439	.00390253	+30
39.15	4.16	256.2404	.00390258	+35
42.25	4.54	256.2203	.00390289	+66

Free period, Set 1, .00390217.

For the purpose of determining the influence of temperature upon the rate of an electrically driven fork, a coil of German silver wire was placed around the fork and heated by means of a current.[1] To avoid convection currents, the whole was surrounded by asbestos. The temperature was taken by means of two thermometers, one near the base of the fork and the other above. To avoid the effect of increased magnetization of the fork, the heating current was stopped while readings were made, the temperature being observed just before.

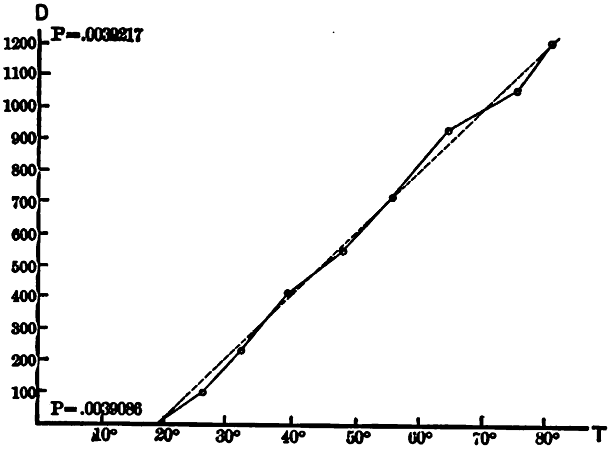

Fig. 2.

It appeared from these experiments just described that for a given current the period was very nearly constant, so that a driven fork, could be used as a standard in the determination of the effect of temperature changes. To make sure, however, that no marked change arose, the standard was frequently compared with a third fork vibrating freely. It is to be observed, however, that the fork becomes magnetized when run by current and this may have a slight effect on the period. By means of a water resistance the current could be varied so as to keep the temperature nearly constant. Readings were taken at intervals of an hour so as to allow the fork to acquire the temperature of the surrounding air as nearly as possible.

The results of a series of observations are given in Fig. 2, the standard being corrected for room temperature under the assumption that its rate of change was approximate to that observed for the other fork. As in the previous cases, *D* is the difference of period in hundred-millionths of a second.

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[1] See this *Review*, Vol. I., p. 147.

The fork heated was made by Koenig, and marked 256 vib. per second. It was about 25 cm. long, 1.85 cm. wide, 1.45 cm. thick. Young's modulus for this fork was determined by method of flexure to be $19.065 \times 10.^{11}$

PHYSICAL LABORATORY OF CORNELL UNIVERSITY,
August, 1893.

ON THE CONTINUOUS SPECTRUM OF SODIUM.[1]

BY BENJAMIN W. SNOW.

BUNSEN and Kirchhoff, in their faithful delineation of the spectra of the various metals, detected in the spectra of certain alkalies a faint continuous background upon which the characteristic metallic lines were superposed. This continuous spectrum, which is so faint as to be barely visible in the case of lithium, is more noticeable in the spectra of sodium and potassium, and becomes a prominent feature in the study of the light emitted by burning cæsium and rubidium. During a recent series of investigations, this continuous spectrum was repeatedly observed, and certain data obtained which seem to point to a possible explanation of this phenomenon.

The experiments referred to consisted in the attempt to detect, by means of the bolometer, the presence of infra-red lines in the spectra of the alkaline metals, when the latter were burned in the flame of an ordinary Bunsen burner. It was proved, however, by further investigation that this source of heat is quite insufficient to cause the infra-red lines to appear, and hence the ends sought were not to be attained in this way. Nevertheless, other results of an unexpected nature were obtained, and these may serve to throw some light upon the subject now under consideration.

The apparatus used for detecting and measuring the thermal radiations was a sensitive bolometer, made from fine platinum wire which had been hammered flat and so wound upon a little frame of mica that three threads in series were exposed to the action of the heat. The resistance of each arm of the bolometer so made was 23 ohms.

In order to obtain a galvanometer of greater sensitiveness than any available for use with this instrument, the heavy astatic system of a Siemens galvanometer was removed, and in its place was hung a delicate needle made of short pieces of magnetized steel mounted upon each end of a thin-walled capillary tube.

A fine spectrometer of the usual form and provided with a prism of dense flint glass served to analyze the radiations to be studied. In place of the cross hairs in the telescope were mounted the sensitive filaments of the bolometer described above, so that these latter served alike for measurements in the visible as well as in the dark heat spectrum. The slit was adjusted in width until its image, as seen in the telescope, was just superposed upon the three threads of the bolometer, and subtended an angle of 20 minutes. This width of slit was retained unchanged throughout the entire series of experiments.

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[1] Read at the American Association for the Advancement of Science, August, 1893.

By means of this apparatus an attempt was made to determine the character of the radiation in the infra-red spectrum of an ordinary sodium flame. To this end a Bunsen burner was mounted immediately before the slit of the spectrometer, and in this flame, a few millimeters below the slit, was held a platinum wire containing a bead of sodium carbonate. So small, however, was the quantity of heat thus emitted, that the bolometer, when allowed to wander through the entire spectrum, was quite incapable of detecting, in the very feeble radiation of the Bunsen flame, an increase due to the presence of the sodium.

In order, therefore, to make this flame more intense, an expedient was adopted, whereby the quantity of the incandescent sodium vapor discharged into the flame was greatly increased. The platinum wire holding the bead of sodium carbonate was replaced by a copper spoon three centimeters in diameter, in which were placed little pieces of metallic sodium weighing about one gram apiece. Each lump of sodium, when ignited by the lamp, burned with great violence, and poured into the flame so copious a flood of the incandescent metallic vapor, that the latter, when examined with the spectrometer, produced a *D*-line of unusual brilliancy.

In that portion of the spectrum of shorter wave length than the sodium line, no thermal effects could be detected by the bolometer. So intense, however, was the radiation from this line alone, that a deflection of several centimeters was produced as soon as this line and the sensitive threads of the bolometers were brought into coincidence.

In the remaining portions of the visible spectrum lying toward the red, the quantity of energy was so small that the galvanometer deflections were hardly appreciable. These, however, increased in magnitude as the extreme red end was approached and passed, and continued to increase as the bolometer was moved on from point to point in the direction of the still longer wave lengths, until a region of about $\lambda = 1\ \mu$ was reached, at which point the deflection equalled those produced by the *D*-line. After this, the deflection again decreased, and became zero at wave length $\lambda = 1.8\ \mu$. The infra-red spectrum thus obtained resembled in every respect that produced by an incandescent solid, except that it presented a more or less jagged appearance, depending upon the size of the lumps of sodium. The presence of so strong a continuous spectrum in the infra-red precluded the possibility of detecting lines in this region, and accordingly this method also was abandoned.

One further attempt in this direction was made, in which the platinum wire with its bead of sodium carbonate was again used, but held now in an oxyhydrogen flame, placed immediately before the slit, instead of the Bunsen burner. Under these conditions, the bolometer showed that the amount of energy contained in the *D*-line was far greater than when metallic sodium had been consumed in the Bunsen burner, but that the energy in the infra-red spectrum had almost entirely disappeared. Beyond the limits of the visible spectrum, the deflections, in fact, nowhere attained a magnitude equal to the one-hundredth part of that produced by the *D*-line.

It was evident, therefore, that the more intense heat of the oxyhydrogen flame had produced a spectrum which differed in two respects from the one arising from burning metallic sodium at a lower temperature. First, the energy of the *D*-line was more than doubled when the more intense source of heat was employed, and second, the continuous spectrum in the infra-red, which had been so prominent when the cooler flame was employed, had almost entirely disappeared.

It has occurred to the writer that a possible explanation of the facts here set forth would be the following: When sodium or a salt of this metal is burned in the flame of a Bunsen burner, the metal unites with the oxygen of the air and produces an oxide which remains suspended in the flame as an *incandescent solid*, at a temperature too low to permit of dissociation. At the higher temperature, however, of the oxyhydrogen flame, the oxide, if, indeed, formed at all, is at once dissociated, and the flame becomes freed from all solid particles which yield a continuous spectrum, and remains filled only with the incandescent metallic vapor, which gives rise to the true line spectrum. If this proposition be granted, it may then be fairly assumed that the faint continuous spectrum, against which the sodium line is so often seen projected, is due, in part at least, to the presence in the flame of particles of incandescent sodium oxide at a temperature too low for dissociation.

PHYSICAL LABORATORY OF THE UNIVERSITY OF WISCONSIN,
October 11, 1893.

NOTE ON SOME SURFACE TENSION EXPERIMENTS.

By Ernest Nichols.

THE writer lately tried to make plain, in a popular lecture, the reasons for some of the phenomena of the surface tension of liquids. There are many ways of showing that a soap-film is elastic and eminently contractile: perhaps the simplest is to show that a soap-bubble is filled with compressed air. This may be done by extinguishing a candle flame with the out-rush of air from a soap-bubble through the bubble-pipe. It is then easy to show some very close analogies between the behavior of soap-films and water surfaces which will make the surface tension of liquids seem a reality to an average audience.

In an interesting experiment with a soap-film described by Van der Mensbrugghe, a loop of moist silk thread is laid on a plane soap-film formed on a ring. The thread enters into the film, and really separates it into two distinct films, — a film within the loop, and a film lying between the loop and the ring. If the film within the loop be destroyed by touching it with the end of a hot wire, the loop is instantly drawn out into circular form by the pull, equal in all directions, exerted by the outer film upon it. The water surface experiment analagous to Van der Mensbrugghe's is shown by laying a thin rubber band on a clean water surface, and then putting a drop of oil on the water surface inside the loop. The band spreads out considerably, because the oil has diminished the surface tension inside the loop, and the surface tension between the walls of the vessel and the rubber band spreads the loop until the elasticity of the

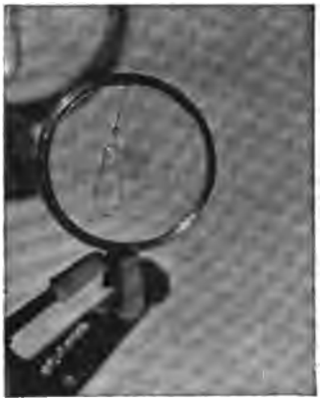
Fig. 1.

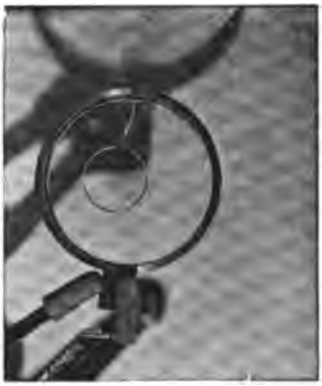
Fig. 2.

rubber striving to restore the old form exactly balances the excess of tension of the outer surface over that of the inner. A drop of oil on the liquid surface outside the loop causes a return of the loop to its former shape. The thinner and longer the band, the more nearly circular will be the expanded loop.

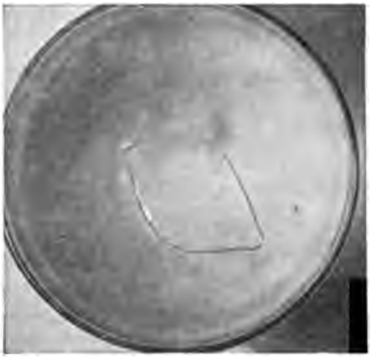

Fig. 3.

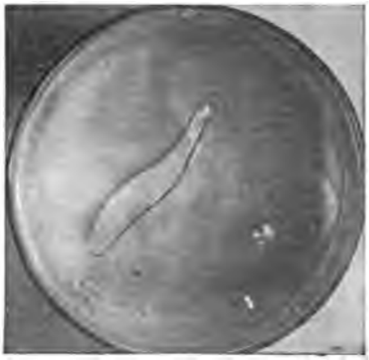

Fig. 4.

Figs. 1 and 2 are from photographs of the two stages in Van der Mensbrugghe's experiment; and Fig. 3 is from a photograph of a rubber band on a water surface, taken after the loop has been expanded by a drop of oil placed inside it. Fig. 4 is the form to which the band returned when oil was placed on the outlying surface as well.

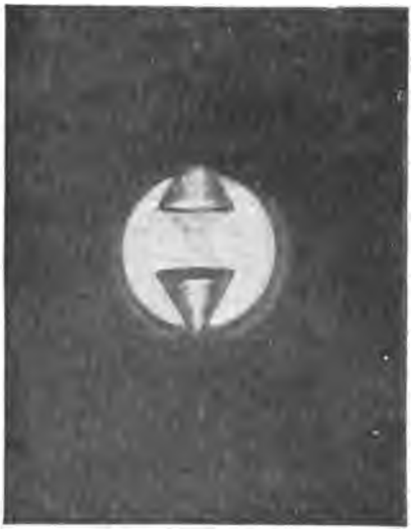

Fig. 5.

Two soap-bubbles blown on glass funnels, and then crowded together, can be made to unite, giving a single film which depends for its form, at any time, on the distance separating the funnels. When the funnels are pulled farther and farther apart, the film becomes spindle-shaped, with a neck more and more constricted, until finally the two bubbles spring apart. This experiment can be repeated in miniature with two drops of water on the ends of two glass tubes. The similarity of behavior of the drops of water and the soap-bubbles, and the resemblance in the curves shown in the process of separation in the two cases are striking.

Fig. 5 is from a photograph of a soap-film, and Fig. 6 shows a stage in the separation of two drops of water. A similar analogy appears between the way in which a soap-bubble leaves the bubble-pipe and the separation of a drop of liquid from the tip of a burette. A water surface seen "edge on," which is supporting a globule of carbon bisulphide colored with iodine is of interest in this connection. This is an experiment of another sort, but the curved outline of the water surface sagging beneath the weight of the heavier carbon bisulphide is a manifestation of surface tension, and the experiment is a significant one. The experiments described are all easily put into the lantern for projection, and are best shown in that way.

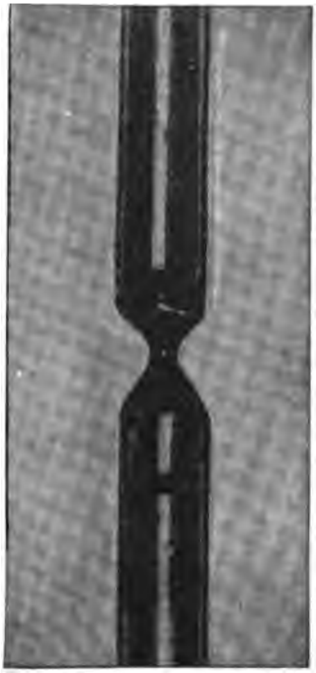

Fig. 6.

Colgate University, Hamilton, N.Y.,
July, 1893.

NOTE.

John Tyndall.—In the passing away of John Tyndall, whose death occurred at Haslemere, England, on the 4th of December, 1893, Physics loses one of her chief votaries. No man of science of his generation has been more widely known than he; very few have exerted a wider influence.

John Tyndall was born on the 21st of August, 1820, at Leighlin Bridge, near Carlow, in Ireland. He was the son of an Irish constable, and a direct descendant of the martyr, William Tyndale, who was burnt at the stake in the sixteenth century. Although in humble circumstances, Tyndall's father was able to keep his son in the local schools until the latter was nineteen years old, at which time he became a member of the Ordnance Survey. In 1843 young Tyndall became a railway engineer in England, which calling he followed for three years, when he resigned his post to become a teacher in Queenwood College. Doubtless his first experience as a teacher made clear to his own mind his deficiencies of preparation. He was already a reader of Carlyle, and it was due to the influence of the latter that he turned his face toward Germany for further study.

Marburg was at that time a seat of great scientific activity. Bunsen was professor of chemistry there, Stegmann of mathematics, and Knoblauch of physics. Under these men he studied and with them became a co-worker. Subsequently, in Berlin, 1851, he worked in the laboratory of Magnus, and made the acquaintance of Riess, Clausius, Wiedemann, and Poggendorff.

It was at the suggestion of Knoblauch, in 1849, that he began the study of diamagnetism, a subject to which he was destined to devote several years of his life. His first paper, published in *Poggendorff's Annalen*, Vol. 79, 1850, and in the *Philosophical Magazine* of the same year, was the result of a joint investigation made in collaboration with that physicist on *the deportment of crystalline bodies between the poles of a magnet.* The publication of this paper had a direct bearing upon Tyndall's subsequent career, for it served as a favorable introduction to Faraday, upon whom he called when he returned to England. Faraday, himself the discoverer of diamagnetism, was impressed with the valuable character of Tyndall's work, and he secured him an invitation to lecture at the Royal Institution.

This lecture was delivered on February 11, 1853, and in July of that year Tyndall was elected Professor of Natural Philosophy in the Royal Institution.

The Royal Institution, from the time of its foundation by Count Rumford to the present day, has been the scene of the most brilliant experimental demonstrations known in the history of physics. It was there that Davy, in 1807, exhibited the first electric arc light; that Faraday announced his many discoveries, demonstrating them to delighted classes. Under Tyndall, who was the follower of Davy and Faraday in this renowned chair, the Royal Institution lost none of its fame.

Tyndall's power as a popular writer of science was so extraordinary that his attainments in this field have largely overshadowed his other work. It is of his lectures on sound and on light; of his books on *Heat as a Mode of Motion*, on the *Forms of Water*, and on the *Glaciers of the Alps;* of his essays as embodied in the *Fragments of Science*, and in the *New Fragments*, that we think when we hear Tyndall's name spoken, rather than of his work as an investigator. And yet, for the quarter of a century during which these books appeared, he was indefatigable in original work. The Royal Society Catalogue gives seventy-two titles of memoirs under his name, previous to 1864, and the supplement covering the ten years following, including 1873, adds forty important papers to the list. The range of subjects is a wide one; but the fields of his activity in research were, roughly speaking, the following: magnetism and diamagnetism, to which he devoted his attention chiefly during the decade 1850–'60; the domain of radiant heat, which occupied his continual attention for ten years following; the study of the atmosphere with particular reference to the formation of clouds and the color of the sky, a theme which led up to the study of properties of smoke and dust in the atmosphere, and so finally to his last great work upon atmospheric germs.

The conclusions which he reached concerning the floating matter in the air were embodied in his famous lecture on *Dust and Disease*, which was delivered at the Royal Institution in 1870. The battle which ensued was long and severe. It resulted in the establishment of the germ theory of disease, an achievement in which Tyndall and Pasteur will always share the honors. During all these years of his scientific activity, Tyndall spent his summers in the Alps. He was one of the most intrepid and successful of mountain climbers, and he was also an enthusiastic student of the phenomena presented by the Swiss glaciers. It is to these vacation studies that we owe some of his most fascinating books, and it is upon them that no small portion of his fame rests.

In addition to his original work in physics and to his studies of the glacier ice, Tyndall was for many years one of England's chief writers in the field of speculative philosophy and of general science. His work in this field deserves to rank with that of his life-long friend, Professor Huxley. First among these essays, perhaps, stands his address as President of the British Association, in 1874. It was in this Belfast address that Tyndall enunciated his view with reference to the origin of life. "Believing as I do in the continuity of nature, I cannot stop abruptly where our microscopes cease to be of use. Here the vision of the mind authoritatively supplements the vision of the eye. By an intellectual necessity, I cross the boundary of experimental evidence and discern in that *matter*, which we in our ignorance of its latent powers, and notwithstanding our professed reverence for its Creator, have hitherto covered with opprobrium, the promise and potency of all terrestrial life."

This daring declaration shook philosophic England to its center. Protests against its materialism arose in many quarters. The discussion which ensued was one of the most notable of that time. Its very echoes have died away now, and if we may judge them by their later utterances the school whose views were embodied in the Belfast address has materially changed its attitude. Tyndall, himself in fuller explanation of his point of view, assumed a much more conservative position.

While Professor Tyndall stands in the first rank of the speculative writers of his time, he stands in another important field entirely alone. As a popular writer upon the phenomena of physics he has had no equal. Remarkable as an investigator and remarkable as a writer of English, it is after all to his wonderful power in describing the beauties of natural philosophy that he owes his chief fame. The aim which he professes in his books is, at first sight, a simple one. In the preface to his *Lectures on Sound*, he says: "In the following pages I have tried to render the science of acoustics interesting to all intelligent persons, including those who do not possess any special scientific culture." The preface to *Heat as a Mode of Motion* begins in almost the same words: "In the following lectures I have endeavored to bring the rudiments of a new philosophy within the reach of persons of ordinary intelligence and culture. . . . My aim has been to rise to the level of these questions for a basis so elementary that a person possessing any imaginative faculty and power of concentration might accompany me." To the fulfilment of this purpose, gifts, more unusual than those of the scientific investigator or those of the writer of speculative essays, are necessary. These gifts Tyndall possessed in the highest degrees, and he has utilized them for the delight and inspiration of an entire generation.

E. L. N.

NEW BOOKS.

Light and Other High-Frequency Phenomena. By NIKOLA TESLA.

Introduction. — Mr. Tesla introduces his lecture about as follows: — In our study of the physical universe we find that our sense of sight is superior to all other senses, because it enables us to penetrate much deeper into the fathomless depths of Nature's mysterious activities. The activity by means of which Nature reveals herself to us through our organ of sight, the activity which we call light, deserves our closest attention. What is light? It is an electromagnetic activity, according to most advanced views; hence the importance of investigating those forms of electromagnetic activities which approach the phenomena of light as near as experimental possibilities of the present day will permit.

The introduction might lead one to expect that Mr. Tesla intends to describe a novel class of high-frequency electrical oscillations for the purpose of illustrating and giving new evidence in favor of the electromagnetic theory of light. In that case one would perhaps feel somewhat disappointed to find that Mr. Tesla's experiments are simply Hertz's experiments in a modified form, a form by which it can be shown that in the first place the Hertzian oscillations enable us to transmit a considerable amount of energy along a single wire, and in the second place that novel vacuum tube effects can be produced by rapid alternations of the electromagnetic field.

But this feeling of disappointment, if it should occur at all, would be soon dispelled and quickly give way to a feeling of admiration for the patient work of an enthusiastic inventor who has brought all the ingenuity of his remarkable engineering skill to bear upon the problem before him, and that problem is contained in the question: — What immediate benefits can modern electrotechnics derive from Hertzian oscillations? This problem Mr. Tesla handles with extraordinary ability.

Method of Conversion. — To produce the oscillations Mr. Tesla connects a direct or alternating current dynamo to an oscillatory transformer. The primary of this transformer, consisting of a small number of turns of stranded wire, corresponds to the Hertzian exciter, and the secondary, consisting of an appropriate number of turns of wire surrounding the primary, corresponds to the Hertzian resonator. Hertz worked with oscillations of about 10^8 periods per second. Mr. Tesla is satisfied with a frequency of 10^6 or even smaller, hence his primary and secondary coils have a much larger self-induction than that of the Hertzian exciters and oscillators. By a suitable arrangement of the ratio of the turns in the primary and secondary, the oscillatory transformer can be an up or a down transformer. The primary spark space is similarly located as in the Hertzian experiments. Capacity is generally added to both the primary and the secondary coils.

Precautions which must be observed in order to render the Primary Impulses very Powerful. — No arc formation should be allowed in the primary spark space. Four different devices for this purpose are described, the purpose of each being to blow out the arc as soon as it begins to form.

1. Spark space is placed between the poles of a sufficiently powerful magnet.

2. Current of hot air sweeps continually through the spark space (especially when dielectrically weak air is required between the discharge knobs).

3. Spark space is placed in a partially exhausted glass tube, the tube being placed between the poles of a strong magnet to prevent arc formation.

4. Two objections against a spark space through a gaseous medium are pointed out: (*a*) the discharges are not sufficiently disruptive; (*b*) there is too much dissipation. To improve both, a liquid dielectric was tried. A hollow pulley was partially filled with oil (the quality of oil not mentioned); the pulley being rotated rapidly, the oil was thrown out centrifugally and compressed against the rim; discharge through oil improved the disruptive character of the primary sparks and diminished losses. These discharges were effective for producing great spark length, but not for luminous effects, since for these, according to Mr. Tesla, a harmonically varying electromagnetic force is preferable.

The best form of generator is then discussed; it is pointed out that the most convenient method is to use a low-tension dyna-motor, to transform direct currents into low frequency alternating currents of low voltage. The low-voltage alternating current thus obtained is then transformed into high tension alternating by a stationary transformer, and this high-tension current is then employed to feed the oscillatory transformer.

Phenomena produced by Electrostatic Force. — In the introduction to these phenomena, Mr. Tesla makes the following statement concerning electrostatic force: "It is the force which governs the motion of the atoms, which causes them to collide and develop the life-sustaining energy of heat and light, and which causes them to aggregate in an infinite variety of ways according to Nature's fanciful designs, and to form all these wondrous structures we perceive around us. . . ."

Evidently Mr. Tesla makes even a broader claim for the electrostatic force than Professor J. J. Thomson does for Faraday's tubes of electrostatic induction. (Compare also H. von Helmholtz's lecture on Faraday.[1])

Experiment 1. — One pole of the oscillatory transformer is touched with one hand, and the other hand approaches the other pole. A brush discharge passes from the free hand to the free pole. The frequency of the current is said to be a million, and the electromotive force $= 2 \times 10^5$ volts. Mr. Tesla calculates that, with four times the frequency, and a potential of 3×10^6 volts, his whole body would be surrounded by a flame, but the current would do him no harm, because the body offers a large cross-section to the high-frequency current, and the current *in its passage through the body distributes itself uniformly*, so that the current density is small at any point. This interpretation takes it for granted that Hertzian oscillations pass through the human body just the same as an ordinary direct current would pass through a good conductor.

Experiment 2. — Illustrates difference between discharges at high and low frequency. Two circular brass plates, each connected to a pole of an oscillatory transformer, are placed at a distance of 12 inches. At high frequency the luminosity of the discharge between the plates is steady and perfectly uniform; at low frequency, the discharge is in the form of numerous, quivering, and noisy spark filaments. Mr. Tesla admits the impossibility of disruption at high-frequency; he observes the powerful chemical action of the high-frequency discharge; he seems, however, to fail to recognize in the difference of the two discharges a confirmation of the generally accepted dissociation theory of electrical conduction through gases.

Experiment 3. — An incandescent lamp bulb with a single straight carbon filament is attached to one pole of the oscillatory transformer. An apparently unipolar discharge takes place. (The luminosity in the immediate neighborhood of the external surface of the lamp is evidently due to the varying condenser charges of that surface. If that surface has any sharp points, the current density there will be large, and the point will be heated.[2]) A bulb drawn out into a point is (therefore) heated and is ultimately broken at that point. Mr. Tesla ascribes this heating to molecular bombardment which, according to his opinion, will take place in *heterogeneous media like gases*, because *the molecules* or atoms in such bodies are free to move under the impulse of electrical forces. Mr. Tesla does not seem to be aware of the fact that nobody has succeeded yet in electrifying the molecules of a perfect gas. Several experiments are then described in support of the bombardment theory, but they do not seem to be convincing.

[1] Vorträge und Reden, Vol. 2, p. 302, 1884.

[2] The reviewer is responsible for the opinion enclosed in parentheses in this paragraph.

On Currents or Dynamic Electricity Phenomena. — In these experiments it is shown that electrical energy can be transmitted along a single open-circuited wire. The *modus operandi* is the following: — One end of a wire is attached to one of the poles of the secondary of an oscillatory transformer; the other end is attached to a large plate. An adjustable coil is also inserted in this circuit. The plate gives this open circuit its principal capacity, the adjustable coil, which the reviewer calls the inertia coil, enables the experimenter to regulate its self-induction or inertia. When the frequency is very high, the capacity of the plate can be dispensed with, the capacity of the remaining part of the circuit being sufficient to bring the circuit in resonance with the impressed electromotive force.

Two methods are then employed to demonstrate the flow in the open circuit and its dependence on the constants of the circuit; namely, capacity, self-induction, and frequency.

Method I. — The circuit of several turns of wire surrounding the auxiliary inertia coil is closed by a 50-volt, 16-C.P. lamp, and the flow studied by the intensity of the glow of this lamp.

Method II. — The flow in the open circuit is transformed in various ways, and various simple devices are then employed to cause a disc to rotate under the electromagnetic or electrodynamic action of the oscillatory currents thus employed. The first method points out the possibility of a distribution of oscillatory currents for electric lighting; the second method demonstrates in a certain way the feasibility of employing such currents for operating electromotors. The difficulties which would necessarily attend such an employment of oscillatory currents might have been discussed a little more fully, in order to prevent a misconception on the part of those who are not familiar with the subject. To show the effects of impedance (a wave distribution of potential energy along a short, stout wire), Mr. Tesla employs Lecher's well-known and generally adopted method of producing very rapid Hertzian oscillations. He devotes, then, twelve pages to a discussion of electrical resonance, and particularly of the bearing of electrical resonance upon the problem of transmitting signals all over the earth by causing the electrical charge of the earth to oscillate.

This discussion is evidently introduced to relieve somewhat the serious character of the rest of the lecture, and seems to bear the same relation to the rest of the lecture as the amusing gravediggers' dialogue in "Hamlet" bears to the rest of the play. In the reviewer's opinion, Mr. Tesla does not wish to be taken *au serieux* in this part of his lecture.

Vacuum Tube Effects produced by Hertzian Oscillations.

1. *Incandescence of a Solid.* — It was observed a long time ago that the electrodes of a Geissler tube, especially the cathode, become very hot, although the current through the tube may be exceedingly small.

Mr. Crookes, it seems, was the first to suggest that this heating was due to molecular bombardment. Mr. Tesla shows that by means of this property of the vacuum-tube discharge carbon filaments and carbon buttons, when enclosed in the vacuum of an incandescent lamp, can be rendered incandescent by an exceedingly small current, provided that the potential of these carbons is very high and the frequency considerable; the higher the frequency, the slower will be the process of disintegration of the carbons. He also ascribes this heating to molecular bombardment, but his arguments in favor of this view are open to the same objections which were brought, long ago, against the molecular bombardment theory of Crookes. Some of the heating in such tubes is very probably due to molecular, or rather atomic, bombardment, but according to the splendid investigations of E. Wiedemann, Schuster, Elster and Geitel, J. J. Thomson, Giese, and others, the chemical process going on in such tubes owing to the action of the electrical forces will account easily for a very large part of the heat. It should also be remembered that the spectrum of an incandescent gas is very rich in ultra-violet light, which part of the luminous energy of the gas never leaves the bulb, being absorbed by the same and transformed into heat. To ascribe all the various heat phenomena in a vacuum tube simply to molecular bombardment means to turn a sharp corner, in order to get out of the way of difficulties instead of forcing the difficulties themselves to get out of the way. The same criticism applies to Mr. Tesla's view of the cause which produces phosphorescence and fluorescence in vacuum tubes. It is, of course, a well-known fact that mechanical shocks can produce phosphorescence and fluorescence; but it is equally well known that no agent is as powerful as ultra-violet light and purely chemical processes to produce these phenomena. Why, then, refer these phenomena in vacuum tubes to molecular bombardment and to nothing else?

It must be observed, however, that the value of Mr. Tesla's work is, of course, independent of the view which he may have formed of the causes of all these various phenomena which he investigated by means of extremely powerful electrical oscillations. The real value of Mr. Tesla's work is due to the fact that he was the first to succeed in constructing a magnificent array of physical apparatus, by means of which these exceedingly powerful Hertzian oscillations can be produced, and to point out a way of attacking old but still puzzling phenomena by weapons with which the refinements of modern electrotechnics can furnish us. It is a work which, for some time to come, will be consulted by the electrical engineer who wishes to become acquainted with the tendencies of recent advances in electricity, and it will be consulted by the physicist who wishes to make himself familiar with the practical bearing of these advances. In the performance of this work Mr. Tesla has, therefore, rendered invaluable service to all those who are interested in the science of electricity.

M. I. PUPIN.

COLUMBIA COLLEGE, New York, Oct. 15, 1893.

Two Treatises on the Theory of Elasticity.[1]

Mr. Love's volume treats the mathematical theory of the elasticity of bodies of finite dimensions. A second volume, according to the plan announced by the aúthor, is to treat the theory of the elasticity of bodies, some of whose dimensions are so small as to be considered infinitesimal. The present volume is well arranged. The order of treatment is as follows: Chap. I., 21 pp., Analysis of Strains; Chap. II., 12 pp., Analysis of Stress; Chap. III., 73 pp., Stress-Strain Relations; Chap. IV., 9 pp., Strength of Materials; Chap. V., 36 pp., General Theorems; Chap. VI., 53 pp., Saint-Venant's Problem; Chap. VII., 33 pp., Curvilinear Co-ordinates; Chap. VIII., 16 pp., General Solutions; Chap. IX., 25 pp., The Problem of an Infinite Solid bounded by a Plane, Boussinesq and Cerruti; Chap. X., 36 pp., The Equilibrium of the Sphere, Lamé; Chap. XI., 22 pp., The Vibrations of the Sphere; Chap. XII., 14 pp., Application of Conjugate Functions.

Chap. III. deserves special mention. It contains an exhaustive treatment of the general theory of elastic constants and a good sketch of crystallography. Chap. IV. is very brief, but, with references given to universally accessible writings, it is essentially complete. Chap. V. gives a brief and satisfactory sketch of the development of the theory of stress-strain relations from Cauchy's point-atom hypothesis, and a number of general theorems concerning possibilities and uniqueness of solutions, energy function, wave motion, and free vibrations of solids. Chap. VII. gives a more than ordinarily clear exposition of curvilinear co-ordinates.

The mathematical developments are very concise and elegant. The notation is simple and straightforward, and is based upon precedent. In arranging the plan of treatment the author has evidently been guided by a consideration of the present state of English literature on the subject, and has avoided extensive treatment of many topics, such, for example, as the natural history of elasticity, and the elaborate theory of types of stress and strain, both of which topics are so admirably treated in the generally accessible writings of Sir W. Thomson (Lord Kelvin). An historical introduction of thirty-four pages is well placed. The author has wisely followed the plan of dividing the text into distinct paragraphs, each with prominent heading, and each strictly limited to a point in question. This fact, together with the exhaustive table of contents and a good index, will go far towards rendering the book readily useful for reference. No problems are given. Mr. Love has introduced but few if any novelties of nomenclature. We cannot refrain from expressing at this point our preference for such expressions as bulk-modulus, slide-modulus, and stretch-modulus, to such expressions as modulus of rigidity, Young's modulus, and the like. Some writers even use the expressions, "coefficient of volume elasticity," "coefficient of simple rigidity." Mr. Love uses the name, bulk-modulus, but not the others. They have been occasionally used, however, and with evident clearness and brevity.

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[1] *A Treatise on the Mathematical Theory of Elasticity.* Vol. I. By A. E. H. LOVE, M.A. 8vo, pp. 354. Cambridge, University Press, 1892.

Leçons sur la Théorie de l'Élasticité. Par H. POINCARÉ, Membre de l'Institut. 8vo, pp. 208. Paris, Georges Carré, 1892.

If Vol. II. come up to the high standard of Vol. I., Mr. Love's treatise will be a valuable addition to the literature on elasticity. It shows no tendency to undue simplification. Its scope is by no means narrow. Its style, being concise, is such as to stimulate the study of other works, and particularly of original memoirs. The realization of such tendency is rendered possible by reason of numerous choice references. On the whole, it would seem that the work must be admitted to first rank in the series of mathematical treatises for which Cambridge is justly celebrated.

M. Poincaré's book is written in the same attractive style as his "Electricité et Optique" and "Thermodynamique." To English readers particularly it must, however, prove less satisfactory. In his preface to "Thermodynamique" the author says, "*Abandonnant les théories ambitieuses d'il y a quarante ans, encombrées d'hypothèses moléculaires, nous cherchons aujourd'hui à élever sur la Thermodynamique seule l'édifice tout entier de la Physique mathematique.*" Now precisely the most serious objection to the "Leçons sur la Théorie de l'Élasticité" is that the treatment of stress-strain relations is made to rest upon Cauchy's point-atom hypothesis — an hypothesis so particular in its assumptions as not to be susceptible of adaptation to broad lines of experimental fact. English readers, being so familiar with Lord Kelvin's writings, cannot now have patience with such a method. Nothing can be more nearly free from misleading particularization than the assumption that the stress at a point in a body is a function of the strain at that point. That part, if any, of the stress which does not vanish with the strain being ignored, then if it be assumed that linear terms and terms of higher order only appear in the expansion of this function, Hooke's law in its most general aspect follows at once for infinitesimal strains, which only are considered at present in the mathematical theory of elasticity. Further, there would be a wide discrepancy between this theory and experience if these assumptions were not in the main legitimate. These assumptions involve Lord Kelvin's definition of perfect elasticity (see *Ency. Brit.* "Elasticity"), which we know to be satisfied with high approximation for many substances through comparatively wide limits of strain. For infinitesimal strains, too, we must remember that the rates of change of the strains are also infinitesimal, so that it is probable that Hooke's law is legitimate for infinitesimal strains for substances which exhibit marked viscosity under varying finite strain. Hooke's law may then be considered a formal axiom justified by experience for infinitesimal strains. Lord Kelvin's application of the laws of thermodynamics in the proof of the existence of the energy function establishes the mathematical theory of elasticity on a thoroughly satisfactory basis.

The notation employed by French writers is perhaps something which no English reader has a right to criticise. Yet English translations of recent French treatises on mathematical physics are often targets in this respect. Witness, for example, the translation of Mascart and Joubert's "Electricity and Magnetism." The "Leçons sur la Théorie d'Élasticité" must be considered decidedly the weakest of M. Poincaré's works in this respect.

Theoretical science has a strong historical perspective. Its present aspects must always be most vital, and remote things must continue to grow less explicitly important. Yet undue haste in disregarding the classic is no less fatal here than elsewhere. A student must not be led to think that any systematic treatise, however well written, can relieve him of the necessity of studying the classics of his subject. M. Poincaré's "Leçons" is not satisfactory in that it is not calculated to incite a student to a vigorous pursuit of the subject. It is poor in its method, narrow in its scope, and fragmentary in its choice of material. It gives no references to original sources. We have nowadays an increasingly large number of books which grow out of the limited experience of a teacher in handling a short course of lectures on a subject. To presume to rank such works with extensive treatises is, of course, preposterous; to judge them by the same standard is manifestly unfair. Some books are written that everybody may know something; some, on the other hand, are written that somebody may know everything. The object of writing is often immediately and apparently the former, but always and really the latter.

W. S. FRANKLIN.

NOTE.—Just as the above is going to press, Volume II. of Love's Treatise[1] appears. A hasty examination produces a very favorable impression, and it seems that the promise of the first volume is fulfilled. An historical introduction to the theory of the elasticity of thin rods and plates, 23 pp., with many choice references, amounts almost to a bibliography of that subject. The arrangement is as follows: Chap. XIII., 25 pp., The Bending of Rods in One Plane; Chap. XIV., 24 pp., The Bending and Twisting of Rods in Three Dimensions; Chap. XV., 21 pp., General Theory of Thin Rods; Chap. XVI., 21 pp., Theory of the Small Vibrations of Thin Rods; Chap. XVII., 21 pp., Resilience and Impact; Chap. XVIII., 30 pp., General Theory of Wires Naturally Curved; Chap. XIX., 25 pp., Elementary Theory of Thin Plates; Chaps. XX. and XXI., 44 pp., General Theory of Thin Plates and of Thin Elastic Shells; Chap. XXII., 44 pp., Applications of the Theory of Thin Shells; Chap. XXIII., Stability of Elastic System.

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[1] *A Treatise on the Mathematical Theory of Elasticity.* Vol. II. By A. E. H. LOVE, M.A. 8vo, pp. 327. Cambridge, University Press, 1893.

The treatment of rods and plates can hardly be expected to be as satisfactory as the treatment of the general theory of elasticity, and it is surely not so well known to students. Chaps. XIII. to XVIII. seem, however, to be thoroughly satisfactory, and Chaps. XIX. to XXII. are perhaps as satisfactory as they can be, considering the difficulties of the theory of plates and shells.

Mr. Love has clearly demonstrated that there was need of a good treatise on the theory of elasticity by writing one.

W. S. F.

The Science of Mechanics. By Dr. E. MACH. Translated by T. J. MCCORMACK. 8vo, pp. x+538. Chicago, The Open Court Publishing Co., 1893.

In looking through a library of text-books one must be struck by the great number of books on mechanics as compared with the number in other departments of physics. This arises from the fact that mechanics is the oldest branch of physics. But there is another and more important reason, and that is the difficulty of stating the fundamental principles, there being a selection of methods of statement. A study of the volume before us will tend to lessen the multiplication of books, as it will show the dissatisfied reader that though possibly he might make another selection, it would not necessarily be an improvement.

Mach's "Mechanics" is unique. It is not a text-book, but forms a useful supplement to the ordinary text-book. The latter is usually a skeleton outline, full of mathematical symbols and other abstractions. Mach's book has "muscles and clothing," and being written from the historical standpoint, introduces the leading contributors in succession, tells what they did and how they did it, and often what manner of men they were. Thus it is that the pages glow as it were with a certain humanism, quite delightful in a scientific book.

Recognizing that mechanical ideas have been evolved from the investigation of simple and special cases of mechanical processes, the method of the book is the historical. The development of the science is shown to cluster around a few great names. Beginning with the investigations of the ancient Greeks concerning equilibrium, the development of the principles of statics is sketched in the first quarter of the volume. The next half of the volume is devoted to an account of the development of the principles of dynamics, and the balance to an account of the more modern extensions of dynamical principles. Here we find D'Alembert's principle, the principle of kinetic energy, of least action, etc., all treated with a minimum of mathematics, but with a maximum of illustration. Throughout the book the method of presentation is to give the discoverer's account of the proposition presented as nearly as possible in his own way. Next follow criticisms of the proposition or of its proof, and perhaps a supplemental proof is given. Then come general observations on the underlying principle, and it may be on the author himself or his methods. Thus, contrasting Stevinus's conception of the inclined plane with that of Galileo, Mach remarks, "It is in this fact that Galileo discloses such scientific greatness: that he had the *intellectual audacity* to see, in a subject long before investigated, *more* than his predecessors had seen, and to trust to his own perceptions." And again, "Newton's reiterated and emphatic protestations that he is not concerned with hypotheses as to the causes of phenomena, but has simply to do with the investigation and transformed statement of *actual facts*, stamps him as a philosopher of the *highest* rank."

There is a very full account of the units used in dynamics though some of the notation is rather antiquated. Surely, for example, it is time to drop *vis viva*, *quantity of motion*, *conservation of force*, and the like. The definitions of mass and weight given are those accepted by physicists, but not by engineers — quite as important a body.

To many the most interesting part of the book, as it is certainly the part most provocative of thought, will be the section in which Newton's laws of motion are discussed, and in which it is urged that the time has come for a restatement of these laws. That there is a necessity for a restatement few will deny, but it is to be doubted if the substitute proposed by Mach will be accepted. Still the discussion is most timely, the subject occupying as it does just now so much of the attention of English and American physicists.

With German thoroughness our author explores not only the highways, but the byways. Hence we have chapters on the relation of mechanics to physiology, and on the theological, animistic, and mystical points of view in mechanics. In the latter is traced the gradual transition from regarding mechanical problems from the theological standpoint to the complete separation of physics from theology, which took place about the beginning of the present century.

The translator merits commendation for the excellent work he has done. A Germanism occurs here and there, it is true, but not enough to mar the reader's enjoyment. His annotations, too, are useful and show care and research. The book is handsomely printed, and deserves a warm reception from all interested in the progress of science.

T. W. WRIGHT.

A Text-Book on Electromagnetism and the Construction of Dynamos. Vol. I. By DUGALD C. JACKSON, B.S., C.E., Professor of Electrical Engineering in the University of Wisconsin. 8vo, pp. 281. New York, Macmillan & Co.

The large number of works upon dynamo-electric machinery and kindred subjects in electrical engineering which are now appearing and about to appear (for we hear from various publishers of further works now in press and in preparation) leads us to inquire the reason for such literature. Most of these books cover identically the same ground, and many of the authors so state in their prefaces. The cause is easily traced to the rapid growth of the electrical industries during the last decade, which has developed a demand for practical information on electrical subjects, and indirectly occasioned a stimulus to scientific investigation and study of electricity from a purely scholarly standpoint. This demand has for the most part been met with general treatises, the authors of which, endeavoring to cover the entire field, have covered what they could, and staked out claims for the remainder. These general treatises often contain a great deal that is of value, but are not in every way satisfactory. It is from an appreciation of this fact, and an honest belief that they may be able to do better, that authors have been led to the production of such books as the one in hand. Many fail; those who succeed generally do so by specializing, and by giving a complete and logical exposition of some one branch of the subject previously treated in a fragmentary manner. It is, therefore, with some apprehension, that we take up a book on "Electromagnetism and the Construction of the Dynamo," fearing that in the same volume we may find Maxwell's theory of magnetism, and a detailed description of forms of dynamos, construction of commutators, etc. However, we are relieved to find the title a misnomer and that the book is on electromagnetism and its relation to the dynamo. The question of design is not taken up; no cuts or description of typical machines appear; details of construction are entirely passed over; and the book is devoted to the principles of the magnetic circuit and of the dynamo.

The treatment of the magnetic circuit in the first chapter is particularly clear, and in a few pages the reader is able to gain a knowledge of permeability, reluctance, magnetomotive force, and magnetization. Simple electromagnets are treated in the second chapter, and magnetic properties of iron in the third. The results of the researches of Ewing, Hopkinson, Steinmetz, and others is given in brief, and illustrated by typical curves. Most of these curves are plotted to show the relation between the permeability and the induction, rather than the relation between induction and magnetizing force, which is more usual, and therefore preferred. Fig. 25, *Miscellaneous Curves*, is to be criticised. In this figure curves are plotted in which the co-ordinates represent B, H, I, κ, and μ. It is only after careful study that one not already familiar with such curves can tell the co-ordinates corresponding to any particular curve.

The explanation of the establishment of electric pressure is well given in a separate chapter, but contains some analytical expressions for the electromotive force which seem hardly called for. The subject of magnetic leakage is well treated, the formulæ of Forbes and others being given and derived for several cases. The determination of the reluctance between two parallel cylinders is given in full and seems a little out of place in an elementary work of this kind, involving as it does the formulæ of Poisson and Laplace.

The subject of armature reactions and the so-called "cross" and "back" turns is given the prominence which it deserves, and various methods of compensation for the cross-turns are discussed, particularly the arrangement of compensating series coils devised by Professor Ryan.

In the chapter on characteristic curves and regulation, the graphical methods of compounding are particularly valuable, not so much perhaps for their practical bearing as for the insight they give into the action of a dynamo. The book closes with a chapter on efficiencies and one on multipolar dynamos.

The author draws largely from S. P. Thompson and Ewing, to whom due credit is given. The reader is usually referred to some standard work for such proofs as are omitted. The explanations are clear and for the most part adequate. Although written primarily for a text-book, the work is useful for reference and is accordingly of value to the engineer. It can be recommended to students in electrical engineering.

F. BEDELL.

Resistance of Ships and Screw Propulsion. By D. W. TAYLOR. 8vo, pp. ix+239. New York, Macmillan & Co., 1893.

Among the various contributions to this subject which have recently appeared, the present may be considered as one of the most notable. It opens with a general description of the main features of ship resistance. Free use is made, and duly credited, of the papers of the Froudes in the transactions of the Institute of Naval Architects. Based on the principles herein set forth, an approximate expression is derived for the natural or tow-rope resistance. This is of two terms, one giving the frictional and the other the wave-making resistance. Two controlled constants are involved, one being the coefficient of frictional resistance, and the other a constant relating the wave-making resistance to the fourth power of the speed. The application of the formula depends, of course, on the proper control of the constants, some precedents for which are given in a later chapter.

Chapter II. is taken up with a new development of one of the methods for the treatment of the theory of the screw propeller.

Such theories in general may be divided into two classes.

(1) Those in which the quantities involved are the mass of water acted on, and the acceleration given to this water.

(2) Those in which the quantities involved are the geometry and movement of the propeller itself, and the amount and distribution of pressure over its surface as it moves through the water.

Theories of both classes are incomplete: those of the first, for the reason that we are not able to ascertain in any given case either the amount of water actually acted on, or the acceleration effected; those of the second, because the amount and distribution of pressure cannot be accurately determined. Those of the first class have been more commonly in use. In 1878, however, the late William Froude published a notable paper in the transactions of the Institute of Naval Architects, in which was sketched from the second standpoint, the foundation of a theory for the action of an element of a screw propeller. This has been followed by a few papers from other writers bearing on the same general topic.

Not until the present work, however, has there been a systematic attempt to develop into a basis of design the theory from this standpoint.

Briefly stated it may be said that the method used involves first the assumption of two controlled constants; one giving the relation between unit normal pressure and velocity, and the other, that between unit frictional resistance and velocity. The torque, thrust, and efficiency may then be expressed as a function of speed, the geometry of the propeller, and these constants. The geometry of the propeller is introduced in two elements, first the diameter as a dimension, and second, certain functions of the geometrical form which are rendered independent of actual dimension, and are denominated "characteristics." The actual surfaces being non-algebraic, these characteristics are expressed in the form of integrals whose values are found by approximate methods from the drawings of the propeller. The performance of any propeller being expressed in terms of these known functions and of the two unknown constants, it becomes possible by comparison with experimental results to determine values of the latter, and thus to obtain the necessary control.

It may be noted that in the computations for characteristics, the driving face only of the propeller is involved. The value of the constants found will, however, include the effect of the thickness on the back, and may therefore be considered as means for the entire surface.

The constants being thus determined, the formulæ become appropriate for the purposes of design under similar conditions.

The same general method provides, also, a means of determining the net resultant stress on the root of the propeller blade, and hence gives the necessary data for the design for strength.

Another important feature of the work is the method developed in Chap. IV. for the analysis of the results of trial trips, and the distribution of the power among the several components. A comprehensive outline of the method would be too long for the present limits, but the results obtained are as follows: —

The assumption and consequent elimination of the load engine friction.

The determination of the initial engine friction, and the consequent elimination of the engine friction entirely.

The determination of the power required to overcome the resistance and friction of the propeller.

The subdivision of the remaining thrust power, among the following components: — (1) That required to overcome the skin resistance of the ship. (2) That required to overcome the wave-making resistance of the ship. (3) The power involved in the thrust deduction.

The value of the wake factor, and the various efficiencies of the propeller also appear in the course of the analysis. The methods used in this analysis are highly ingenious, and with proper care will undoubtedly give valuable information relative to the distribution of power. From the very nature of the case, however, the analysis involves certain assumptions and several controlled constants. The propriety of the former and proper values of the latter can only be determined by experience in the application of the method to the analysis of varying cases.

As a whole, the mode of analysis is, so far as I know, the first by means of which the indicated horse power, as given by experimental results, may be subdivided in so great detail.

In Chap. V., an account is given of various methods for estimating the power required for the propulsion of a ship at a given speed. The different methods in common use are explained, and a series of tables and graphical data are given for the ready application of Froude's law of comparison. The method by the use of the Admiralty $D^{\frac{2}{3}}$ coefficient is characterized as very untrustworthy. It may be assumed that reference is had to the method of use without proper means of control for the constant; for, as it is well known, the use of this formula with the *same* values of the constant for *similar* ships at *corresponding* speeds, will give results identical with those obtained by Froude's law of comparison as explained by the author. As a matter of fact, the use of the $D^{\frac{2}{3}}$ formula with the constant thus controlled is but one of the many forms in which this law may be expressed.

The most general of the law of comparison is as follows: — Let the resistance of a ship consist of such terms that it is capable of expression in the following form: —

$$R = \Sigma A L^{3-\frac{n}{2}} v^n;$$

where A is a constant for any one form of ship, L is a linear dimension of the ship, v the speed, and n may have, within certain limits, any value, either whole or fractional. Such conditions being fulfilled, it is readily shown that similar ships (those wherein only L varies) at corresponding speeds (those in the ratio of the square roots of the linear dimensions) have resistances as the cubes of their linear dimensions. In fact, one of the simplest conceptions of the law of comparison is that it virtually assumes the resistance of a ship to be capable of expression by the sum of a series of terms as above.

The use of the $D^{\frac{2}{3}}$ formula may be readily seen to be merely a special case of the application of this general form.

In addition to these methods, the author gives another called the "independent" estimate method. This is based on the formula for resistance developed in Chap. I., together with certain assumptions relative to propeller efficiency, engine friction, etc. This method, provided the constants are properly controlled, would give results valuable for comparison with those obtained by the use of Froude's law.

As a whole, the work is an exceedingly valuable contribution to the subject. Many of the methods used are characterized by originality combined with great ingenuity and elegance. As aids in the practical application of the theories and methods developed, the book abounds throughout with tables and graphical data. In connection with this application, the only caution to be observed is one inseparable from the nature of the subject; that is, that any applied theory resting on a series of assumptions, not capable of *a priori* proof, and involving a considerable number of controlled constants, must necessarily be used with care and only applied to the extent justified by the amount of control to which the constants have been subjected.

W. F. DURAND.

Alternating Currents: An Analytical and Graphical Treatment for Students and Engineers. By Frederick Bedell, Ph.D., and Albert C. Crehore, Ph.D. New York, W. J. Johnston Co.; London, Whittaker and Co. *Second edition.*

The second edition of this book is not materially different from the first, a review of which appeared in the Physical Review, No. 1.

HEINRICH HERTZ.

Volume I. *March–April, 1894.* *Number 5.*

THE

PHYSICAL REVIEW.

THE ELECTRICAL CONDUCTIVITY OF COPPER AS AFFECTED BY THE SURROUNDING MEDIUM.

By Henry S. Carhart.

THE following investigation was undertaken at my suggestion by two students in my laboratory, Mr. Hugh Rodman and Mr. Fred L. Keeler, for the purpose of confirming or refuting the conclusion reached by Professor Fernando Sanford[1] respecting the dependence of the electrical resistance of copper upon the surrounding medium.

The resistance, which Professor Sanford measured under varying conditions, consisted of a copper tube 120 cm. long and 2.5 cm. in diameter, and a copper wire stretched along the axis of the tube. The tube was closed at the ends by copper plates, each provided with a stopcock for emptying and filling the tube. The wire was connected electrically to the plate at one end and passed out through an insulating bushing in the other. The combined resistance of the wire and tube in series was about 0.0335 ohm.

Measurements of this resistance were made by means of a Wheatstone's bridge with proportional coils of 1 and 1000 ohms, and with resistances divided to 0.1. A change of resistance equal to 0.0001 ohm was indicated plainly by a deflection of the galvanometer, while a visible deflection could be obtained for a resistance change of one-fifth the above by reversing the current several times so as to cause the effect to be cumulative. Accordingly it would appear that a variation of 0.00002 ohm was the very smallest that the apparatus could indicate.

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[1] Leland Stanford Junior University Publications. Studies in Electricity, No. 1; Phil. Mag., January, 1893, Vol. 35, No. 212, p. 65.

The tube was filled with various dielectrics, — air, petroleum, wood alcohol, benzine, and the like, — and measurements of the resistance of the wire and tube were made at various temperatures and the results were plotted. Professor Sanford concludes that the dielectric present affects the conductivity of the copper, and that the conductivities in the various liquids are as follows compared with the conductivity in air as unity: —

Petroleum	1.0018
Carbon bisulphide and oil of turpentine	1.0009
Wood alcohol	0.9998
Wood alcohol and benzine	0.9985
Absolute alcohol	0.9981
Wood alcohol and petroleum	0.9973

Professor Sanford also investigated the effect of certain gaseous dielectrics upon the same copper resistance, with the result of finding that the conductivity of copper in alcohol vapor, chloroform vapor, gasoline vapor, and sulphuric ether vapor was lower than in air.

It seemed to me necessary to try the effect of liquid dielectrics only, since a confirmation of Sanford's results with those would establish a strong probability in favor of the others, and *vice versa.*

The resistance apparatus of Messrs. Rodman and Keeler was substantially the same as that of Professor Sanford, except that a well paraffined stopper was used at one end for the wire to pass through. This stopper could be removed for the purpose of cleaning the tube and of examining its interior. The resistance was about 0.0468 ohm at 21°.1 C., or 40 per cent higher than Professor Sanford used. But the method employed for measuring resistance was at least two and one-half times as sensitive, since a change of 0.000008 ohm was very apparent with the galvanometer used, a Thomson astatic of about 7000 ohms resistance, made by Elliott Brothers.

The following were the dimensions: —

Length of tube	86.3 cm.
Diameter of tube	2.5 cm.
Diameter of wire	0.7 mm.
Length of wire	90 cm.
Length of tube for thermometer . .	7 cm.
Diameter of tube for thermometer .	1 cm.

No advantage would be gained in using a smaller copper resistance with the method employed, since the sensibility was in no way affected, except that the error arising from ignorance of exact temperature would be larger, not relatively, but in ohms. The difference in resistance between the tube-wire combination and that of a short link of quarter-inch copper rod was measured by means of the Carey Foster method. It is quite certain that no other method exceeds this one in accuracy, if indeed any other equals it. The bridge wire was a Kirchhoff-Wheatstone, wound on a marble cylinder, a well-known form made by Hartmann and Braun. The resistance of this wire is nearly 20 ohms, but it was shunted with a German silver wire of about 0.079 ohm resistance, which was kept in a bath of petroleum so that its temperature could be accurately determined. The bridge wire is divided directly into 1000 parts; reading easily, however, to 10,000 parts. Each division of the bridge wire was equal to about 0.000008 ohm., at 22° C. Whether this was a little more or a little less at 22° C. was immaterial, since only comparative results were aimed at, and not absolute measurements. This value of the resistance of unit length of the bridge wire was obtained, however, by means of two standard ohms, one of them with a Cavendish Laboratory certificate, and having a known difference between them which had been previously determined many times over. Assuming the temperature coefficient of German silver wire to be 0.00044, an allowance was made of 0.00001 ohm in the calculated resistance of the copper wire and tube for each half degree variation from 22° C. If a coefficient of 0.0004 or 0.0005 had been used, the difference in the result would have been only 2 in the sixth decimal place. The resistance measured is equal to that of 5852 parts of the shunted bridge wire at 21°.1 or to 0.0468 ohm. A change of one degree in the shunt would make a change therefore of $0.0468 \times 0.00044 = 0.00002$ ohm, or 0.00001 ohm per half degree. Any small error in reading this temperature or in the value of the assumed temperature coefficient is practically negligible, since the results are carried to five decimal places only. The bridge wire itself is also German silver. Its temperature was always very nearly the same as that of the shunt. An error of one degree in the temperature of the bridge wire itself entailed an error of only 5 in the eighth decimal place.

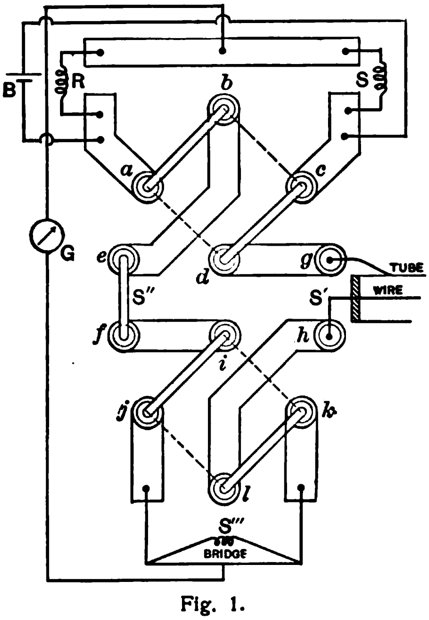


Fig. 1.

The comparison between the copper link referred to and the tube-wire resistance was easily made by means of the auxiliary apparatus attached to the bridge and represented diagrammatically in Fig. 1. S''' is the German silver shunt immersed in petroleum, S'' is the copper link, while the tube and wire are shown at S'. To the end of the tube and the wire were soldered heavy copper wires, and their amalgamated ends dipped into mercury cups, *g* and *h*. The link dipped into corresponding cups *e* and *f* on the opposite side of the auxiliary apparatus. The several mercury cups shown are of copper, brazed to heavy copper bars arranged as shown. The connections of the link and the resistance to be measured to the ends of the bridge wire are reversed by a quarter turn of the two commutators, so that the links take the position of the dotted lines. Since this commutation reverses the direction of the current through the wire, a reversing key, not shown, was placed in circuit with the battery, in order that the balance could be made for both positions of the tube with respect to the bridge with the current through it in the same direction. The auxiliary resistances R and S, of one ohm each, were wound together in the same box, and their relative resistances were assumed not to change during the time of a single comparison, which required about thirty seconds. It is not probable that any change whatever in their relative resistance took place, since they were made of wire cut from the same spool.

The temperature of the copper resistance was made to vary by means of a hot-water bath, but in no case was the tube allowed to come in contact with the water. Means were employed to make the temperature of all parts of the tube as nearly alike as possible. A thermometer graduated to half degrees was placed with its bulb in the tube next to the wire.

With these appliances a series of measurements were made with the tube filled first with kerosene, then alcohol, then air, and the results were plotted with temperatures and resistances as co-ordinates. The three lines were straight and practically coincident. The tube was then thoroughly cleaned, and after a week the experiments were repeated in the order, air, alcohol, kerosene, as the dielectrics. Again the results were coincident lines. The measurements were repeated several times, using the same dielectrics in various orders, also a mixture of alcohol and kerosene, without once detecting any difference too large to be accounted for by errors of observation.

The Tables I. to VI. give the data for the measurements with the tube containing air, alcohol, kerosene, respectively. a_1 and a_2 are the bridge readings in the two positions of the commutator, b the temperature of the bridge wire and shunt, R the computed resistance of the wire-tube combination, and t its temperature. Two tables belong to each dielectric; one made with the current flowing in one direction through the tube wire, and the other with the current reversed. The circles in the corresponding diagrams show the points obtained with one series of determinations, and the crosses the other. There is no difference between them, showing that there was no thermo-electromotive force present to introduce

Table I. — DIELECTRIC AIR.

a_1	a_2	$a_1 - a_2$	b	R	t
7973	1988	5985	20°	0.04784	27°.6
67	90	77	"	78	.3
58	99	59	"	63	26°.4
54	2000	54	"	59	.25
53	02	51	"	57	.0
51	04	47	"	54	25°.9
43	10	33	"	42	24°.8
42	12	30	"	40	.2
40	15	25	20°.5	36	.1
35	20	15	"	29	23°.9
34	20	14	"	28	.65
32	22	10	"	25	.2
29	26	03	"	19	.0
27	28	5899	21°	16	22°.7
24	30	94	"	13	.3
19	35	84	"	05	.0
16	40	76	"	0.04699	21°.8
13	41	72	"	96	.6
10	44	66	"	91	.3
03	51	52	"	80	.1

Table II. — DIELECTRIC AIR.

a_1	a_2	$a_1 - a_2$	b	R	t
7975	1985	5990	20°	0.04788	28°.0
70	90	80	"	80	27°.25
80	99	59	"	63	26°.4
54	2001	53	"	58	.2
53	02	51	"	57	.0
51	05	46	"	53	25°.9
43	11	32	"	42	.1
42	13	29	"	39	.0
40	15	25	"	36	24°.8
35	20	15	20°.5	29	.2
33	21	12	"	27	.0
32	22	10	"	25	23°.9
29	28	01	"	18	.45
27	30	5897	"	15	.15
24	31	93	"	12	.0
19	37	82	21°	04	22°.65
16	40	76	"	0.04699	.2
13	41	72	"	96	.0
10	44	66	"	91	21°.8
03	50	53	"	80	.2

TABLE III. — DIELECTRIC ALCOHOL.

a_1	a_2	a_1-a_2	b	R	t
7988	1967	6021	21°	0.04815	29°.5
79	77	09	"	05	28°.75
76	78	5998	"	0.04796	.3
70	83	87	"	86	27°.7
62	92	70	"	74	.0
58	96	62	"	68	26°.7
56	2000	56	"	63	.35
53	01	52	"	60	.2
49	07	42	"	52	25°.85
47	11	36	"	47	.2
41	15	26	"	39	.0
39	17	22	"	36	24°.8
34	21	13	"	30	.35
31	22	09	"	25	.0
27	28	5899	"	17	23°.7
25	31	95	"	14	.3
23	32	91	"	11	.0
06	46	60	"	0.04686	21°.5

TABLE IV. — DIELECTRIC ALCOHOL.

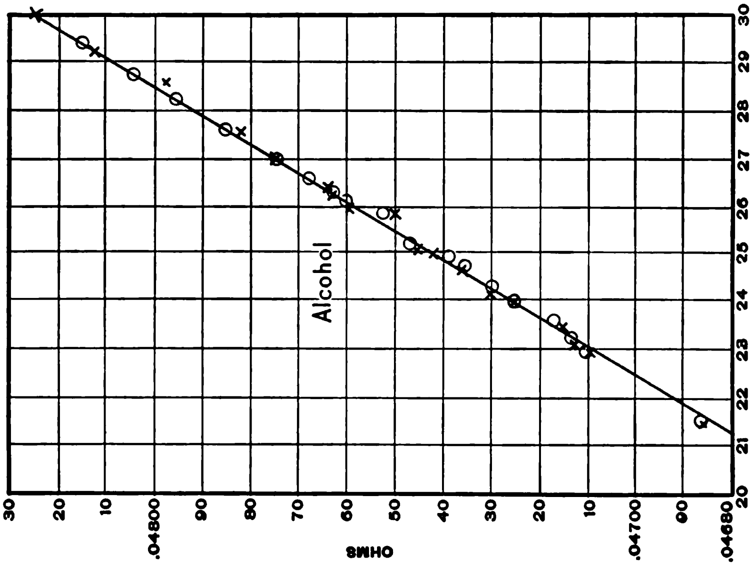
a_1	a_2	a_1-a_2	b	R	t
7996	1962	6034	21°	0.04825	30°.0
88	69	19	"	13	29°.3
78	78	6000	"	0.04798	28°.6
76	79	5997	"	96	.2
69	89	80	"	82	27°.6
62	92	70	"	74	.0
58	2000	58	"	64	26°.5
56	2000	56	"	63	.3
52	03	49	"	59	.0
48	08	40	"	50	25°.8
46	12	34	"	45	.15
41	13	28	"	41	.0
39	18	21	"	35	24°.75
35	20	15	"	30	.25
31	23	08	"	24	.0
26	30	5896	"	15	23°.55
24	31	93	"	13	.2
22	33	89	"	09	.0
06	47	59	"	0.04685	21°.5

TABLE V. — DIELECTRIC KEROSENE.

a_1	a_2	a_1-a_2	b	R	t
7996	1958	6028	21°	0.04820	30°.0
87	67	20	"	14	29°.35
83	70	13	"	08	.3
76	78	5998	"	0.04796	28°.3
72	83	89	"	89	27°.8
65	93	72	"	76	.0
57	97	60	"	66	26°.45
52	2003	49	"	57	.0
50	05	45	"	54	25°.5
46	11	35	"	46	.2
42	15	27	"	40	24°.7
34	21	13	"	30	.3
25	20	5895	"	14	23°.2
19	36	83	"	06	22°.7
15	40	75	"	0.04698	.2
02	49	53	"	82	21°.25

TABLE VI. — DIELECTRIC KEROSENE.

a_1	a_2	a_1-a_2	b	R	t
7992	1963	6029	21°	0.04821	29°.8
87	70	17	"	12	.15
80	77	03	"	0.04800	28°.6
76	81	5995	"	0.04794	.2
70	89	81	"	83	27°.7
62	94	68	"	72	26°.9
56	99	57	"	64	.0
52	2006	46	"	55	25°.7
48	10	38	"	48	.2
42	13	29	"	41	.0
39	18	21	"	35	24°.5
31	23	08	"	24	.0
22	35	5887	"	08	23°.0
17	39	79	"	0.04700	22°.4
14	43	71	"	0.04695	.0
02	53	59	"	82	21°.2


Alcohol
TEMPERATURE C.
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OHMS
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40
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20
10
.04700
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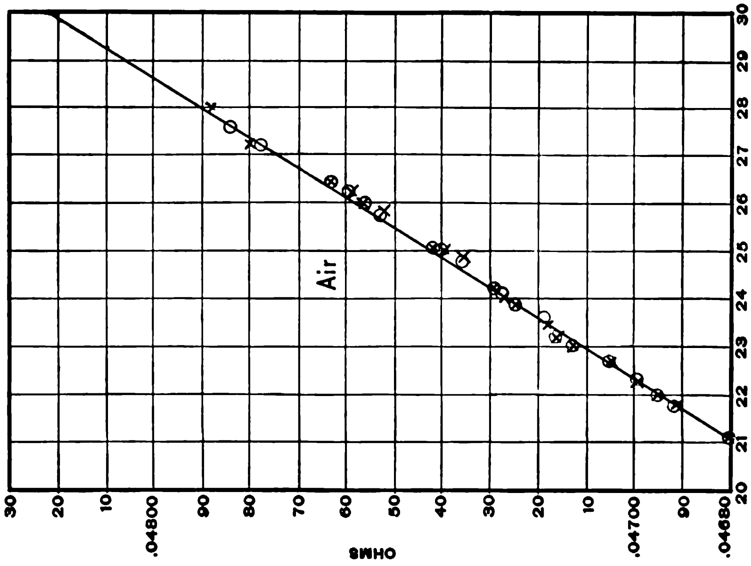
.04680

Fig. 3.


Air
TEMPERATURE C.
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OHMS
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20
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.04700
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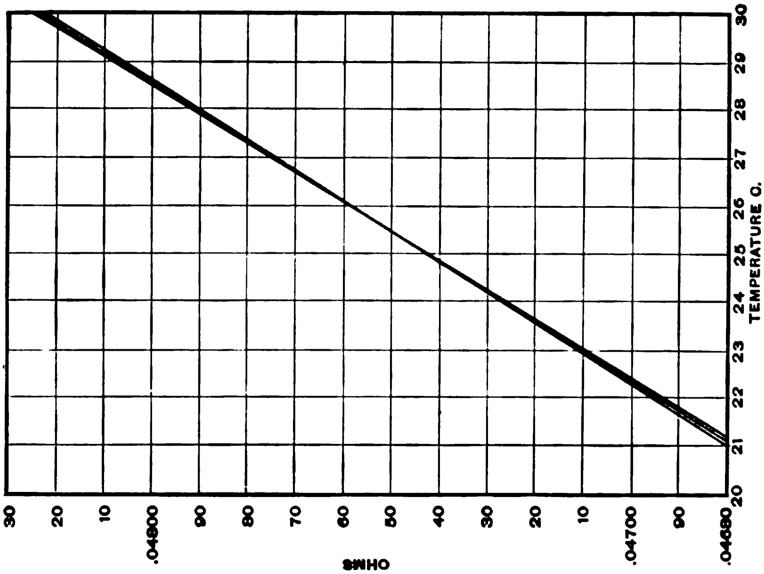
.04680

Fig. 2.


TEMPERATURE C.
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OHMS
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.04800
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20
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.04700
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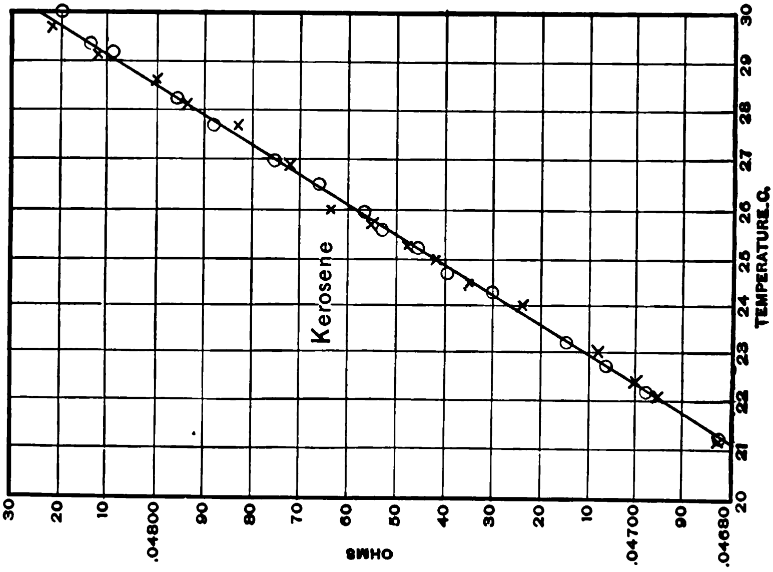
.04680

Fig. 5.


Kerosene
TEMPERATURE C.
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OHMS
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.04800
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.04700
90
.04680

Fig. 4.

an error. Figs. 2, 3, and 4 were plotted directly from the tables, and Fig. 5 shows the three curves transferred to one sheet. It is hardly too much to say that a single line would represent the entire six sets of observations. Certainly there is no difference here which may not properly be ascribed to errors of observation, and there are no such parallel lines at a sensible distance, as Professor Sanford found. Fig. 6 has been plotted from his observations with air and kerosene. The vertical distance apart of the two lines is six or seven parts in the fifth decimal place. This is equivalent to eight parts on the bridge wire in our investigation, a difference which would have made a large deflection on our galvanometer. Professor Sanford's air-line lies above that of kerosene. Ours cross at a very minute angle, and if they were forced into parallelism, the air-line would fall below that of kerosene.

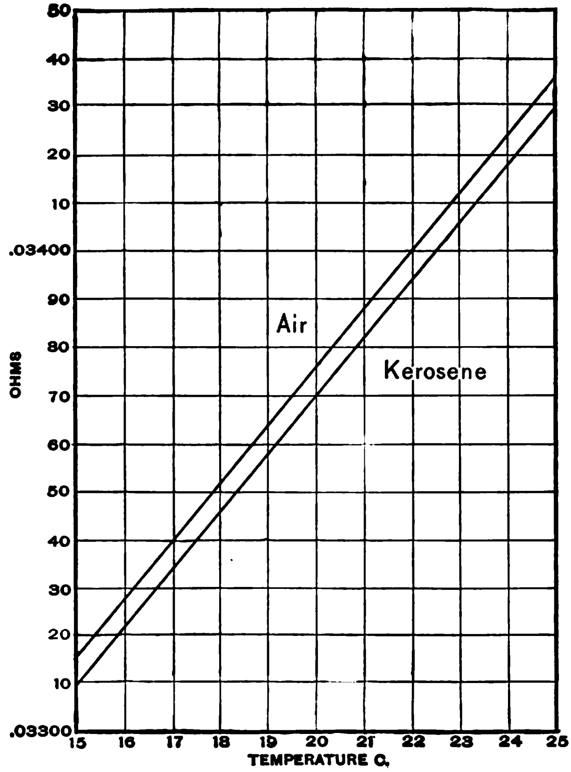


Fig. 6.

TABLE VII.

DIELECTRIC AIR.

a_1	a_2	$a_1 - a_2$	b	R	t
8188	1724	6464	21°.5	0.05169	29°.4
86	28	58	"	64	.2
83	29	54	"	61	.0
82	31	51	"	59	28°.85
78	33	45	"	55	.55
75	37	38	"	48	.4
73	40	33	"	45	.05
70	43	27	"	41	27°.9
64	48	16	22°.0	33	.55
61	50	11	"	29	.4
60	54	06	"	25	.1
58	56	02	"	22	26°.9
52	58	6394	"	15	.6
50	62	88	"	10	.4
47	63	84	"	07	.2
46	66	80	"	04	.0
42	68	74	"	0.05099	25°.8
40	71	69	"	95	.5
38	73	65	"	92	.4
35	77	58	22°.5	87	.0
30	81	49	"	80	24°.7
27	84	43	"	75	.5
25	88	39	"	72	.4
23	89	34	"	68	.1
16	93	23	"	59	23°.7
12	97	15	"	53	.4
06	1802	04	"	44	22°.8
05	06	6299	23°.0	41	.7
02	08	94	"	37	.5
8099	11	88	"	32	.3
96	13	83	"	28	.1
94	15	79	"	25	21°.9
93	18	75	"	22	.6

TABLE VIII.

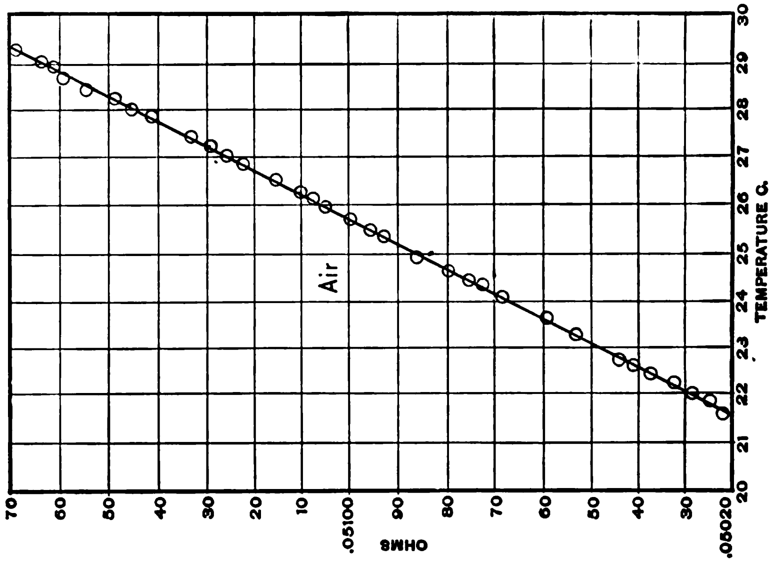
DIELECTRIC KEROSENE.

a_1	a_2	a_1-a_2	b	R	t
8185	1724	6461	21°.5	0.05168	29°.95
85	28	57	"	65	.4
84	30	54	"	62	.0
74	35	39	"	50	28°.55
74	37	37	"	49	.4
73	39	34	"	46	.2
65	41	24	"	38	27°.85
60	48	12	"	29	.55
60	52	08	"	25	.3
59	54	05	"	22	.0
52	55	6397	"	17	26°.6
51	62	89	"	10	.4
48	67	81	"	04	.1
46	69	77	"	01	25°.9
39	73	66	"	0.05092	.4
33	76	57	22°	86	.05
33	84	49	"	79	24°.7
32	87	45	"	76	.4
20	89	31	"	65	.0
15	99	16	"	53	23°.5
11	1800	11	"	49	.4
11	02	09	"	47	.0
08	02	06	"	45	22°.9
02	04	6298	"	38	.8
01	11	89	21°.5	32	.4
8098	15	83	"	27	.15
97	19	78	"	23	21°.9

TABLE IX.

DIELECTRIC ALCOHOL.

a_1	a_2	a_1-a_2	b	R	t
8184	1724	6460	22°.5	0.05170	29°.4
82	28	54	"	64	.2
79	31	48	"	59	28°.9
77	37	40	"	53	.55
72	38	34	"	48	.35
70	42	28	"	43	.15
68	43	25	"	41	27°.9
66	46	20	"	37	.7
63	49	14	"	32	.4
60	54	06	"	26	.2
58	58	00	"	21	26°.95
53	62	6391	22°	13	.5
46	65	81	"	05	.2
45	66	79	"	03	.1
42	68	74	"	0.05099	25°.9
40	71	69	"	95	.75
38	73	65	"	92	.5
36	77	59	"	87	.35
33	79	54	"	83	.0
29	79	50	"	80	24°.7
24	79	45	"	76	.5
22	90	32	"	66	.2
17	88	29	"	63	23°.9
16	89	27	"	62	.7
15	1800	15	"	52	.4
11	03	08	"	46	.2
05	07	6298	"	38	22°.8
03	09	94	"	35	.5
8099	12	87	"	30	.1
97	15	82	"	25	21°.9
95	19	76	"	21	.7


Air
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TEMPERATURE C.
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.05100
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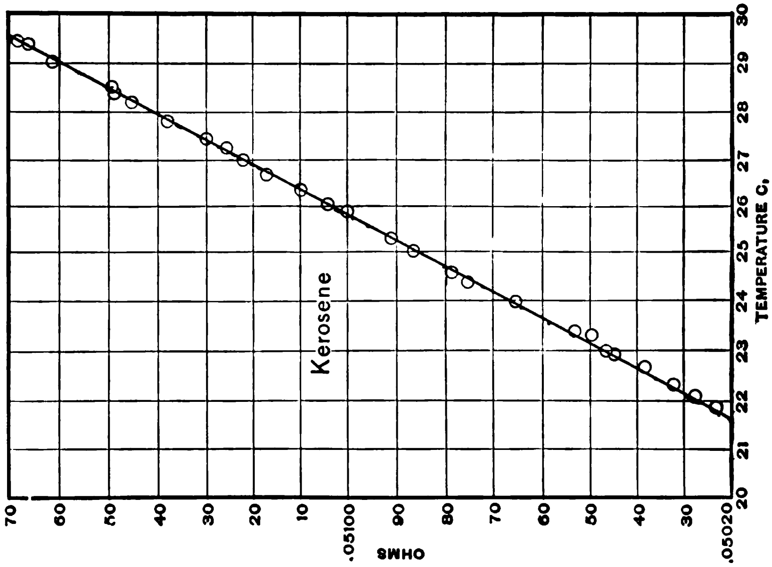
.05020
OHMS

Fig. 7.


Kerosene
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TEMPERATURE C.
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.05100
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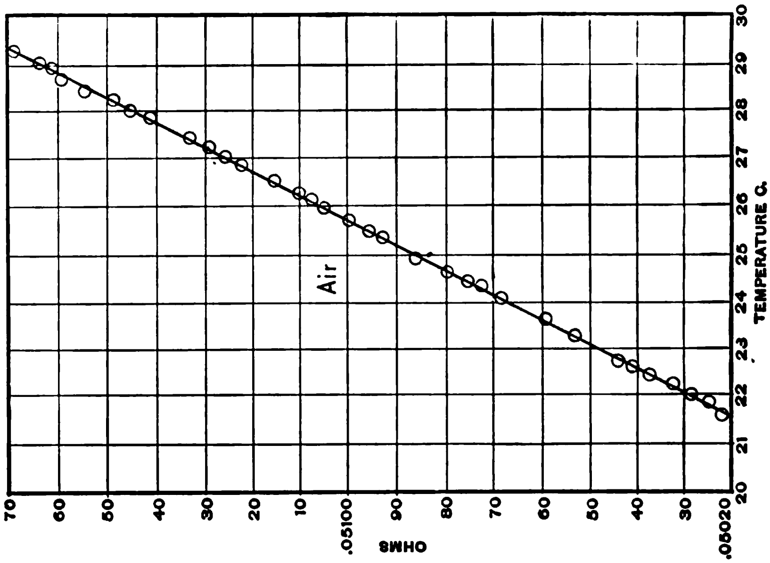
.05020
OHMS

Fig. 8.

Table IX.

DIELECTRIC ALCOHOL.

a_1	a_2	a_1-a_2	b	R	t
8184	1724	6460	22°.5	0.05170	29°.4
82	28	54	"	64	.2
79	31	48	"	59	28°.9
77	37	40	"	53	.55
72	38	34	"	48	.35
70	42	28	"	43	.15
68	43	25	"	41	27°.9
66	46	20	"	37	.7
63	49	14	"	32	.4
60	54	06	"	26	.2
58	58	00	"	21	26°.95
53	62	6391	22°	13	.5
46	65	81	"	05	.2
45	66	79	"	03	.1
42	68	74	"	0.05099	25°.9
40	71	69	"	95	.75
38	73	65	"	92	.5
36	77	59	"	87	.35
33	79	54	"	83	.0
29	79	50	"	80	24°.7
24	79	45	"	76	.5
22	90	32	"	66	.2
17	88	29	"	63	23°.9
16	89	27	"	62	.7
15	1800	15	"	52	.4
11	03	08	"	46	.2
05	07	6298	"	38	22°.8
03	09	94	"	35	.5
8099	12	87	"	30	.1
97	15	82	"	25	21°.9
95	19	76	"	21	.7


Air
TEMPERATURE C.
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OHMS
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.05100
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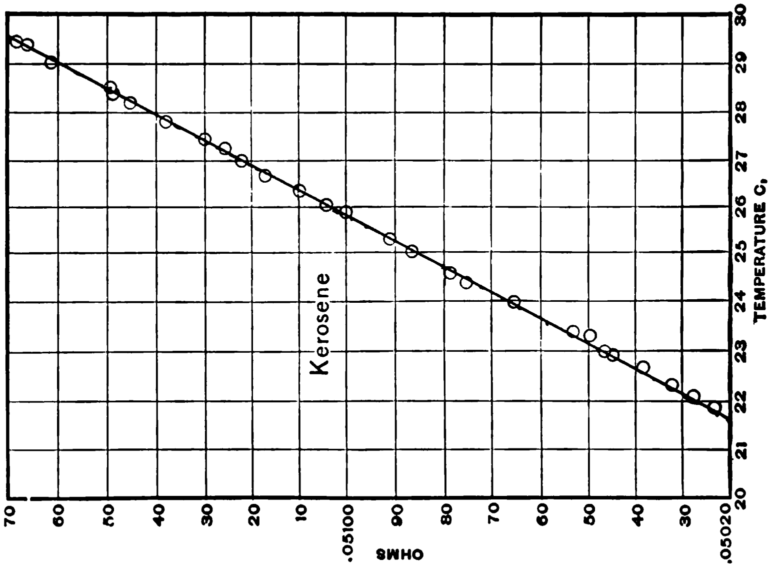
.05020

Fig. 7.


Kerosene
TEMPERATURE C.
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OHMS
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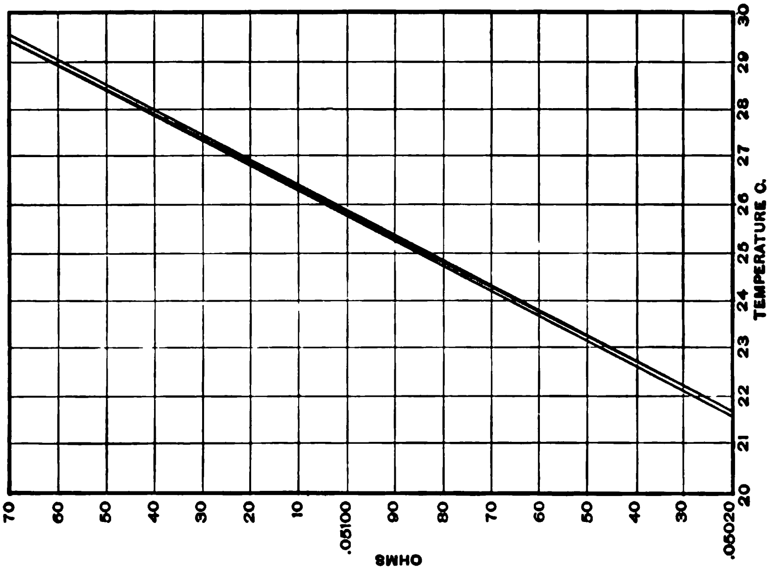
.05020

Fig. 8.


.05020
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OHMS
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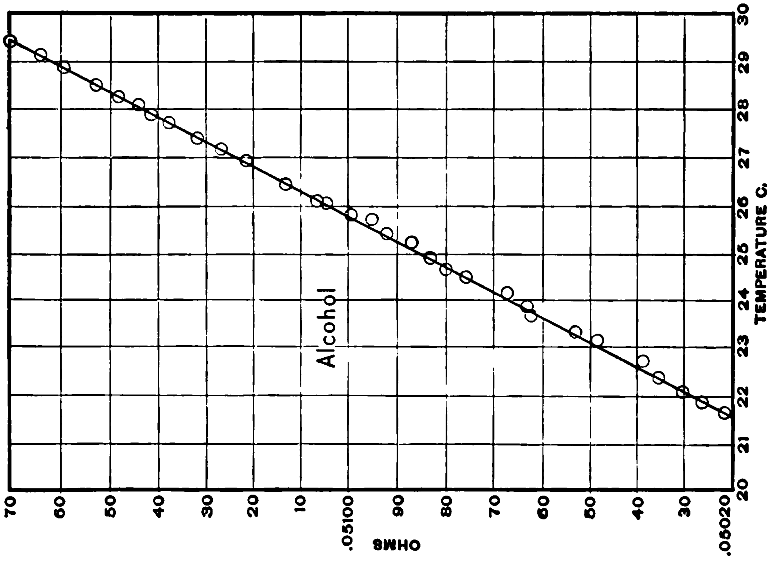
TEMPERATURE C.

Fig. 10.


Alcohol
.05020
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60
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80
90
.05100
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20
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40
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60
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OHMS
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TEMPERATURE C.

Fig. 9.

Such series of observations as the above were taken a number of times, using the dielectrics air, alcohol, kerosene, in various orders; also a mixture of alcohol and kerosene as already stated; but in no case was a difference in resistance detected which was due to the presence of the dielectric.

The procedure was also varied by measuring the resistance of a helix of copper wire immersed in air, alcohol, and kerosene, in succession. The observations are contained in Tables VII., VIII., IX., and are plotted in the curves of Figs. 7, 8, and 9. The three are drawn for comparison in Fig. 10. Any such difference as Professor Sanford found would have been detected in our measurements with this resistance as readily as if it had been as small as his. Doubling the current made no difference in the result. The heating effect of keeping the key closed for twenty or thirty seconds was tried, and in no case did the resistance increase more than 0.00001 ohm.

Professor Sanford's lines are distinctly apart; ours are practically coincident. On the supposition that his are correct, it would be a strange error indeed that would in all cases bring ours together, especially when our method has the great advantage of increased sensibility. We can only conclude that his measurements are affected by some systematic error which he has not discovered.

Mr. Rodman and Mr. Keeler have devoted a good deal of time to this investigation, and have made all the observations. I am indebted to them not only for the tables, but also for the drawings from which the curves have been made.

THE USE OF THE ROTATING SECTORED DISC IN PHOTOMETRY.

ERVIN S. FERRY.

FROM the fact that in photometric measurements the minimum error of observation is introduced when the photometer reading is at the middle of the bar, and the error rapidly increases as the reading is displaced toward one end, the comparison of two lights of different luminosities is attended with considerable uncertainty. The most obvious plan to bring the photometer reading into the region of least error is to introduce into the path of the more intense ray an absorbing medium whose coefficient is constant for all wave lengths. A pair of Nicol's prisms is sometimes employed for the same purpose, and the law of Malus used to compute the percentage of light transmitted; but the ordinary method is to use a rapidly rotating disc from which radial sectors have been removed. The assumption here made is that the ratio of the amount of light transmitted to the total incident illumination, is equal to the ratio of the aperture of the disc to the entire disc. Several experimenters have observed, however, that the candle power of an arc light as obtained by the use of the rotating sectored disc is sometimes considerably less than the value obtained without it. It was in the study of this anomaly that the following experiments were undertaken.

The apparatus consisted of an ordinary 1000 division photometer bar, having near one end the sectored disc rotated by a small electric motor. The disc was on a hinged shelf arranged so that it could be quickly moved out of the path of the ray of light. Various photometers were used according to their especial fitness for the particular conditions of the experiment. The Bunsen was found best for the comparison of lights of considerable difference in color; the Lummer-Brodhun was used when the two lights were of the same quality, while a modification of

Ritchie's photometer gave the best results where the two lights differed slightly in quality. The Nichols horizontal slit spectrophotometer was used in the comparison of monochromatic lights.

As the Nichols-Ritchie photometer has not yet been described, it may not be out of place to here give a brief description of it. The principal object in its construction was to have the two luminous spots one above the other so as to eliminate any error due to a difference between two eyes. It consists of two vertical mirrors (Fig. 1), one above the other, at right angles to each other, and each at 45° to the direction of the photometer bar. These mirrors are placed in a box having open ends, and a circular aperture in the front side covered with a thin sheet of celluloid or other translucent material. From this arrangement, it is evident that the upper half of the translucent screen will be illuminated by the light at one end of the bar, while the lower half will receive its illumination from the other end. The reading consists in moving the photometer along the bar till the line separating the two halves of the disc becomes invisible. By exercising proper care in fitting the mirrors together and in selecting the translucent screen, this photometer can be made as sensitive to differences of color and luminosity as the Lummer-Brodhun, and as convenient to use as the Bunsen.

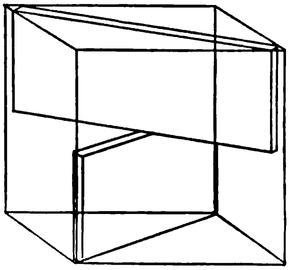

Fig. 1.

The first experiment made was to determine if the error of the disc depends upon the size or arrangement of the separate apertures. For this purpose, discs were made containing 12, 8, 4, 2, and 1 equal and equally spaced segments; also others having the segments in each disc of different sizes, and arranged unsymmetrically. All of these discs were double, so that quite a range of total apertures could be obtained with each pair. When they were revolved with sufficient rapidity to produce a steady illumination of the photometer screen, it was found that the error of the disc was constant for all arrangements of the segments, so long as the total aperture was the same.

From the preceding experiment, the error appeared to be a function of the total aperture only.

The relation between the error and the total aperture was determined as follows: A lime light was placed at the disc end of the photometer bar, and a 16 C.P. incandescent lamp was used as a standard at the other end. The double disc, containing twelve fifteen-degree sectors, was used in this and the succeeding experiments. This disc had a graduated scale on its edge, so that the aperture of each segment could be varied from zero to fifteen degrees. The method of observation was as follows: a photometer-reading was taken with the rapidly rotating disc in the path of the stronger ray; the shelf containing the disc was then quickly raised out of the path of the ray, and another reading taken. The aperture of the disc was then changed, and another pair of readings taken. Denoting by I, the light ratio when the disc is in the path of the stronger ray; I_0, the light ratio without the disc; a, the angular aperture of the disc; and $a+b$, the whole circumference of the disc, we have the equation.

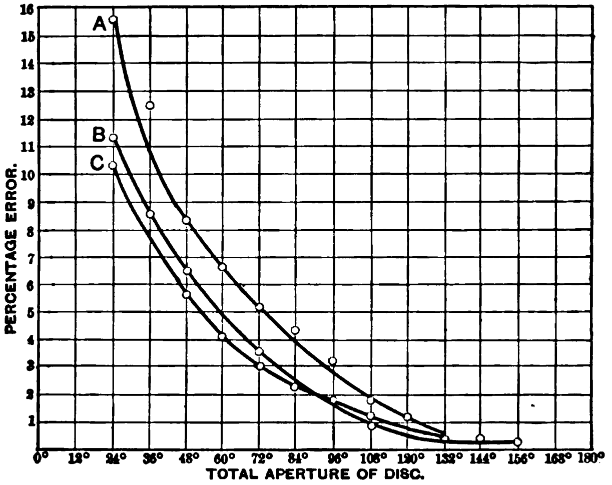

Fig. 2.

$$I = \frac{a}{a+b} I_0.$$

The relation between I obtained in this way and by direct observation will then give the error introduced by the disc. The observations and deductions thus obtained are given in the following table, while the results are graphically shown in Curve C, Fig. 2.

Table I.

COMPARISON OF LIME LIGHT WITH 16 C.P. INCANDESCENT LAMP.

Observations.			Deductions.			
Aperture.	With disc.	Without disc.	I_0.	I calculated.	I observed.	Per cent error.
24°	546	227	11.596	0.77	0.69	10.4
36°	493	227	11.596	1.16	1.06	8.6
48°	453	227	11.596	1.55	1.46	5.8
60°	424	227	11.596	1.93	1.85	4.1
72°	403	229	11.335	2.27	2.20	3.0
84°	385	230	11.208	2.61	2.55	2.3
96°	371	232	10.958	2.93	2.87	1.7
108°	358	223	10.836	3.25	3.21	1.2
120°	347	234	10.716	3.57	3.54	0.8
132°	339.5	237	10.365	3.80	3.79	0.2
144°	345	250	9.000	3.60	3.60	0
156°	342	255	8.536	3.69	3.70	0
168°	335	256	8.446	3.94	3.94	0
180°	328.5	257	8.358	4.18	4.18	0

Several attempts were made to obtain a similar curve for the case of the arc light; but these attempts were all unsuccessful with ordinary arc lamps, on account of their unsteadiness. But finally, through the courtesy of Mr. L. B. Marks, an "incandescent-arc" lamp[1] was obtained, which met the required condition of steadiness excellently. Table II. and Curve A, Fig. 2, give the disc's error in the case of the arc light.

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[1] For description, see Electrical World, Sept. 9, 1893.

TABLE II.

COMPARISON OF ARC LAMP (65 VOLTS AND 8.2 AMPERES) WITH 16 C.P. INCANDESCENT LAMP AT 110 VOLTS.

Observations.			Deductions.			
Aperture.	With disc.	Without disc.	I_0.	I calculated.	I observed.	Per cent error.
24°	521	205	15.039	1.002	0.846	15.6
36°	458	200	16.000	1.600	1.400	12.5
48°	410	195	17.042	1.136	2.071	8.2
60°	373	190	18.175	3.029	2.826	6.7
72°	350	190	18.175	3.635	3.449	5.1
84°	336	193	17.484	4.079	3.905	4.2
96°	320	193	17.484	4.662	4.516	3.1
108°	318	202	15.606	4.682	4.600	1.7
120°	297	195	17.042	5.681	5.603	1.2
132°	290	198	16.407	6.016	5.994	0.3
144°	279	197	16.615	6.646	6.678	0
156°	261	188	18.655	8.083	8.017	0
168°	256	190	18.175	8.481	8.446	0
180°	249	190	18.175	9.088	9.097	0

When a 16 C.P. incandescent lamp was compared with a standard Methven slit, Curve B, Fig. 2, was obtained.

Two 16 C.P. incandescent lamps were then compared. Two were selected that were as nearly alike in every respect as possible, and connected in multiple to a storage battery circuit. Their spectra were examined photometrically, color for color, and found to be nearly alike in luminosity throughout. They were then mounted on the photometer bar, and light ratios obtained with and without the sectored disc. The electromotive force of the lamps was varied from 70 to 120 volts, and the aperture of the disc varied throughout its range, without any appreciable difference being observed between the calculated and observed value of the light that passed through the sectored disc. This experiment shows that the error introduced by the disc is not a function of the absolute illumination of the photometer screen.

Two incandescent lamps giving nominally 150 C.P. and 16 C.P., respectively, were then mounted on the photometer bar. They were connected to separate circuits, and the included resistances varied till they were of the same color. Again, little or no error was observed, even when the disc was nearly closed, but if the lamp at the disc end of the bar was made to burn at a voltage much greater than normal, the error again appeared. This showed that the error of the disc is not dependent upon the luminosity ratio of the two lights and suggested the possibility that the color of the light might be the cause of the discrepancy. It is to be noticed that in the above four cases where a marked error occurred, the light at the disc end of the bar was of a bluer quality than the standard.

If, after the two lights have been brought to the same color, a sheet of glass slightly tinted with blue be interposed in the path of the ray from the disc end of the bar, error values are obtained similar to those shown in Fig. 2. This seems to prove that the error of the disc is dependent upon the color of the light that it interrupts.

The phenomena described in these experiments can probably be explained from considerations of retinal inertia. The luminosity of ordinary light is the integral of the luminosity of all the wave lengths that affect the retina. The distribution of luminosity in the spectra of ordinary illuminants proceeds from a maximum in the orange or yellow, to zero at the ends of the spectrum. Both the intensity of the retinal sensation[1] and the time light must act upon the retina in order that it may be seen[2] are direct functions of the luminosity. Hence, if the light acts upon the retina for but a very short time, the elements of low luminosity — *i.e.* at the ends of the spectrum — will not produce their maximum impression. Therefore the value of the candle power deduced from the observation with the disc will be less than the total luminosity of the light. If the curve of luminosity with respect to wave length of any light be known, and also the curve of time necessary to receive a retinal impression with respect to the same range of luminosity, the error of the disc can be prophesied for any aperture.

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[1] Fechner, Revision der Hauptpunkte der Psychophysik, p. 184.

[2] J. M. Cattell, The Inertia of the Eye and Brain, "Brain," Part XXXI.

After the disc has attained a speed sufficient to cause an unflickering illumination of the screen, any increase of speed produces no difference in the photometer reading. But it was noticed that if the disc did not revolve rapidly enough to produce a perfectly steady illumination of the photometer screen, *more* light appeared to go through the sectored disc than theoretically should. This can possibly be explained, as has been an analogous phenomenon by Ernst Brücke[1] from the consideration of a building up of the separate impressions upon the retina.

Conclusions.

I. While it is physically true that the proportion of light transmitted by a rapidly rotating sectored disc to the total incident illumination is equal to the ratio of the total aperture of the disc to the entire disc, yet the effect of this light upon the retina will not always be proportional to the ratio of the total aperture of the disc to the entire disc.

II. With mixed light containing elements of different luminosity shining upon the retina, a rotating sectored disc will appear to not cut off all the elements in equal proportion, but will intercept most strongly the elements of low luminosity.

III. With any given light, the error introduced by the use of the rotating sectored disc increases as the aperture of the disc diminishes.

IV. With ordinary illuminants, the error is negligible when the total aperture of the disc is more than one-half the entire disc, but rapidly increases as this aperture is diminished.

NOTE UPON THE ACTION OF INTERMITTENT LIGHT UPON THE RETINA.

From the study of rotating cardboard discs painted black and white in sectors, von Helmholtz deduced the law "if a point of the retina is excited by a light which undergoes regular and periodic variations, and which has the duration of its period sufficiently short, it produces a continuous impression, equal to that produced if the light emitted during each period were distributed uniformly throughout the duration of the period."[1] Plateau, Fick, and Dove have repeated the experiments of von Helmholtz and confirmed this law; and yet it was thought useful to test the law by using monochromatic light throughout a wide range of luminosity and wave length.

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[1] "Ueber den Nutzeffect intermitterenden Netzhautreizungen," Sitzungsberichte, Wiener Akademie, 1864, Vol. 49, II. p. 128.

The method employed was to compare photometrically, color for color, two spectra seen side by side in the eye-piece of a spectrometer, one spectrum being produced by a ray coming directly from the source of light, and the other by a ray from the same source, but made rapidly intermittent by a revolving sectored disc. Sunlight, daylight, and the incandescent electric were used to produce the spectra. A very accurate double slit with micrometer adjustments at the object end of the collimator gave the means of varying the luminosity of the two spectra. The method of transmitting the two rays into the collimator is shown in the figure. Observations were taken throughout the entire spectrum and of luminosities varying 1000 per cent. A difference of luminosity of 2 per cent could have been detected by this method, but no deviation from the law of von Helmholtz was observed.

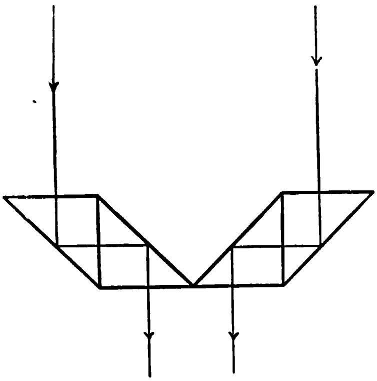

Fig. 3.

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[1] Physiologische Optik, 2d ed., p. 483.

FATIGUE IN THE ELASTICITY OF STRETCHING.

By Joseph O. Thompson.

ALTHOUGH twenty-eight years have passed since the publication of Lord Kelvin's article, in which he describes the phenomenon of fatigue in the elasticity of torsion, no one to my knowledge has yet demonstrated the existence of a similar fatigue in the elasticity of stretching. In some of Lord Kelvin's experiments, it will be remembered, this phenomenon was very marked. In one experiment[1] he used two exactly similar wires stretched by vibrators of equal weight and equal moment of inertia. No. 1 had been kept at rest for nine days, while No. 2 had been kept oscillating more or less every day during that time. In an experiment then made, No. 1 subsided from an amplitude of 20 scale-divisions to 10, after 100 vibrations in 242 seconds, making a period of 2.42 seconds. No. 2 subsided through an equal angle, after 44 or 45 vibrations, and the period was 2.495 seconds. The increase in the period is thus a shade over 3 per cent, and this means a diminution of over 6 per cent in the modulus of rigidity. While the phenomenon of fatigue in the elasticity of stretching is much less marked, it gives us, nevertheless, a new fact concerning the properties of matter, and deserves careful attention.

The observations noted below were made in the spring of 1891 in the Physical Institute of the University of Strassburg. The object then immediately in view was to demonstrate the invalidity of Hooke's law, and to point out a method of determining the Young's modulus of an unstrained wire. Consequently the phenomena described below were unlooked for, and at that time received merely a passing glance. Then, on account of lack of time, no attempt was made to subject the phenomena to an extended investigation; and since that time there have not been at the disposal of the writer place and apparatus suitable for experiments demanding such a high degree of accuracy.

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[1] Mathematical and Physical Papers, Vol. III., p. 26.

The apparatus used was able to give exceptionally accurate results. Since all its details, together with the method of experiment, have been fully described in an earlier article,[1] extended description of the same is omitted here. Suffice it to say that the wires used were about 23 m. in length and 0.25 mm. in diameter; that the temperature was remarkably constant, and could be measured to $\frac{1}{80}°$; and that the constant use of two cathetometers enabled me to keep the true elastic lengthening separate from the elastic after-effect (*elastische Nachwirkung*) and all other disturbing influences.

A careful examination of my results showed that the elastic lengthenings in the first of any series of measurements were uniformly less than those caused by the same stretching weights later in the day, the temperature remaining either exactly or approximately the same. This is shown clearly in the table below, where the first column shows the weights added to the pan; the second, the lengthenings produced in an unfatigued steel wire, as given by the averages of three sets of observations; the third, the lengthenings produced in the fatigued wire, as given by the averages of six sets of observations: —

Kilograms.	Millimeters.	Millimeters.	Difference.
0.2	6.993	7.009	0.016
0.4	14.065	14.085	0.020
0.6	21.188	21.210	0.022
0.8	28.353	28.366	0.013
1.0	35.548	35.569	0.021
1.2	42.792	42.807	0.015

It cannot be said that these differences in the last column are due to a diminution of the Young's modulus on account of a rise in temperature; for, after each one of the three sets of measurements, the averages of which are given in the second column, I immediately took two other sets, and thus I get the six sets, the averages of which appear in the third column. In every such series of three sets of measurements, the temperature was entirely free from fluctuation. The rise during the first series was $\frac{1}{4}$°. It was the same in the second, and was $\frac{1}{20}$° in the third; and in this third series the fatigue was as marked as in either of the others.

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[1] Wiedemann's Annalen, Vol. 44, No. 11, 1891, and Am. Jour. Sci., Vol. XLIII., January, 1892.

The probable error in the measurements was small. For example, when the weight added to the pan was 0.4 kg. the elastic lengthenings produced were as follows: —

	May 12.		**May 13.**		**May 15.**		**Mean.**
Unfatigued wire .	14.065		14.06		14.07		14.065
Fatigued wire .	14.08	14.08	14.08	14.09	14.085	14.095	14.085

The increase in the measurements, made May 15, is due chiefly to the fact that on that day the temperature of the tower in which the observations were made was nearly $\frac{1}{2}$° higher than on the other two days.[1]

Brass wire gave phenomena exactly similar to those described above. In the following table are given the results of the first four sets of measurements, made April 25, between 8.58 and 10.57 A.M. The rise in temperature in this time was perfectly uniform, and amounted to $\frac{1}{4}$°; and, according to Dr. Miller, this would not lower the modulus by more than $\frac{1}{8000}$.

Kilograms.	**Millimeters.**	**Millimeters.**	**Millimeters.**	**Millimeters.**
0.2	7.10	.13	.12	.11
0.4	14.25	.265	.26	.28
0.6	21.47	.50	.50	.50
0.8	28.76	.775	.78	.77
1.0	36.11	.125	.14	.145
1.2	43.53	.57	.56	.55
1.4	51.05	.07	.065	.07
1.6	58.66	.71	.685	.71
1.8	66.33	.355	.335	.36

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[1] According to the measurements of Dr. Andreas Miller, of Munich, an increase of $\frac{1}{2}$° in temperature lowers the Young's modulus of steel by $\frac{1}{3000}$.

The lengthenings obtained in the first set are evidently less than the others. Whether the fatigue is progressive, that is, whether it is a function of the time the wire is kept in use, my measurements do not indicate.

A similar fatigue was distinctly noticed in a silver wire.

In copper wire the after-effect was troublesome, and obscured the phenomenon of fatigue, and indeed made it uncertain whether in this particular metal the phenomenon existed. I consider it probable, however, that it does exist, just as in the other metals mentioned.

In answer to the query, Why is the fatigue in the elasticity of stretching so comparatively slight, the following may be said: In Lord Kelvin's torsion experiments the elastic forces had been kept in almost constant action for several days. In my experiments, on the other hand, measurements were made at intervals of two minutes, and the stretching weight was allowed to remain on the pan only twelve or thirteen seconds, so that even when observations were being made the elastic forces were in action only 10 per cent of the time, and this was about 1 per cent of the entire day. Under these circumstances, it is hardly surprising that the fatigue is slight. It will be borne in mind that if I had kept the elastic forces continually in action, I should have had a superposition of after-effects, and this after-effect was the one thing which I was at that time particularly anxious to avoid.

Reference should here be made to an article by Dr. Miller,[1] entitled "Der primäre und sekundäre longitudinale Elastizitätsmodul und die thermische Konstante des Letzteren." Dr. Miller states that he found in his measurements that the lengthening of his wire was greater in his first measurement than in the succeeding ones, a result exactly the reverse of mine. Still he states that the Young's modulus of the wire in its original condition, or its primary modulus (E_p), as he terms it, is greater than its modulus after it has been subjected to repeated stresses, — its secondary modulus (E_s). This statement accords with my result, and is doubtless true, although his measurements give him no ground for making it. He correctly says that $E_p = \frac{\pi_1}{l_0}$ and $E_s = \frac{\pi_2}{\lambda}$, and then goes to work to prove that $E_p > E_s$. In his demonstration, however, he in one place makes the incorrect assumption that $E_s = \frac{\pi_1}{l_0 + l}$, where l is what he terms the *Nachwirkungsrückstand*, a lengthening due chiefly to the after-effect. So we would here remark, in the first place, that in computing the modulus, no one is justified in combining this l with the true elastic lengthening; and, in the second place, that if we assume $E_p = \frac{\pi_1}{l_0}$ and $E_s = \frac{\pi_1}{l_0 + l}$, we see at a glance that of necessity $E_p > E_s$ (l being in every case a positive quantity), and the author's laborious proof is superfluous.

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[1] Aus den Abhandlungen der k. bayer. Akademie der Wiss., II., Cl. XV., Bd. III., Abth.

The elastic lengthenings in my measurements are affected to an inappreciable extent by the after-effect, and not at all by the thermal effects within the wire, as has been demonstrated in my article, already cited.

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A STUDY OF THE POLARIZATION UPON A THIN METAL PARTITION IN A VOLTAMETER. II.

By John Daniel.

IN this paper two questions will be discussed: first, the passage of ions through a gold-leaf partition in a voltameter; second, the minimum current strength at which the ions are deposited visibly upon the partition for various electrolytes. This will be called the "critical current." This paper is a continuation of the work done last spring in Berlin in the quantitative measurement of the polarization upon metal partitions ranging in thickness from 0.0001 mm. to 0.02 mm. for various current strengths in a 30 per cent H_2SO_4 voltameter. In those experiments there was no development of gas nor polarization on a gold-leaf partition (0.0001 mm. thick) for the highest current used, which was four-tenths to five-tenths ampere.

The present apparatus consists essentially of a glass voltameter vessel with platinum electrodes separated by the metal partition under investigation, so that there is no path for the current except through this partition, an accurate instrument for measuring current, and a strong, steady battery. The voltameter consists of an outer glass jar 8 cm. high, 8 cm. wide, and 8 cm. long; and an inner glass jar 8 cm. high, 5 cm. wide, and 5 cm. long, placed inside the first jar. A platinum cathode, suspended by a platinum wire, is placed inside the inner jar; and a similar electrode serves as anode in the larger jar, though a copper anode was sometimes used when the electrolyte was $CuSO_4$. A hole 2 cm. in diameter was bored in one side of the smaller jar. Glass plates 4 cm. wide, 6 cm. long, and 1 mm. thick were bored with a hole 1.5 cm. in diameter. Powdered sealing-wax was placed around the edge of this hole and melted; the gold-leaf was secured to the melted sealing-wax. This plate was then securely sealed over the hole of the inner jar, so that there was no possible leakage. The voltameter was then carefully filled, keeping the liquid inside and outside on the same level so as not to break the gold-leaf partition, and was then ready for use.

To measure the current, a Thomson composite electric balance and a Weston double-scale ammeter were used in series with the voltameter. The two instruments were found to agree so well that the Weston ammeter was used alone for most of this work, as it was much more convenient to read.

The battery consisted of 25 accumulators of 350 ampere-hours capacity, and an electromotive force of 50 volts. The current strength was adjusted to any desired value by resistance in series, which could be varied at pleasure between zero and 12,000 ohms. For sealing-wax, pure rosin and beeswax (without coloring-matter) was used, mixed in such proportion as to give a low melting-point. The gold-leaf used was bought in Nashville, and is known to the trade as "XX." It is about 0.0001 mm. thick. Careful selection was made of such parts of the gold-leaf as were found, upon holding up to the light, to be free from small holes. It has already been observed, in Part I. of this paper, that when $CuSO_4$ was used as the electrolyte, and the current was over 0.3 ampere, copper was deposited on the rim of the gold-leaf, which was necessarily larger than the hole in the glass plate. In this work it was found necessary to remove this gold quite close up to the edge of the hole by scraping it off, as was first tried, or by covering it carefully with sealing-wax, which was found both easier and better. This was accomplished by melting the sealing-wax over the gold with a hot brass hammer of peculiar shape made for the purpose. The wax could thus be made to flow quite close to the edge of the hole. This left only that part of the gold exposed which was immediately over the hole. To neglect this was in all cases to reduce the critical current, the deposit of the cathion appearing first on that part of the gold-leaf which was nearest the anode and farthest from the opening.

Passage of the Ions through the Gold-leaf Partition.

To test this, a 17 per cent $CuSO_4$ solution (*i.e.* 1 g. $CuSO_4$ to 5 c.c. H_2O) was used in the outside vessel (anode side of the voltameter), and H_2SO_4 (30 per cent solution, Sp. Gr. 1.23) in the inside vessel (cathode side of the voltameter). These solutions being separated by the gold-leaf partition, the appearance of the ions upon the electrodes and upon the partition, when the current was closed, was noted.

The first method of observation was to close the circuit upon the voltameter, read the current strength by the ammeter, and, at stated intervals, weigh the copper deposited on the cathode. This gave Table V., where it may be seen that the amount of copper deposited was very small at first, not more than 2 per cent or 3 per cent the first hour, but increased rapidly with the time. This did not settle the question as to whether the current caused the copper to pass through the gold-leaf partition or not.

The second method was to set up two exactly similar voltameters at the same time, close the circuit on one, leaving the other open, and, at stated intervals, to weigh the copper deposited on the cathode of the voltameter through which the current had passed, at the same time making a quantitative analysis of the solution on the cathode side of both the open and the closed voltameters. This was done by extracting 10 c.c. from each with a pipette, and depositing the copper electrolytically in two similar platinum crucibles, connected in series. Equilibrium was maintained in the voltameter by adding 10 c.c. of the 30 per cent H_2SO_4 solution to replace the 10 c.c. thus removed. Knowing the volume of solution in each voltameter, these analyses were sufficient to determine the total amount of copper that had passed through the partition during the same interval for each voltameter. Table VI. gives the results. Here it was observed that imperceptible differences in the specimens of gold-leaf were sufficient to cause enough difference in the diffusion to leave the question unsettled. The amount passing the partition of the open voltameter was as often *greater* than that of the closed voltameter as it was *less*. It was now evidently necessary to test one and the same gold-leaf partition for diffusion, with circuit open and closed successively. This leads to the third method, the results of which are shown in Table VII.

It is thought that this method is entirely free from objection or serious error; it leads to the conclusion that the current does not sensibly affect the diffusion of $CuSO_4$ and H_2SO_4 through the gold-leaf partition. This method, in detail, was to set up two similar voltameters at the same time, leaving both open and making quantitative analyses at stated intervals; then, closing the current through both in series, to again make quantitative analyses of the solution on the cathode side, and to weigh the copper on the cathodes; finally, to break the circuit and again repeat the analyses. Curves plotted (see Fig. 5) from these results show *no break nor change of slope* for the intervals during which the current was passing. In the foregoing experiments there was no deposit of copper on the gold-leaf partition, provided the current was less than about 0.3 of an ampere. The time curves for the

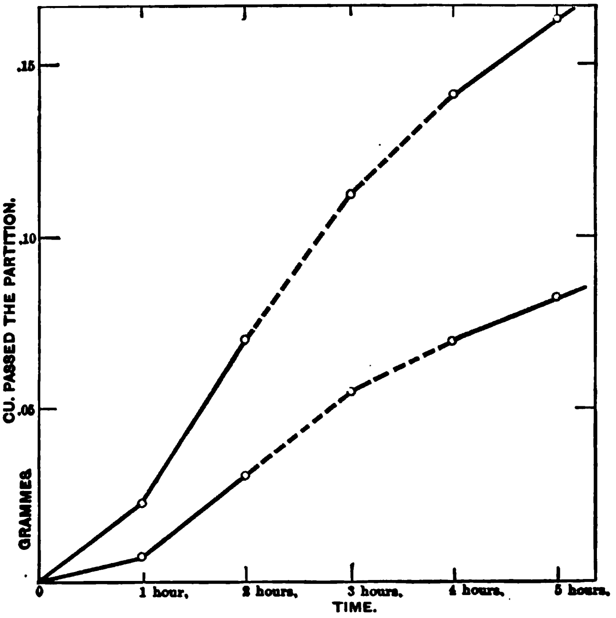

Fig. 5.

two specimens of gold-leaf are not identical, showing a different rate of diffusion, but are quite similar in shape. Using a copper anode, there was no visible gas on the anode, but the anode was dissolved by the current in the usual manner. Using a platinum cathode and a platinum anode, there was, of course, an escape of gas from the anode proportional to the current strength. Gas always escaped from the platinum cathode, but proportionally less as the amount of copper deposited increased. In the case where the partition was removed and the solutions thoroughly mixed, there was no gas, but the full amount of copper deposited on the cathode. Thus it seems that the passage of the current in such a voltameter is not accomplished by the passage of the copper through the gold-leaf partition. Some of the $CuSO_4$ passes the partition by diffusion, and then does its proportional part of the conduction of the mixed solution of CuSO and H_2SO_4, which accounts for the increased per cent of copper deposited on the cathode, as the interval, from the time the voltameter was filled, increases.

In the time curves which may be constructed from Table VII. (see Fig. 5) it will be noticed that the amount of copper at the end of the first interval seems too small, causing a similar bend in both curves. This is doubtless due to the fact that the current was not continued long enough to deposit all the copper in the 10 c.c. sample. The amount of $CuSO_4$ present being very small, it would be deposited very slowly; and the circuit should have remained closed several hours instead of less than an hour, as was the case. A similar bend occurs in both curves, because the specimens were analyzed in series and the same defect applies to both alike. If the current was increased beyond about 0.3 ampere, with $CuSO_4$ and H_2SO_4 as the solutions, separated by a gold-leaf partition, there was a development of gas and copper on the partition. This brings us to the second problem, the determination of the *minimum current strength* at which the ions are deposited visibly on the gold-leaf partition for various electrolytes.

Critical Current Density for Gold-leaf.

This part of the investigation consists simply in closing the circuit on the voltameter with a high auxiliary resistance, slowly increasing the current by decreasing the resistance, and observing at what current strength bubbles of gas or deposit of metal *begin* to appear upon the partition. Table VIII. shows these values for various solutions used as electrolytes, with a gold-leaf partition. Table IX. gives the results in 30 per cent H_2SO_4 for aluminum-leaf, platinum-leaf, and gold-leaf, respectively. Table IX. shows the same for gold-leaf partition in H_2SO_4 of various concentrations from 100 per cent (?) to 0.5 per cent. Unfortunately the importance of the concentration of the solution in affecting the value of the "critical current" was not noted until after several of the solutions had been tested and thrown away without measuring their concentration or specific gravity, which therefore are not given for those cases. The specific gravity of those solutions for which it is given was determined by a very sensitive and accurate set of hydrometers reading directly to 0.001. Table IX. gives the results for 30 per cent H_2SO_4, with platinum partitions of various thicknesses; also of a "thick" platinum partition perforated with 1, 2, 3, and 4 holes successively, each 0.5 mm. in diameter. This is important in showing that the very small holes which may have existed in the carefully selected gold-leaves used as partitions, could not have had much, if even a sensible, influence on the value of the critical current. Table X. shows the interesting case of a 0.02 mm. paladium partition in 30 per cent H_2SO_4. The values given in the tables are in most cases the mean of several determinations which differed but slightly.

The importance of having the edges of the partition covered well up to the edge of the hole in the glass was emphasized in this work on account of the fact that the anode was usually placed on one side, and sometimes even at the back of the inner vessel, instead of in the front, in order not to obstruct a free inspection of the partition, and to avoid the disturbance of the gas escaping from the anode, as this was considerable with the strongest currents used; *i.e.* 10 to 12 amperes. Thus one edge of the exposed partition being nearer to the anode than the opening, the deposit would always begin first on the nearest edge.

Inspection of the tables will show that in 30 per cent H_2SO_4 the gold-leaf partition will pass about ten amperes before gases are developed upon it. Platinum-leaf and aluminum-leaf showed sensibly the same "critical current," rather less for the aluminum, which was, however, thicker though not so free from holes.

TABLE V.

A 17% SOLUTION OF $CuSO_4$ WITH COPPER ANODE SEPARATED BY A GOLD-LEAF PARTITION FROM A 30% SOLUTION OF H_2SO_4 WITH PLATINUM CATHODE.

Time in minutes.	Current in amperes.	Cu deposited on cathode in grams.		Ratio of Cu to current.		
30	0.175	0.0030		3%		
33	0.205	0.0765		58%		After standing from Saturday to Monday.
40	0.206	0.0920		57%		
40	0.127	0.0700		70%		
45	0.054	0.0380		80%		
		No. 1.	No. 2.	No. 1.	No. 2.	
30	0.132	0.0020	0.0766	2.6%	99.0%	Voltameters in series.[1]
30	0.147	0.0270	0.0850	31.0%	99.0%	After some hours.
52	0.152	0.0453	0.0950	47.6%	100.0%	
65	0.035	0.0022	0.0026	5.0%	5.7%	Fresh solutions.[2]

REMARKS. — [1] The partition in No. 2 broke before the current was closed. [2] Partition of No. 1, single gold-leaf; of No. 2, double.

A solution of CdI_2 showed regular deposit of both cadmium and iodine upon the partition for the weakest current to 0.001 ampere. This case was peculiar in having both ions *solids*, and indicated as a consequence that neither would pass through the gold-leaf partition.

Using 9 per cent $AgNO_3$ solution, Ag_2O_2 was deposited on the anode, but this came about by a secondary action at the anode; and though we have in this case also two solids deposited on the electrodes as the result of the electrolysis, one of them is not primarily a solid, and the critical current is of the order of magnitude of that for other metallic salts. A similar case was found in that of $MnSO_4$ solution, where MnO_2 was deposited on the anode. A solution of lead acetate gave PbO_2 on the anode, but showed a very low "critical current."

TABLE VI.

TO TEST DIFFUSION OF 17% $CuSO_4$ AND 30% H_2SO_4 WITH CURRENT PASSING IN VOLTAMETER NO. 1, AND NO CURRENT PASSING IN NO. 2.

Time in minutes.	Current in amperes.		Cu on cathode.		Per cent of Cu to current.		Cu in 10 c.c. solution.		Total Cu passed the gold.	
	No. 1.	No. 2.	No. 1.	No. 2.	No. 1.	No. 2.	No. 1.	No. 2.	No. 1.	No. 2.
92	0.0505	0.0	0.0058	0.0	6.4	0.0	0.0020	0.0023	0.0308	0.0300
87	0.0530	0.0	0.0143	0.0	15.8	0.0	0.0040	0.0050	0.0643	0.0650
Fresh solutions and single gold-leaf partitions.										
45	0.1500	0.0	0.0022	0.0	1.7	0.0	0.0043	0.0040	0.0550	0.0480
65	Broke.	0.0	—	0.0	—	0.0	—	0.0060	—	0.0760
18 hr.	—	0.0	—	0.0	—	0.0	—	0.0256	—	0.3172
Fresh solutions and gold-leaf partitions.										
55	0.0770	0.0	0.0022	0.0	5.3	0.0	0.0014	0.0033	0.0190	0.0410
60	0.0770	0.0	0.0090	0.0	20.0	0.0	0.0035	0.0065	0.0540	0.0850
60	0.0790	0.0	0.0142	0.0	30.8	0.0	0.0038	0.0084	0.0797	0.1148

Began the above at 12 hr. 55 min. 30 sec.; closed at 3 hr. 30 min. 30 sec.

An interesting fact is the dependence of the "critical current" upon the concentration of the electrolyte, as shown in Table IX., indicating that the "critical current" is proportional to the conductivity of the electrolyte. I have made some experiments to determine whether variation of the temperature has the same effect upon the "critical current" as upon the conductivity, although the results are as yet hardly sufficient to justify a conclusion. I have been entirely unable anywhere to get a series of platinum partitions of thicknesses suitable to repeat the quantitative measurement of polarization which were made in Berlin for the gold plates. Table X. for 0.02 mm. palladium is interesting.

TABLE VII.

THIRD AND BEST METHOD.

Fresh solutions and gold-leaf partitions.

Time in minutes.	Current in amperes.		Cu on cathode.		Per cent of Cu to current.		Cu in 10 c.c. solution.		Total Cu passed the gold.	
	No. 1.	No. 2.	No. 1.	No. 2.	No. 1.	No. 2.	No. 1.	No. 2.	No. 1.	No. 2.
60	0.0	0.0	0.0	0.0	0.0	0.0	0.0019	0.0006	0.0228	0.0075
60	0.0	0.0	0.0	0.0	0.0	0.0	0.0055	0.0024	0.0679	0.0306
Then the circuit was closed.										
45	0.10	0.10	0.0154	0.0060	35.0	13.7	0.0074	0.0036	0.1116	0.0540
45	0.10	0.10	0.0205	0.0100	47.0	22.8	0.0080	0.0038	0.1467	0.0701
Again, leaving the circuit open, the following was obtained: —										
60	0.0	0.0	0.0	0.0	0.0	0.0	0.0090	0.0048	0.1667	0.0864
Filled the voltameters at 12 hr.; last test for Cu at 5 hr.										
16 hr.	0.0	0.0	0.0	0.0	0.0	0.0	0.0310	0.0187	0.5387	0.3200

Either gold, platinum, or aluminum as thick as this behaves like a very thick plate, *i.e.* shows gas at once in amount corresponding to the current strength. When first set up, the 0.02 mm. palladium partition showed a "critical current" of 0.08 ampere. After considerable use, the "critical current" has gradually, though not slowly, increased to 0.3 in 30 per cent H_2SO_4. This seems to be quite distinct from the well-known property of palladium of occluding gases, especially hydrogen, for this plate will pass 0.2 amperes or more for hours without showing gas, and develops gas at once for a little stronger current. The gas on the partition of palladium will cease to escape if the current be again reduced below the "critical" value. Currents of several amperes have several times been passed through this partition, developing profuse gas, and it was thought probable that the palladium was

TABLE VIII.

SHOWING MEAN MINIMUM CURRENT AT WHICH GASES DEVELOPED VISIBLY ON A GOLD-LEAF PARTITION, WITH PLATINUM ELECTRODES.

Electrolyte.	$CuSO_4$.	$AgNO_3$.[1]	CdI_2.	$ZnSO_4$.	$MnSO_4$.[2]	$FeSO_4$.
Sp. Gr.	17%	9%	5%	saturated		1.10
Current	0.17	0.40	0.0001 –	0.19	0.15	0.32
Current density . . .	0.10	0.23		0.11	0.08	0.18

Electrolyte.	$SnSO_4$.	$NiSO_4$.	$HgSO_4$.	NH_4NO_3.		
Sp. Gr.				1.05	1.10	1.135
Current	0.30	0.13	0.17	0.65	2.80	3.00
Current density . . .	0.17	0.07	0.10	0.37	1.60	1.75

Elec.	Benzoate of Na.	Sulphate of		Acetate of			
		Am. & Fe.	Am. & Ni.	Pb.[3]	Am.	Am.	Na.
Sp. Gr. . .				1.10	1.06	1.09	sat'd
Current . .	0.75	0.18	0.31	0.018	1.10	0.70	0.75
Cur. density .	0.43	0.10	0.18	0.01	0.63	0.40	0.43

REMARKS.—[1] AgO_2 formed on anode. [2] MnO_2 formed. [3] PbO_2 formed.

A 96% solution of acetic acid was found to have too high a resistance to give a sensible current with 50 volts, electromotive force. Benzoic acid dissolved in the same was also an insulator.

being dissolved by the 30 per cent H_2SO_4 under the action of the current. Accordingly a test of the solution was made for palladium by adding ammonia until alkaline, and then adding ammonium sulphide, which gave no indication of the presence of palladium.

It has been observed that platinum is perfectly constant, showing the same "critical current" as often as tested. Gold is oxidized the first time the "critical current" is attained, and its value decreases on repeating the test (with the same specimen).

Efforts were made to test the critical currents for four thicknesses of gold from 0.0004 mm. to 0.0018 mm., in ammonium nitrate, hoping that this would not oxidize the gold; but the partitions were oxidized by currents strong enough to develop gas.

Table IX.

THE CRITICAL CURRENT AS AFFECTED BY STRENGTH % OF H_2SO_4, AND NATURE OF PARTITION.

Strength.	100 (?)	55	43	30			22	15	4	0.5
Partition .	Au	Au	Au	Al	Au	Pt	Au	Au	Au	Au
Current .	0.55	4.5	8.0	8.0	10.0	10.0	8.0	6.0	1.8	0.23
Cur. dens.	0.31	2.57	4.57	4.57	5.71	5.71	4.57	3.43	1.03	0.13

30 % H_2SO_4. CRITICAL CURRENT AFFECTED BY THICKNESS OF PLATINUM PARTITION IN MILLIMETERS AND BY PUNCTURES 0.5 mm. IN DIAMETER.

Thickness.	0.00333	0.0050	0.02				
Holes	—	—	0	1	2	3	4
Current	0.090	0.002	weakest	0.07	0.13	0.20	0.27
Current density .	—	—	—	0.04	0.07	0.115	0.154

Table X.

INCREASING CRITICAL CURRENT OF 0.02 mm. PALLADIUM IN 30 % H_2SO_4.

Date.	**January**		**February**					
	31	31	4	8	17	17	18	18
Current	0.08	0.10	0.15	0.17	0.25	0.26	0.27	0.27
Current density	0.045	0.057	0.086	0.10	0.143	0.148	0.154	0.154

A new specimen of the same palladium is being tested, and is showing the same phenomenon of increase of the critical current. This last specimen was carefully selected, and has *no* holes. Its critical current at first was 0.016; after some use, it was 0.043 ampere.

I have begun to use a solution of sodium acetate, with the hope that it will not oxidize the gold. I can already say that it is at least very much less active than sulphuric acid. It also shows a decided *temperature coefficient* (see Fig. 6) for the critical current. This part of the investigation will be resumed when opportunity offers.

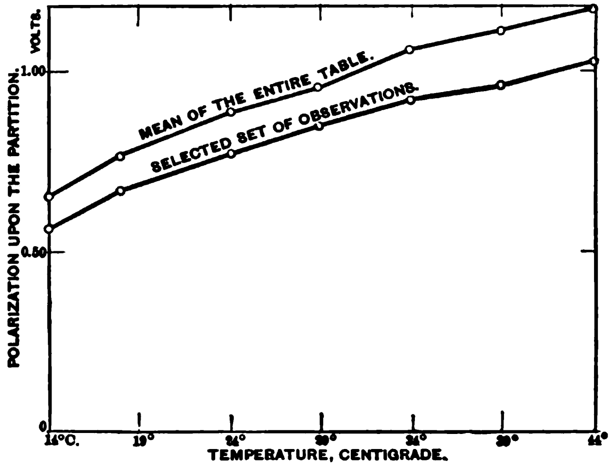

Fig. 6.

Vanderbilt University, Nashville, Tennessee,
February 22, 1893.

Note. — More satisfactory specimens of palladium have not yet been obtained, so the matter is left for the present with the seemingly anomalous results, recorded above, as I find them in my note-book. As regards the temperature coefficient of the "critical current density," I have had Mr. R. W. Clawson, a student in the Laboratory, make some measurements, which are tabulated below. It may be well to state that the method of observation, that of observing the first appearance of *visible* gas, is not very sensitive, because this development of gas on thin partitions is not quite sudden and sharply defined, but begins gradually, especially with very thin partitions. The probable error of a reading is, however, I think, not more than 5 per cent. Only "XX" gold-leaf has been used in a solution of sodium acetate. The method was to set up the voltameter with gold-leaf partition, close the circuit, and run the current up until the "critical current density" was reached. Current and temperature were recorded, the current slightly reduced and allowed to pass till the temperature had

Table XI.

(Double lines indicate a new partition.)

Sodium acetate; Sp. Gr. = 1.06. Gold-leaf partition.									
Temperature . . .	22	24	26	29	30	33	35	24	27
Current	.65	.70	.76	.80	.85	.92	.96	.53	.64
Temperature . . .	24	26	22	23	24	24	33	41	45
Current	.54	.60	.53	.57	.59	.77	1.18	1.33	1.48
Temperature . . .	15	17	19	20	23	26	30	33	37
Current	.30	.37	.45	.50	.65	.74	.83	.96	1.02
New solution sodium acetate; Sp. Gr. = 1.13.									
Temperature . . .	23	27	23	28	24	30	35	38	—
Current	.70	.80	.80	1.0	.95	1.05	1.20	1.29	—
Temperature . . .	14	19	24	29	34	39	44	—	—
Current	.57	.67	.78	.85	.93	.98	1.05	—	—
Temperature . . .	30	36	41	46	25	30	35	40	25
Current	.87	.91	.99	1.1	.71	.76	.86	.94	.60
Temperature . . .	30	35	40	45	25	30	35	40	45
Current	.65	.69	.74	.82	.72	.85	.92	.98	1.05
Temperature . . .	25	30	35	40	45	—	—	—	—
Current	.72	.85	.92	.98	1.05	—	—	—	—

risen, say 5 degrees, when the current was again increased to the "critical" value for this new temperature. The limit was reached at about 45° C., when the sealing-wax softened. Since both "critical current density" and conductivity of the electrolyte increase with temperature, effort was made to learn whether both these quantities have the same temperature coefficient, by raising the current just to the "critical" value, and, without altering the outside resistance, to observe if the increasing current, due to increasing conductivity of the electrolyte, just sufficed to maintain an incipient escape of gas at the partition. In every case the gas soon ceased to escape on being left alone, after adjusting the current to a given temperature, requiring always some reduction of the outside resistance to regain the critical current for the higher temperature. A quantitative correction for the temperature coefficient of the outside metallic resistance in the circuit would probably account for the discrepancy.

The curve plotted from the mean of all the above results gives almost a straight line. .

APRIL 15, 1893.

THE QUANTITATIVE DETERMINATION OF RADIANT HEAT BY THE METHOD OF ELECTRICAL COMPENSATION.[1]

By Knut Ångström.[2]

I.

THE older methods for the absolute determination of radiant heat, the methods of the pyrheliometer and actinometer, had two principal faults: they were neither accurate nor sensitive enough. They were, moreover, almost exclusively intended for the measurement of the radiation from the sun, and were not well adapted for the precise determinations frequently demanded at the present day in the physical laboratory. The great progress which has been made within the last fifteen years in the construction of instruments for relative measurement, such as the thermopile, bolometer, and the radiomicrometer, have been followed by very important investigations upon radiant energy. It is necessary to recall only the fundamental labors of Langley and the brilliant series of memoirs by Julius, Robert von Helmholtz, Rubens, Snow, and others. The investigation of radiant heat has taken an important place in the physical progress of the present day, and for that reason the need of a good method of quantitative measurement is strongly felt.

Such a method, by means of which relative measurements may be converted into quantitative ones by simultaneous observations of a given source of heat with two instruments, I described in 1885.[3] I have made use of the same continually since that time in my bolometric work, and can but emphasize its exactitude and reliability. In the comparative studies of different actinometric methods carried on by Mr. Chwolson,[1] the advantages of this method have been clearly demonstrated. While, on the one hand, I regard, therefore, the problem of the quantitative measurement of radiation as solved, it must be said, on the other, that differential pyrheliometers leave something to be desired as regards simplicity and convenience of operation. In order to compare this, my older method, with the one to be described below, permit me to refer briefly to the former.

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[1] From the Transactions of the Royal Society of Sciences, Upsala, 1893.

[2] Translated for the Review by E. L. N.

[3] K. Ångström, Acta Reg. Soc. Ups., 1886. See also Bihang till K. Vet-Akad. Handlingar, 15, No. 10, 1889, and Wiedemann's Annalen, 39, p. 294, 1890.

Two circular plates of massive copper serve as calorimeters; the blackened faces of the same are exposed alternately to radiation, and one observes the time at which the difference of temperature (θ) changes sign. This difference of temperature is determined by means of a thermo-element and a mirror galvanometer. If W is the water equivalent and A the absorbing surface of the calorimeter plates, then it is plain that the quantity of heat which is absorbed by a unit of surface is expressed by the equation

$$Q=\frac{2\,\theta W}{AT},$$

and that by this arrangement of observations the consideration of rate of cooling is eliminated.

II.

The principle of the new method is briefly as follows: Given two thin strips of metal, A and B, Fig. 1, which are as nearly as possible identical. The sides of these, which are exposed to the source of heat, are blackened, and the strips are arranged in such a way that it is possible to determine accu-

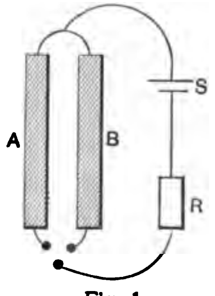

Fig. 1.

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[1] O. Chwolson, Rep. für Meteorologie, 15, pp. 1–166; also 16, pp. 1–150. The former treatise contains a theoretical and critical discussion of actinometric procedure; the latter, a full account of my own method, including some attempts to devise a convenient apparatus, making use of the same, for meteorological purposes. Although both researches have as their chief aim the measurement of the radiation of the sun, they are of great interest from the point of view of general study of radiant energy. The success of the author's modifications of the method need not concern us here. Upon another occasion I hope to discuss both of these important papers in detail.

rately when they are of the same temperature. These strips are so placed that a current of any desired strength can be sent through them. If one of them, for example, *A*, is exposed to the source of heat, while *B* is protected by a screen, we may restore the balance of temperature which has been disturbed by the absorption of heat on the part of *A*, by sending a current of proper intensity through *B*. When the temperatures are the same, then the amounts of energy which *A* and *B* have received are equal to one another. Let l and b be the length and width of the strips, r the resistance of the same, and i the current. Then since the heat absorbed by *A* is the equivalent of that produced by the electric current in *b*, we may write

$$qlb = \frac{ri^2}{4\cdot 2}, \text{ or } q = \frac{ri^2}{4\cdot 2\, lb},$$

where q is the radiant energy received by a unit of surface. In order to counteract the inequality of the strips, they are interchangeable, *B* being illuminated and the current sent through *A*. In order to compare the two methods, the operations necessary in each to a complete measurement of radiation are tabulated below.

A. Constants to be determined once for all.

THE OLD METHOD. (*The Differential Pyrheliometer.*)	THE NEW METHOD. (*Method of Compensation.*)
1. Determination of the water equivalent of the calorimeter plates.	1. Determination of the electrical resistance of the strips.
2. Determination of the absorbing surface.	2. Determination of the absorbing surface.

B. Observations to be made during each experiment.

1. Determination of the time within which the temperature differences of the calorimeters change sign.	1. Adjustment of the two strips to equal temperature.
2. Calibration of the thermo-element in degrees.	2. Measurement of the current.

This comparison appears to me to be favorable to the new method. The measurement of resistance is more rapidly and conveniently carried out than the determination of the specific heat of the plates of the calorimeters, and as regards the determinations which are to be made during each experiment, it is much easier to make the adjustment to equal temperatures than to carry out the tedious determination of the time. The last operation requires a more skilled observer. The determination of current strength is doubtless of about the same difficulty as the calibration of the thermo-element in degrees. Since both of these measurements are ultimately dependent upon the constant of the instrument for the measurement of current, it seems to me that here also the new method is to be preferred, because that instrument does not need to be of so high a degree of delicacy, and consequently its figure of merit is more easily determined.

III.

In the practical application of this principle of compensation, one may follow various methods. The equality of temperature may be determined in a variety of ways. If thermo-elements are used for this purpose, one needs a sensitive galvanoscope, and the measurement of the current can be made upon an instrument of ordinary delicacy. It is possible, however, to carry on the investigation without any difficulty, using only one galvanometer, a plan which I pursued in the case of the first apparatus which I constructed. Fig. 2 gives a diagram of the connections,

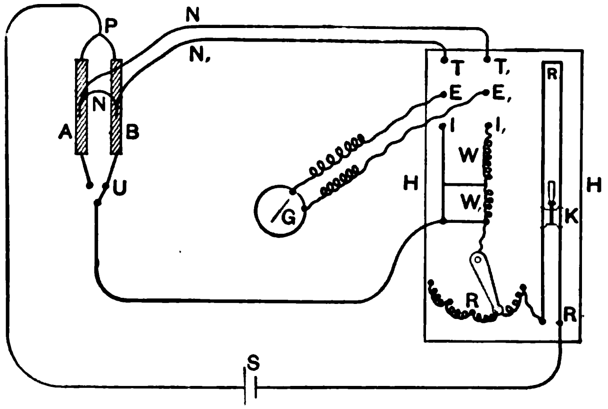

Fig. 2.

while Figs. 3, 4, 5, and 6 show more in detail the arrangement of the apparatus. The metallic strips, *A* and *B*, cut simultaneously from two thin sheets of platinum laid one upon another, are 0.154 cm. wide and 1.80 cm. in length. They are blackened in the usual way upon the side exposed to radiation, and are mounted side by side in a frame of ebonite (Fig. 4). This frame is inserted in a tube, as is shown in Fig. 6. Upon the back of the strips are laid two exceedingly thin leaves of mica, upon which the very minute junctions of copper and German silver, NN_1, are inserted (see Fig. 4). To hold the mica and the thermo-junctions together, I used marine glue in as thin a layer as possible. Mounted upon the block, Fig. 2, is a rheostat, a resistance WW_1, which serves as a shunt for the galvanometer *G*, and the commutator *TEI*. The rheostat has a sliding contact, *K*, by means of which the resistance can be varied at will. In this way the current from the battery, *S*, usually a Daniell's cell, is indirectly controlled.

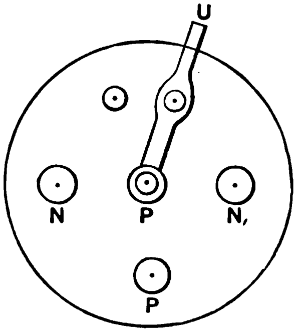

Fig. 3.

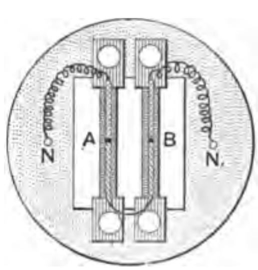

Fig. 4.

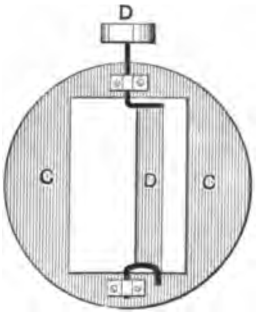

Fig. 5.

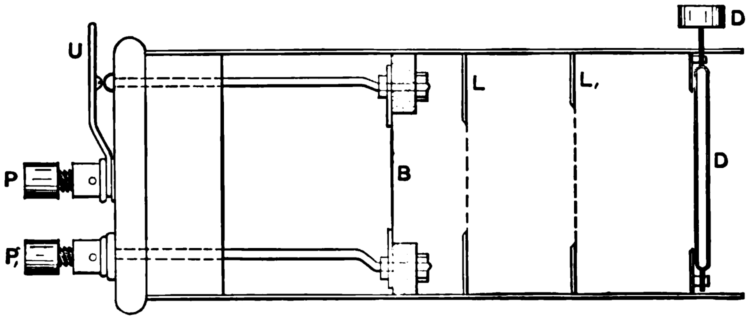

Fig. 6.

One of the strips is subjected to radiation; the circuit is then closed through the other one; the thermo-element is brought into circuit with the galvanometer by means of the commutator; the sliding contact is adjusted until the galvanometer stands at zero; the commutator is then reversed, and the strength of the current used in heating the strip is determined. The switch, U (Fig. 3), is then reversed, the shutter is placed in front of the other strip, and the setting and current measurement are repeated. There are two other ways in which one can use this apparatus for the measurement of radiation, viz.: —

First Variation of the Method. — One of the strips, for example, A, is exposed to radiation, while the other is screened. One notes the deflection of the galvanometer, which becomes constant in about fifteen seconds. A is then also screened, and by means of the current is brought to the same temperature to which radiation had previously brought it. The strength of the current producing this rise of temperature is then measured. The advantage of this arrangement is that the same strip is warmed by means of radiation and then through the agency of the current, so that it is not necessary to be so painstaking as regards the identity of the two strips.

Second Variation of the Method. — One of the strips, for example, A, is exposed to radiation, the other, B, is screened; the deflection of the galvanometer is observed after the thermo-current has become constant. The quantity of heat which A has received is calculated from Newton's law of cooling,

$$Q = blq = k\theta,$$

where θ is the difference of temperature indicated by the galvanometer, and k is the constant of cooling of the strip. If we now send a current through A, without making any change of conditions, the difference of temperature will become greater still. This will be indicated by the resulting deflection θ_1. The strength of the current producing this additional heating effect may be measured in the manner already described. The amount of heat added to the strip is

$$Q_1 = blq + \frac{ri^2}{4\cdot 19} = k\theta_1,$$

from which equations we find

$$q=\frac{i^2r\theta}{4\cdot 19(\theta_1-\theta)bl}.$$

When the experiment is performed in this way, it is not necessary to have an arrangement for the adjustment of the intensity of the current. The greatest advantage is, however, that the use of a screen may be avoided. This is perhaps the only method by means of which the screen, the use of which is always open to suspicion, can be eliminated.

IV.

As an example of the application of the new method, the following determinations of the radiation from an argand lamp may be of interest. In front of the lamp a double screen was set up so that the rays could reach the apparatus only through a hole 2.4 cm. in diameter. The distance between the strips and the middle of the flame was 60 cm. The other constants of the apparatus were as follows: Absorbing surface of each strip, .277 $\overline{\text{cm.}}^2$; resistance of A, 0.172 ohms; resistance of B, 0.176 ohms; reduction factor of the galvanometer (with shunt), 0.00122 amperes.

(*a*) *Experiment using the zero-method.*

Heating current of compensation:

(1)	When A was subjected to radiation,	49.8 s.d.
(2)	" B " " " "	49.7 "
	Mean,	49.75 "

This reading corresponds to 0.0607 amperes, from which the radiation may be computed from the equation

$$q=\frac{\overline{0.0607}^2\times 0.174}{4.19\times 0.277},$$
$$=0.000552 \text{ gram-calories per second per } \overline{\text{cm.}}^2$$

(*b*) *Experiment following the first modification.*

The strip A was subjected to radiation. The deflection of the galvanometer due to the thermo-current was 44.0 s.d. The strength of current necessary to bring about this rise of temperature was 49.5 s.d. = 0.0604 amperes, from which the following value of q was computed, viz.: —

$$q = 0.000541 \text{ gram-calories per second per } \overline{\text{cm.}}^2$$

(*c*) *Experiment following the second modification.*

The strip A was subjected to radiation. The galvanometer gave a deflection, due to the thermo-current, of 45.5 s.d. Upon closing the circuit of the heating current, the deflection rose to 91.5 s.d. The intensity of the current was 50 s.d. = 0.0610. The radiation was computed by means of the equation

$$q = \frac{\overline{0.0610}^2 \times 0.172 \times 45.5}{4.19 \times 0.277 \times 46.0},$$

from which we find $q = 0.000546$ gram-calories per second per $\overline{\text{cm.}}^2$

From what has been said it will be seen that the determination is an extraordinarily simple one. When the constants of the instrument are known and the figure of merit of the galvanometer, the whole measurement can be made with great exactness in a few minutes. If it is desired to take into consideration the change in resistance due to heating, it is only necessary, after an adjustment to zero, to open the circuit with the strip still exposed to radiation. From the deflection then produced, the temperature of the strip can be determined with sufficient exactitude. Thus far we have considered only the heat which is received by the instrument, without taking into account the absorbing power of the surface. In the method just described, as in the case of all other methods, it is necessary for exact measurements to determine the coefficient of absorption. No very good method for estimating this constant exists, but it is possible to obtain surfaces which possess the same absorbing power within the errors of observation; that is to say, within about 98 per cent. In most cases, therefore, the determination of this quantity may be neglected. In conclusion, I hope that the method outlined in this paper will be found to afford a means for the quantitative measurement of radiation, sufficiently simple, rapid, exact, and sensitive. In this brief note I have merely indicated the principle of the method; later, after a more thorough investigation, I purpose to give a fuller description of it.

MINOR CONTRIBUTIONS.

An Apparatus for the Generation of Oxygen and Hydrogen by Electrolysis.

By E. L. Nichols and G. S. Moler.

ALTHOUGH it is not possible to produce oxygen in large quantities by electrolytic action so cheaply as by other methods, there are certain conditions under which electrolysis is to be preferred. Whenever, for instance, it is desirable to have pure hydrogen gas instead of the illuminating gas which is ordinarily furnished, as well as oxygen, — that is to say, in our chemical and physical laboratories everywhere, where the gases are used not only in the oxyhydrogen blowpipe, but also for various analytical purposes, — the electric current will be employed.

Nearly all physical laboratories, at the present day, are provided with steam or water power and with shunt-wound dynamos, so that the production of oxygen and hydrogen by means of the current may be conveniently undertaken. The advantage of having a store of the pure gases at command is very great, not only for use with the lantern, but in a variety of other operations, for many of which the compressed gases furnished in cylinders do not suffice. For about seventeen years the Department of Physics in Cornell University has manufactured its own oxygen and hydrogen gas by electrolysis. It is the purpose in this paper to give a brief sketch of the various modifications which the methods there pursued have undergone, pointing out the technical difficulties which have been met and how they have been overcome, and then to describe, with sufficient detail to enable the present very satisfactory plan to be duplicated, the method at present in use.

Early Forms of Gas Generators. — In 1875 and 1876, Professor William A. Anthony constructed a series-wound Gramme dynamo for experimental purposes. This machine, which was probably the first of any considerable size made in this country, was exhibited at the Centennial Exhibition. In the following year experiments were made upon the production of gas for use with a magic lantern, in which experiments one of the present writers (Moler) took part.[1]

[1] For accounts of apparatus designed or in use elsewhere for the electrolysis of water on a large scale, see Giltay, Electrical Review (London), Vol. 18; Latchinoff, British

The first form of electrolytic apparatus consisted of twelve cells placed in series. The cells were of the following form: The lower part of each was of wood well filled with paint, forming an oblong box with a bar across the middle. The upper edge of the box and the cross-bar were grooved, and into these grooves were cemented two inverted boxes filled with sheet lead (Fig. 1). In the top of the boxes were the necessary holes and tubes for filling with acidulated water and for carrying off the gases. The twelve cells were set side by side in a trough of water, it having been found that the heat generated by the current would otherwise melt the cement.

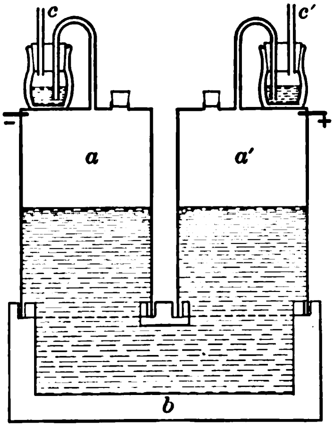

Fig. 1.

The twelve cells were connected in series in the circuit of the Gramme dynamo, and an automatic safety switch was placed in the circuit to prevent the counter-electromotive force of the cells from reversing the polarity of the dynamo when the latter was not in motion. This switch was so constructed as to shift the current to an outside wire of proper resistance whenever the current fell below a certain strength, and to shift it back again as soon as the electromotive force was great enough to overcome the electromotive force of the cells. Later on, the dynamo was separately excited, and this switch was dispensed with. Each leaden box was connected to an outside system of pipes, to which the gas was so conducted away through a separate wash-bottle. The bottles were necessary, in order that one might see that the cells, interiors of which were not visible, were working properly.

It was found to be impossible to keep the cells tight for any great length of time, and they were finally abandoned. Others, constructed of stone china, were tried with better results. The form of these china cells is shown in Fig. 2. They were shaped something like a U-tube with an enlarged base, being made in one piece. In the top of each branch of the cell were holes for corks, and through these the electrodes were inserted. These consisted of lead pipes, into the ends

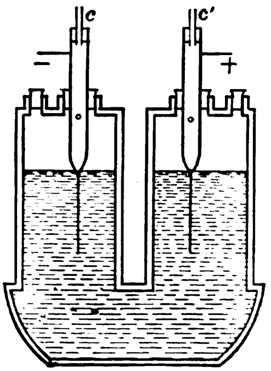

Fig. 2.

Patents, Specifications of Inventions, Vol. 80, No. 15935 (1888); also, Fortschritte der Elektrotechnik, Vol. 4, p. 536; Renard, La Lumière Electrique, Vols. 39 and 40.

of which the platinum plates were fused. Two holes through the walls of the pipes allowed the gas to enter, and these pipes served as delivery tubes for the oxygen and hydrogen. They also conveyed the current to the platinum electrodes. The pressure within these cells was great enough to force the gas into gasholders, which were weighted down sufficiently to enable one to operate the jet of the lime light without further compression. This form of generator was in use for a considerable period, but was finally supplanted by a set of electrolytic cells made entirely of glass, the form of which is indicated in Fig. 3. The essential features of this, which is the type at present in use, are as follows: A tall cylindrical jar, six inches in diameter and thirty inches high, contains the acidulated water to be decomposed. Two long conical glass tubes, about one inch in diameter at the top and two and one-half inches at the base, are clamped side by side in a wooden holder, which rests upon the rim of the outer cylinder. These reach to within about two inches of the bottom of the latter. These tubes are fitted above with paraffined corks, through which lead pipes are inserted, care being taken to secure a thoroughly gas-tight joint. These pipes, as in the earlier forms, serve as conductors for the current and as delivery tubes for the generated gases. The electrodes are attached to the lower ends of these pipes.

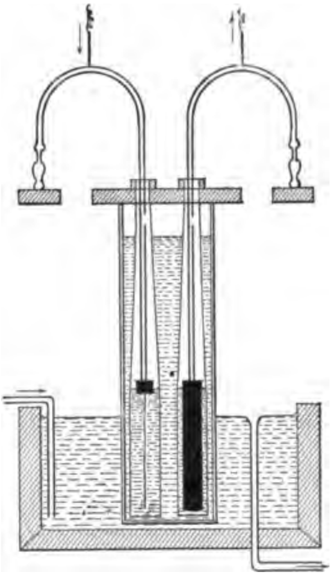

Fig. 3.

For a number of years platinum electrodes of the size already described were made use of, but it was found that upon the hydrogen side they suffered rapid disintegration, due to some local chemical action, the nature of which has not been investigated. On account of the expense and trouble of replacing these, experiments were made with the view of substituting other substances for platinum.

Following a suggestion by Renard, iron plates in an electrolyte consisting of a dilute solution of caustic soda were tried, but they were very short-lived. Carbon plates, such as are used in the cells of primary batteries, were also tested, the liquid being the usual acidulated water; but owing probably to the impure character of the carbon, these plates were found to disintegrate with great rapidity, especially while the generator was in operation, and they had to be abandoned. Lead plates were then tried, these consisting of blocks of cast lead, into the end of which, while still molten, the extremity of the leaden delivery pipe had been inserted, forming a continuous piece. It was known that these would oxidize upon one terminal, forming a plate like that of the Planté battery, and that more or less of spongy lead would be produced at the other pole; but no further inconvenience than that arising from the very considerable counter-electromotive force was anticipated.

So much confidence was felt in the permanence and reliability of this form of electrode, that when a new generator was to be installed in the Chemical Laboratory of Cornell University, such leaden plates were introduced. The result proved to be disastrous. The new apparatus had been in use only a very short time when an explosion occurred of sufficient violence to destroy the entire generator. It was found upon investigation that the *oxygen* tank had been blown up, and that the mixing of gases which led to this explosion was due to an entirely unforeseen cause. When the current is passed through the leaden plates of such a generator, the electrode at the surface of which oxygen is set free is rapidly oxidized. This oxide is a good conductor of electricity. It is in great part adherent, but not entirely so; and in this case small portions became detached from the surface of the electrode and fell to the bottom of the cylindrical jar (Fig. 3). Thus in a short time a film was formed which covered the entire bottom of the cell. The conducting power of this layer was considerably greater than that of the electrolyte itself, and as a consequence double electrolysis took place. Hydrogen was generated at the hydrogen pole as before, and oxygen gas upon that portion of the film which lay immediately beneath the open mouth of the hydrogen-collecting tube. Hydrogen was generated again at that portion of the film which was situated under the mouth of the oxygen tube, and this mingled with the oxygen generated at the cathode of the apparatus. In this way mixed gases were delivered to both gasometers, and as soon as one of these reached the proper proportions for explosion and became ignited, the accident took place. Further study showed that this troublesome disintegration of the electrode was confined wholly to the oxygen pole, and that the spongy lead formed at the other terminal was completely adherent.

Now, as has been already stated, the difficulty with platinum electrodes lies with the plate at the surface of which hydrogen was generated, while the oxygen electrode remained bright and unimpaired. It became evident, then, that by using platinum plates as cathodes and lead plates as anodes, a generator could be constructed which would be free from the difficulties previously encountered with both of these metals; and when, in 1891, a new generator was to be installed in the Physical Laboratory, this plan was followed. The result was very satisfactory, the apparatus in question having been in almost continuous operation for two years with entire freedom from the annoying, troublesome, and oftentimes dangerous experiences which had been met with in the attempt to use the earlier forms.

Test of the New Generator. — In June, 1893, a test of the new generator was made for the purpose of determining the efficiency of this method of producing oxygen and hydrogen gas. After the apparatus had been in continuous operation for a number of hours in order to bring it into its normal condition, measurements were begun.

At the beginning of the test the temperature of the cells and the density of the acidulated water which they contained were determined. The results are given in the following table: —

TABLE I.

Cell.	Temperature of electrolyte.	Density of electrolyte.	Cell.	Temperature of electrolyte.	Density of electrolyte.
1	30° C.	1.095	7	27° C.	1.085
2	31°	1.115	8	28°	1.080
3	28°	1.112	9	29°	1.095
4	32°	1.115	10	31°	1.130
5	34°	1.155	11	31°	1.090
6	30°	1.160	12	33°	1.149

According to this table, the average temperature of the electrolyte within the twelve cells was 30°.4 C., the coolest cell indicating 27°, and the hottest 34°. The average density of the electrolyte was 1.138, with a range in different cells from 1.080 to 1.160.

The temperature of the water entering the cooling trough surrounding the cells was found to be 21°.3; that of the water in the efflux tube was 23°. The rate at which water was supplied to the trough was determined by weighing that received during a given time (235 seconds). The amount was 60.34 kilograms, or 256.7 grams, per second.

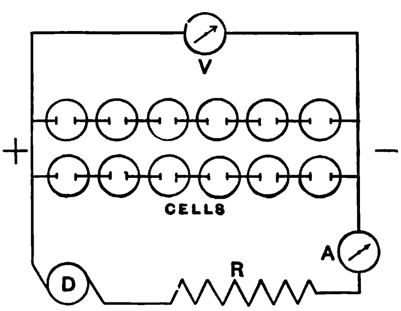

Fig. 4.

The cells of the generator were connected in two groups of six, in parallel (see Fig. 4). They were in series with a rheostat (*R*); the whole being in the circuit of a shunt-wound dynamo (*D*), giving 120 volts at the brushes. The rheostat was adjusted so as to secure a total current of about 49 amperes. *A* and *V* in the diagram are the ammeter and voltmeter.

The gases were collected over water in two large gasometers. These were partly filled previous to the beginning of the test, and the water through which the gases passed, as well as that within the generator cells, was thoroughly saturated. The amount of gas collected during the continuous run of 9000 seconds was computed from the vertical distance through which the tanks were lifted, their contents per centimeter of vertical height, the pressure, and the temperature. The following are the essential data: —

TABLE II.

Hydrogen tank; circumference (inside)	383.540 cm.
Hydrogen tank; area of cross-section	11706.000 sq. cm.
Hydrogen tank; total rise	28.880 cm.
Hydrogen tank; gas received in 9000 seconds . . .	338.071 cu. cm.
Oxygen tank; circumference (inside)	384.050 cm.
Oxygen tank; area of cross-section	11733.000 sq. cm.
Oxygen tank; total rise	14.450 cm.
Oxygen tank; gas received in 9000 seconds	169.541 cu. cm.
Temperature of H gas, within the tank	24°.0 C.
Temperature of O gas, within the tank	24°.0 C.
Pressure of H gas, within the tank	1 atm. + 2.683 cm.
Pressure of O gas, within the tank	1 atm. + 2.683 cm.

The identity in the temperature of the two gases was doubtless the result of their passage through the water of the gasometers, which, standing within the two tanks, undisturbed for many months, in a room subjected to slight and very gradual changes of temperature, was naturally in the same condition in the two gasometers. The pressure was adjustable by means of water ballast in a cylindrical compartment at the top of each tank. The exact identity of the two readings for pressure is, of course, a mere coincidence.

Throughout the course of the test, frequent readings of voltage and total current were made, as follows: —

TABLE III.

Time.	Amperes.	Volts.	Time.	Amperes.	Volts.
1 h. 44 m.	48.8	115.0	3 h. 10 m.	49.1	114.0
1 54	49.3	116.0	3 18	49.0	114.0
2 09	49.0	115.0	3 24	49.2	114.5
2 15	48.7	114.5	3 38	49.4	114.5
2 25	49.0	115.0	3 47	48.6	113.5
2 35	50.0	117.0[1]	3 55	49.4	114.5
2 40	48.5	113.0[1]	4 05	49.5	115.0
2 46	48.8	114.0	4 15	49.0	114.5
3 00	50.0	116.0			

The average current was taken from these readings to be 49.2 amperes: the average voltage, 114.1 volts; and the total number of watts, 5614. The initial and final measurements of the positions of the gasometer cylinders were made at 1 h. 44 m. and 4 h. 14 m., respectively, which gave for the total length of run 2 h. 30 m. = 9000 s.

From the above data the capacity and efficiency of the generator are readily computed. Considering, for convenience, each pair of cells in the double series of six as a single cell, we have as the output per pair at 0° and 76 cm. (with correction for vapor tension) : —

Liters of hydrogen in 9000 seconds	51.443
Liters of oxygen in 9000 seconds	25.721

Production in each of the twelve cells in 1 second: H, 2.8413 cu. cm.; O, 1.4290 cu. cm.

The ratio of these volumes is

$$\frac{51.443}{25.721} = 1.9940,$$

a value which is as near to the theoretical ratio as one could expect with any ordinary water voltameter.

The volumes of gas which should be produced by 49.2 amperes of current in 9000 seconds, assuming the electro-chemical equivalent of hydrogen to be 0.0000104 grams per second per ampere, are: —

Hydrogen: 52.110 liters in 9000 seconds;
Oxygen: 26.055 liters in 9000 seconds.

The agreement of the observed volumes with these values is within the errors of our knowledge of the pressure and temperature of the gases.

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[1] A temporary value holding only for a few seconds.

The Efficiency of the Generator. — The efficiency of the generator may be readily computed from the mechanical equivalent of the heat of combination of the gases which are produced. Assuming the heat of combustion of hydrogen to be 34,200, we get as the output of the generator 48.74 kilogrammeters per second. The energy corresponding to 5614 watts is, however, 572.3 kilogrammeters per second, the ratio of which gives for the efficiency of the apparatus —

$$\frac{48.74}{572.3} = 0.0851.$$

This is a lower efficiency than that obtained by Professor Anthony with the porcelain cell generator in 1884.[1] He obtained, with an expenditure of 3104 watts, gas enough to account for 429 watts, which corresponds to an efficiency of 0.138. With a much smaller output the efficiency rose to 0.24. The test which has been described in this paper was purposely made with the apparatus at its maximum of production, which is the condition of its daily use.

No attempt was made to make determinations of the losses, but roughly speaking, they may be accounted for as follows : —

Estimating the amount of water delivered to the cooling trough and its rise of temperature, from data given in a previous paragraph, we find a large amount of energy taken away. During the test, also, the temperature of the electrolyte rose rapidly in spite of the cooling bath which surrounded the lower portions of the cells. Some time previous to the close of the experiments measurements showed the various cells to have risen to temperatures between 50° and 60°. The amount of water in the twelve cells was approximately 157 kilograms, and it is probable that the average rise in temperature was about 30°.

We may therefore estimate roughly the expenditures as follows : —

(1) Expended in heating the cooling bath	212	kgm. per second
(2) Expended in heating the electrolyte	222	kgm. per second
(3) Expended in generating gas	48.74	kgm. per second
Total	482.74	

This estimate leaves only about one-sixth of the total energy unaccounted for, an amount which would probably be found in the loss of heat by various parts of the apparatus to the surrounding air.

Under the conditions which these results indicate it is not strange that the manufacture of oxygen and hydrogen, which at one time appeared to be likely to take on commercial importance, has been superseded by other and more economical methods.

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[1] Anthony, Proceedings of the American Association for the Advancement of Science, Vol. 33, p. 115 (1884).

June, 1893.

THE FREEZING-POINTS OF DILUTE SOLUTIONS.—A CORRECTION.

By E. H. Loomis.

IN the curves of results published on page 287 of the preceding number of the Physical Review in connection with the article on the "Freezing-Points of Dilute Solutions," the curve which gives the *theoretical* values for magnesium-sulphate was found to be incorrect after the proofs were returned to the publisher. It was thought better to indicate the error in a subsequent number of the Review than to change the curve itself.

The correct numerical data from which the curve was inaccurately constructed are as follows: —

m	$\frac{\Delta}{m}$
0.01	2.98
0.025	2.81
0.05	2.69
0.10	2.59
0.25	2.46

THE EFFECT OF EXTREME COLD ON MAGNETISM.

By Louis W. Austin.

SOME years ago Professor Trowbridge[1] stated, in an article in the *American Journal of Science*, that steel bars magnetized at ordinary temperatures lose the greater part of their magnetism when subjected to the temperature of solid carbon dioxide and ether. This statement has remained, as far as I know, until now unchallenged. A recent experiment carried on in the Physical Laboratory of the University of Wisconsin would, however, seem to place it in doubt.

This experiment was as follows: A steel knitting-needle, 22 cm. long and 0.16 cm. in diameter, was magnetized to saturation, and, after being smartly tapped a few times to rid it of its temporary excess of magnetism, was placed in a wooden trough, being sprung into place in such a way as to be held fast by its ends about half-way between the bottom and the top. This was placed at right angles to the magnetic meridian in front of a magnetometer, whose deflections were read by the usual mirror and scale method. The trough was then filled with solid carbon dioxide, and ether was poured upon it.

Three series of observations were taken.

The first time that the freezing mixture was applied there seemed to be a slight decrease (about 1 per cent) in the strength of the magnet.

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[1] J. Trowbridge, American Journal of Science, Vol. 21, p. 316 (1881).

The results in this first series were very uncertain, however, on account of the fatigue of the silk suspension of the magnetometer needle. This was, therefore, replaced in the other two series by a quartz fiber, which proved much more satisfactory in its action.

The results of the third series, which agreed almost perfectly with the second, are given in the following table: —

Time.	Deflections.	
3 o'clock, 12 min.	39.9 cm.	Magnet in place.
13	40.0	CO_2 applied.
15	40.1	Ether added.
17	40.4	
20	40.4	
25	40.5	
30	40.6	
35	40.7	
40	40.9	
4 o'clock, 00	41.3	Freezing mixture removed.
15	41.3	

At the end of these observations the zero of the magnetometer was found to have moved from 0 to 0.6 cm. This was probably due to the change in the magnetic condition of the building on account of the cooling of the iron steam pipes. The results would indicate that the magnetic moment of a permanent magnet, magnetized at ordinary temperatures, becomes slightly greater when the temperature of the magnet is reduced to that of solid carbon dioxide and ether.

These conclusions are in marked contrast to those of Professor Trowbridge, who found that a bar magnetized at 20° C. lost 66 per cent of its magnetic strength after being placed for forty-three minutes in a bath of the same freezing mixture as that used in my experiment.

Physical Laboratory of the University of Wisconsin,
January, 1894.

NOTE.

Heinrich Hertz. — On the first of January, 1894, died Heinrich Hertz, foremost of the younger physicists of Germany. Hertz was born in Hamburg in 1857. He received his early training in the engineering schools, but at the age of twenty-one he decided upon an academic career, and entered the University of Berlin as a pupil of von Helmholtz and Kirchhoff.

Of the many gifted students of physics who have come forth from the celebrated Berlin laboratory, there are no others who have become so famous as Heinrich Hertz became before the end of his too short life. His qualities as an investigator were speedily recognized by von Helmholtz, who urged him, while still a student, to undertake the solution of the prize problem proposed by the Berlin Academy of Sciences in 1879.

The subject presented by the Academy was *the experimental establishment of a relation between the electro-dynamic forces and the dielectric polarization of insulators.* The solution seemed at that time unattainable, and Hertz selected for his thesis, presented in the following year upon receiving the degree of doctor of philosophy, another subject, viz. *Induction in Rotating Spheres.* The fact that his attention was called to the Academy theme, although fruitless at the time, was to be of the greatest significance; for it appears that he carried the matter in his mind continually for seven years,[1] and ultimately built upon it, as it were, his crowning work.

From 1880 to 1883 Dr. Hertz was an assistant in von Helmholtz's laboratory; he then lectured for two years as *privat docent* at Kiel. From 1885 to 1889 he was professor of physics in the Polytechnicum of Karlsruhe. In the latter year Clausius died, and Hertz was selected as his successor in the University of Bonn, where he spent the few remaining years of his life.

Hertz's career as an investigator covers a period of scarcely more than ten years, during which time he published thirty-six papers. Of these, a series of thirteen, which appeared in *Wiedemann's Annalen,* were upon the subject with which his name will forever be connected, the laws of the propagation of electro-magnetic induction through space. Of this great work, which afforded a complete experimental verification of the Maxwellian theories concerning electro-magnetism and the relation of electricity to light, there is no need to speak. We have all witnessed the interest which it aroused, and have shared in the enthusiasm, unparalleled in the recent annals of physics.

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[1] See the introduction to his work, Die Ausbreitung der Elektrischen Kraft, p. 1.

The importance of Hertz's contributions to this great subject received instant recognition. It would, indeed, be difficult to find any other instance in which researches bearing upon a most subtle and difficult question, and absolutely devoid of all elements of a utilitarian or even of a popular character, have secured to their author such sudden fame. In addition to the recognition of those who were able to appreciate his work, Hertz received the acclamations of the entire world of thought. Fortunately he possessed a nature of such complete simple-mindedness that his sudden rise into a position akin to notoriety had no effect upon him. The unassuming bearing which had always characterized him remained with him to the end.

In delightful harmony with the genuine and simple nature of the man were his surroundings in the quiet university town of Bonn. His laboratory was situated in the apartments formerly occupied as a dwelling by Clausius in a wing of the old palace. Since electricity has become utilitarian, we find it associated everywhere with moving machinery, and with the rush and bustle of modern industrial life, but in Hertz's laboratory there was nothing to suggest electro-technics. The place seemed to breathe that spirit of academic repose which to the inmates of most of the electrical laboratories of the present day must have seemed to have vanished altogether from the world. What might not such a man, in such an environment, have been able to mature and bring to fruition had he lived!

In 1892, at the request of the publishers of *Wiedemann's Annalen*, Hertz's researches upon electrical waves were gathered together in a volume, under the title "Untersuchungen ueber die Ausbreitung der Elektrischen Kraft." These papers, as they appeared, had been translated into English by De Tunzelmann. They were published in the *Electrician*.[1] Almost on the day of his death another excellent translation of Hertz's researches appeared. This volume[2] presents them in their final form, with the interesting introduction written by the author for the German version, a preface by the translator, Professor D. E. Jones, and a note by Lord Kelvin.

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[1] The Electrician. London. Vols. 21 to 25.

[2] Electric Waves: being researches on the propagation of electric action with finite velocity through space. By Heinrich Hertz. (Authorized English translation by D. E. Jones, B.Sc.) Macmillan, 1893.

The following is a chronological list of the works of Heinrich Hertz:—

1880.

1. Ueber die Induction in rotirenden Kugeln. *Inaugural Dissertation*, Berlin, 93 pp.
2. Versuche zur Feststellung einer oberer Grenze für die Kinetische Energie der electrischen Strömung. *Wiedemann's Annalen*, 10, p. 414.

1881.

3. Ueber die Vertheilung der Electricität auf der Oberfläche bewegter Leiter. *Wiedemann's Annalen*, 13, p. 266.
4. Obere Grenze für die Kinetische Energie der bewegten Electricität. *Wiedemann's Annalen*, 14, p. 581.

1882.

5. Ein neues Hygrometer. *Physikal. Gesellschaft Berlin*, pp. 18–19 (1882).
6. Ueber die Berührung fester elastischer Körper. *Crelle's Journal*, 92, pp. 156–171.
7. Ueber die Härte der Körper. *Physikal. Gesellschaft Berlin* (1882).
8. Ueber die Spannung des gesättigten Quecksilber Dampfes. *Physikal. Gesellschaft Berlin* (1882); *Wiedemann's Annalen*, 17, p. 193.
9. Ueber die Berührung fester elastischen Körper. *Verein zur Beförderung des Gewerbfleisses*, November, 1882. [Contains a recapitulation of No. 5, and experiments in verification of the results therein obtained.]

1883.

10. Ueber die Vertheilung der Druckkräfte in einem elastischen Kreiscylinder. *Zeitschrift für Math. Physik.*, pp. 125–128 (1883).
11. Ueber die continuirlichen Ströme, welche die Flutherragende Wirkung der Gestirne im Meere veranlassen muss. *Physikal. Gesellschaft Berlin* (1883).
12. Dynamometrische Vorrichtung von geringem Widerstand und verschwindender Selbstinduction. *Zeitschrift für Instrumentenkunde*, 3, p. 17.
13. Eine Erscheinung beim Ueberschlagen des Funken eines Inductoriums oder einer Leydener Flasche durch mässig verdünnte trockene Luft. *Physikal. Gesellschaft Berlin* (1883); *Wiedemann's Annalen*, 19, p. 78.
14. Versuche über die Glimmentladung. *Wiedemann's Annalen*, 19, p. 782.
15. Ueber das Verhalten des Benzins als Isolator und als Rückstandsbildner. *Wiedemann's Annalen*, 20, p. 278.

1884.

16. Graphische Methode zur Bestimmung der adiabatischen zustandsänderungen feuchter Luft. *Meteorologische Zeitschrift* (1884), pp. 421–431.
17. Gleichgewicht schwimmender elastischen Platten. *Wiedemann's Annalen*, 22, p. 449.
18. Beziehungen zwischen den Maxwell'schen elektrodynamischen Grundgleichungen und den Grundgleichungen der gegnerischen Elektrodynamik. *Wiedemann's Annalen*, 23, p. 84.

1885.

19. Dimensionen des Magnetischen Pols in verschiedenen Maasssystemen. *Wiedemann's Annalen*, 24, p. 114.

1887.

20. Sehr schnelle electrische Swingungen. *Wiedemann's Annalen*, 31, pp. 421 and 543.
21. Ueber einen Einfluss des Ultravioletten Lichtes auf die Electrische Entladung. *Berliner Berichte* (1887), pp. 487–490; *Wiedemann's Annalen*, 31, p. 983.
22. Ueber Inductionserscheinungen hervorgerufen durch die elektrischen Vorgänge in Insolatoren. *Berliner Berichte* (1887), pp. 885–896.

1888.

23. Ueber die Ausbreitungsgeschwindigkeit der Elektrodynamischen Wirkungen. *Berliner Berichte* (1888), pp. 197–210; *Wiedemann's Annalen*, 34, p. 551.
24. Ueber Strahlen elektrischen Kraft. *Berliner Berichte* (1888), pp. 1297–1307.
25. The Forces of Electric Oscillations, treated according to Maxwell's Theory. *Nature*, Vol. 38, pp. 402–404, 450–452, and 547–548.
26. Einwirkung einer geradlinigen Schwingung auf eine benachbarte Strombahn. *Wiedemann's Annalen*, 34, p. 155.
27. Inductionserscheinungen durch die electrischen Vorgänge in Isolatoren. *Wiedemann's Annalen*, 34, p. 273.
28. Electrodynamische Wellen im Luftraume und deren Reflexion. *Wiedemann's Annalen*, 34, p. 609.

1889.

29. Ueber die Beziehung zwischen Licht und Electricität. 27 pp. Bonn, E. Strauss.
30. Electrische Schwingungen nach der Maxwell'schen Theorie. *Wiedemann's Annalen*, 36, p. 1.
31. Strahlen electrischer Kraft. *Wiedemann's Annalen*, 36, p. 769.
32. Fortleitung electrischer Wellen durch Drähte. *Wiedemann's Annalen*, 37, p. 395.

1890.

33. Ueber die Grundgleichungen der Electrodynamik für ruhende Körper. *Göttinger Nachrichten*, pp. 106–149; *Wiedemann's Annalen*, 40, p. 577.
34. Grundgleichungen der Electrodynamik für bewegte Körper. *Wiedemann's Annalen*, 41, p. 369.

1891.

35. Mechanische Wirkungen electrischer Drahtwellen. *Wiedemann's Annalen*, 42. p. 407.

1892.

36. Ueber den Durchgang der Kathodenstrahlen durch dünne Metalschichten. *Wiedemann's Annalen*, 45, p. 29.

E. L. N.

NEW BOOKS.

Utility of Quaternions in Physics. By A. McAulay, M.A., Lecturer in Mathematics and Physics in the University of Tasmania. pp. xiv, 107. London, Macmillan & Co.

The volume before us is an essay that was submitted in December, 1887, in competition for the Smith's Prizes at the University of Cambridge, under the title of "Quaternions as a Practical Instrument of Physical Research." An article bearing the original title of the essay appeared in the *Philosophical Magazine* for June, 1892, and another extract was printed in the *Proceedings of the Royal Society of Edinburgh*, 1890–91, p. 98, under the title of "Proposed Extension of the Powers of Quaternion Differentiation." The present volume contains the complete essay, with a short preface and some foot-notes in addition.

The essay opens as follows: "It is a curious phenomenon in the History of Mathematics that the greatest work of the greatest mathematician of the century which prides itself upon being the most enlightened the world has yet seen, has suffered the most chilling neglect." In further description of this phenomenon, it is stated that the work has been neglected alike by pure analysts and mathematical physicists with very few exceptions, the grand exception being Professor Tait; that it is not studied at the University of Cambridge except by a few, and only as a non-commutative algebra, not as a geometrical method; that there is a solid and well-nigh universal scepticism as to its utility in original physical investigations, and that the physicists who have studied it are satisfied with Maxwell's paradoxical position: "I am convinced that the introduction of the ideas, as distinguished from the operations and methods of quaternions, will be of great use to us in all parts of our subject." To complete the description of the phenomenon, I may add that a Scottish mathematician, on reading Hamilton's *Quaternions*, first formed the alternative conclusion that either he himself was a dull stupid or the book sheer nonsense, but on reading further was able to arrive at the more comforting alternative; that a German mathematician declared the method to be "an aberration of the human intellect"; and that a French mathematician gave the verdict, "Quaternions have no sense in them, and to try to find for them a geometrical interpretation is as if one were to turn out a well-rounded phrase, and were afterwards to bethink oneself about the meaning to be put into the words."

How does the author explain the curious phenomenon? As follows: "Workers naturally find themselves, while still inexperienced in the use of quaternions, incapable of clearly thinking through them and of making them do the work of Cartesian geometry, and they conclude that quaternions do not provide suitable treatment for what they have in hand. They then grow rather disgusted with these vexatious quaternions, and, consoling themselves with the reflection that Maxwell, before penning the above extract, had had more experience than themselves, decide that the subject only requires a superficial study to be rendered of as great utility as it is capable." But the author admits that there is a veritable stumbling-block in the way, and to remove it is the object of the essay. He says, p. 2: "The fact is that the subject requires a slight development in order readily to apply to the practical consideration of most physical subjects. The first steps of this, which consist chiefly in the invention of new symbols of operation and a slight examination of their chief properties, I have endeavored to give in the following pages." The author's development consists in an extension of quaternion differentiation, pp. 12–24. According to Hamilton and Tait, the symbol $\nabla = i\frac{d}{dx} + j\frac{d}{dy} + k\frac{d}{dz}$ is merely an operator, and therefore should be written immediately to the left of the operand; but, according to Mr. McAulay, it is a *symbolic vector*, and therefore is capable of any position, whether before or after the variable. He denotes the tie between the symbolic vector and the variable, not by juxtaposition to the left, but by a common suffix. In the third edition of his *Treatise*, Professor Tait allows separation, but he stickles for separation to the left only. A new symbolic vector Δ is introduced; it applies to all the variables in the term in which it appears. The symbol ∇ with a prefix, as $\sigma\Delta$, means $\frac{d}{du}i + \frac{d}{dv}j + \frac{d}{dw}k$, σ being $ui + vj + wk$. Similarly, ϕ being a linear vector function of any vector whose co-ordinates are $a_1b_1c_1$, $a_2b_2c_2$, $a_3b_3c_3$, $\phi\square$ is defined as a symbolic linear vector function, whose co-ordinates are $\frac{d}{da_1}\frac{d}{db_1}\frac{d}{dc_1}$, $\frac{d}{da_2}\frac{d}{db_2}\frac{d}{dc_2}$, $\frac{d}{da_3}\frac{d}{db_3}\frac{d}{dc_3}$. Finally, a symbolic vector ζ is introduced, which is such that $Q\ (\alpha, \beta)$ being linear in each of the vectors α, β,

$$Q(\zeta, \zeta) = Q(\nabla_1, \rho_1) = Q(i, i) + Q(j, j) + Q(k, k).$$

The remainder of the essay consists of an application of the quaternion analysis so developed to the theories of elastic solids, electricity and magnetism, and hydrodynamics. It is almost wholly a translation into quaternion notation of known results: the author, however, has endeavored to advance each of the theories chosen in at least one direction. The work shown is designed to make good the following statements: *first*, that quaternions are in such a stage of development as already to justify the practically complete banishment of Cartesian geometry from physical questions of a general nature; and *second*, that quaternions will in physics produce many new results that cannot be produced by the rival and older theory. But in the preface, the author now states that he delayed publication until he could by a more striking example than any in the essay show the immense utility of quaternions; this he believes has been done by a paper published in the *Philosophical Transactions* for 1892. At the time of writing the essay he possessed little more than faith, and he felt that something more than faith was needed to convince scientists. In conclusion he exhorts mathematical physicists to study quaternions seriously, and he looks forward to the time when quaternions will appear in every physical text-book that assumes the knowledge of elementary plane trigonometry.

I agree with the author in his estimate of the value of Hamilton's quaternion researches: they constitute, in my opinion, the greatest mathematical work of the century. They contain what was long sought after — *a* veritable extension of algebra to space: I do not say *the*, for I believe that there is more than one. The Cartesian analysis is also an extension of algebra to space, but it is fragmentary and incomplete; whereas the quaternion analysis is the true spherical trigonometry in which the axis of an angle as well as its magnitude is considered.

But I cannot agree with the author in his explanation of the comparative neglect which the work has hitherto received. The notation has been a stumbling-block. Familiar functions, such as the cosine and sine, are replaced by new selective symbols S and V; Greek letters are used alike for axes, vectors, versors, and functions of these. As a consequence, the notation is contracted; to make it more expansive, Maxwell introduced German capitals for vectors, Mr. Heaviside used black letters instead, and in the work before us we have a rather incongruous mixture of Greek and black letters. The notation has divorced the method from the rest of analysis, so much so that Mr. McAulay believes that it is an independent plant which cannot be grafted on the old tree of analysis (*Philosophical Magazine*, Vol. 33, p. 479).

The identification of vectors and quadrantal versors has been a stumbling-block. Quaternion analysis is nothing but spherical trigonometry in which the axis of the angle is explicitly denoted; this will become clear to any one who studies the fundamental rules and the manner in which Hamilton arrived at them. It is the analysis of directed ratios. The identification mentioned assumes that it is also the analysis of lines, areas, volumes, and other physical products; it leads to the paradox that the square of a vector is essentially negative, and to a total disregard of the dimensions of physical quantities. Here no doubt we have the reason why Maxwell accepted the ideas but not the operations and methods. The analysis which the physicist mostly requires is not identical with, but complementary to, the analysis of ratios.

The largely *symbolic* character of the method has been a stumbling-block. Neither the definitions nor the rules have been placed on a clear and unambiguous basis. The fundamental rules are based, sometimes on the addition of quadrantal arcs, sometimes on the quadrantal rotation of an axis. The notation ∇ is presented as symbolic, so are the author's $\overline{\nabla}$, ζ, and Ɑ ; at most they are shorthand rather than systematic and logical notation. Need we wonder that competent mathematicians cannot think clearly through quaternions, for the original writers do not pretend to do so.

The rival or antagonistic attitude towards the Cartesian analysis has been a stumbling-block. Not only does the quaternion plant, according to Mr. McAulay, require independent sowing, but he would have us pull up the old Cartesian tree with its multitude of branches and far-spreading roots, in order to make room for the new plant. But when he comes to consider more specifically how much should be pulled up, he encounters a difficulty. Thus, p. 3, he says, "For particular problems, such as the torsion problem for a cylinder of a given shape, we require of course the various theories specially constructed for the solution of particular problems, such as Fourier's theories, complex variables, spherical harmonics, etc. It will thus be seen that I do not propose to banish these theories, but merely Cartesian geometry." If, then, the quaternion analysis fails, and the problem is turned over to the theory of complex variables (as at p. 49), it is important that these two branches of analysis should be logically harmonious and free from contradiction in matters of convention. If they are logically harmonious, it will be easy for a student or analyst to pass from the one to the other; but, as a matter of fact, the conventions are contradictory. Is not this the very meaning of the author's metaphor of the independent plant that cannot be grafted on the already flourishing tree? In several papers recently published I have aimed at showing how this logical harmony may be brought about, and one space-analysis be developed which shall embrace algebra, trigonometry, complex numbers, Cartesian analysis, Grassmann's method, and quaternions. Till this harmony is established the ideas and methods of Hamilton will not bring forth the great results which exist in them potentially.

ALEXANDER MACFARLANE.

Notes on Recent Researches in Electricity and Magnetism: intended as a sequel to Professor Clerk Maxwell's Treatise on Electricity and Magnetism. By J. J. THOMSON, M.A., F.R.S. Oxford, Clarendon Press, 1893.

The object of this volume is to give an account of the advances made in electricity and magnetism since the appearance of Maxwell's *Treatise;* thus the student may retain that treatise as the source from which to obtain the fundamental principles of the science and yet have a treatment which is thoroughly up to date. There is much in the book that is original; it opens with an entirely new view of the electromagnetic field, a physical theory which satisfies Maxwell's equations, and yet differs from the displacement theory. The importance and interest of the subject justify a brief sketch of it.

It is now accepted that if an electrified body moves through a dielectric, it produces the same external effects as an electric current. Of course tubes of electrostatic force — or, as our author calls them, Faraday tubes — terminate on this electrified body, and move with it. We have then a magnetic field accompanied, and apparently caused, by the motion of electrostatic tubes of force. May it not be that a magnetic field is nothing more nor less than the *motion of electrostatic tubes of force?* This is the essence of the view advanced by Professor Thomson.

The ether is supposed to be filled with Faraday tubes, which are regarded as incapable of creation or destruction. Some of these tubes form closed curves; others extend from an atom having a small positive charge to one having an equal negative charge. The existence of the closed tubes is necessary to account for the presence of a magnetic field in the absence of electrostatic charges, and moreover it is impossible to explain Rowland's celebrated experiment on convection currents without some such aid.

In speaking of the termination of a Faraday tube the word *atom* was used advisedly. All tubes have the same strength; the charge at each end is that which we associate with a univalent element in electrolysis. We certainly meet with no fractional part of this unit in electrolysis, and it is assumed we do not elsewhere. When the length of a Faraday tube joining two atoms is comparable with the distance between the atoms of a molecule, these atoms are said to be chemically united. The seat of an electric charge is always a dissociated atom, never a molecule. All the Faraday tubes the atoms of a molecule can accommodate are needed to hold them together in the molecule. The physical phenomenon of electrification differs from chemical combination only in the length of the Faraday tubes. In the case of liquids and gases there is good experimental evidence that the molecule cannot be charged, that the seat of a charge of electricity always a dissociated atom. A digest of this evidence is given in Chapter II.

It is supposed that there is a motion of rotation inside and around the Faraday tubes, even when they themselves are at rest. The energy of this motion constitutes the potential energy of the electrostatic field. When the tubes are in motion, we have another form of energy, that of the magnetic field. Though these tubes are nowhere so called, one inevitably thinks of them as vortices. The interpretation of Maxwell's equations on this theory shows that a straight Faraday tube, moving at right angles to itself, generates a magnetic force at right angles both to itself and to its direction of motion.

Enough has been said to give a general idea of the theory. The reader must be referred to the book itself to learn how the inertia of the Faraday tubes causes an induced current, how, in case of two parallel currents, each shields the other from the bombardment of the tubes so that attraction results, the tubes playing a part similar to that of Le Sage's ultra-mundane particles. The action of a galvanic cell in producing a current is described, as well as the process by which light is transmitted. This resembles Newton's Emission theory, his symmetrical particles being replaced by Faraday tubes, thus making possible the explanation of polarization phenomena. This theory is of great value, not only because it gives a clear picture of what may be taking place in electromagnetic phenomena, but also because it sounds a loud note of warning against accepting any such view as an accurate representation of what actually does take place, as a photograph, so to speak, of the mechanism involved. In his *Modern Views of Electricity*, Oliver Lodge is guarded in his statement, speaking of electricity as "a form or rather a mode of manifestation of the ether." Yet doubtless more than ninety-nine out of every hundred of his readers have regarded the identity of electricity and the ether as practically proved. According to the view of Professor Thomson, electricity is the end of a tube of ether moving in a peculiar way; and to-morrow may bring forth another view which will fit the phenomena just as well. The theory advanced by our author is rich in promise, especially because it gives us a mental picture of chemical affinity.

Let us glance hastily at the rest of the book. In Chapter II. is an excellent summary of experimental work on the electrical behavior of gases; a field which is full of promise and has received altogether too little attention. Then comes a discussion of conjugate functions and of electrical waves and oscillations. Naturally following this is a description of the experiments of Hertz, and of the work which they have inspired. The literature of this subject is already bulky: hardly a number of Wiedemann's *Annalen* comes out without at least one paper which depends on the work of Hertz.

Such a summary as we have here, which gives the salient points of these researches in logical sequence, is of great value. Chapter VI. contains Lord Rayleigh's treatment of the distribution of rapidly alternating currents, and the last chapter treats of the electromotive intensity in moving bodies.

When possible, it is wise for the student to depend on libraries for many of his books; this volume does not belong to that class. In order of choice among electrical works it should come next to Maxwell's *Treatise.*

NORTHWESTERN UNIVERSITY. HIRAM B. LOOMIS.

Physikalisch-chemische Tabellen. Edited by HANS LANDOLT and RICHARD BÖRNSTEIN. Second Edition. Large 8vo, pp. xi + 563. Berlin, Julius Springer, 1894.

In spite of the great advances which have been made in all branches of physical and chemical science during the ten years which have passed since the first appearance of these well-known tables, this book has continued to maintain its position as the most useful and comprehensive of its class. Frequent revision in the case of a book of this kind is, however, a matter of absolute necessity. New and more reliable determinations of important constants are continually being made, and no book of tables which does not contain the results of these determinations can be in the highest sense useful for reference. It is not surprising, therefore, that the need of revision in the case of Landolt and Börnstein's Tables has for some time past been strongly felt. The great labor involved in the preparation and revision of a work of this kind is, however, a sufficient explanation of the delay in the appearance of the second edition. Indeed, were it not for the assistance of the seventeen collaborators whose names appear on the title-page the appearance of the book in its present form would doubtless have been altogether impossible.

In comparing the present edition with that of 1883, the reader will first be struck with the great increase in the size of the book. The number of pages has in fact been more than doubled. This increase has been brought about in three ways; viz. (1) the tables which appeared in the original edition have been enlarged by the addition of new data; (2) a number of new tables, dealing with subjects not treated in the first edition, have been added; (3) the number of references to the literature of each subject considered has been very largely increased. As stated in the preface, the book now contains not only a reference to the authority for each number given in the tables, but also references to numerous other determinations, whose results have for various reasons not been quoted. These references are given at the close of each table, or group of tables, and constitute by no means the least valuable part of the book.

To mention in detail the changes which have been made in the tables of the first edition would be a tedious and unnecessary task. Suffice it to say that the book has been thoroughly modernized by the insertion of the results of the most recent and reliable determinations. The following examples will serve to indicate how complete this revision is, and how greatly the tables have been enlarged.

In Table 1 (atomic weights) the results collected by Meyer and Seubert, by Ostwald, and by F. W. Clarke, are given in the tables proper, while the results of numerous determinations made in 1892–93 are given in the foot-notes. The atomic weights are referred both to $H = 1$ and to $O = 1$, thus enabling the reader to avoid the annoying uncertainty as to the atomic weight of oxygen. In the list of references following this table are collected the results of some twenty determinations of the atomic weight of oxygen, with a statement in each case of the method employed.

Tables 38–42, dealing with condensed gases, have been greatly enlarged. The latter table now contains not only the values of the critical temperature and pressure, but also the corresponding values of the volume and density. Data are given in this table for 150 substances. The following tables may also be mentioned as showing special enlargement: 74–84, on the Specific Gravity of Solutions and Mixtures; 118–121, on the Viscosity of Liquids and Gases; 142, 143, on Heats of Combustion; and 165, on Indices of Refraction for Organic Compounds. In the last two cases the increase is from two to fifteen pages, and from three to fourteen pages, respectively.

In Table 133, on the Specific Heat of Water, the recent determinations of Bartoli and Stracciati (1891) are added. The reader will be somewhat surprised, however, to observe that Rowland's results have been omitted.

The table which shows the most complete revision is perhaps No. 152, which deals with the Wave Lengths of the Spectral Lines. This table has been prepared by Professor Kayser, and is so thoroughly modernized that it might almost be considered new. For some ninety lines of the visible and ultra-violet spectrum, the results of Rowland's most recent determinations (1893) are given. A smaller table contains a comparison of the results of Fraunhofer, Ångström, Cornu, and Rowland. The value of this table naturally lies chiefly in its historical interest. Following these are tables containing the ultra-violet lines of iron (Kayser and Runge, 1890) and the more important lines of hydrogen (Ames, 1890). Next come lists of the principal lines in the spectra of the alkalies, and similar lists for thirteen other metals. These tables, which are highly satisfactory as far as they go, might well have been considerably enlarged. For example, the infra-red lines of the alkalies could to advantage have been added, while a table dealing with the absorption spectra of the rare earths, and in fact with absorption spectra in general, would have been in the highest degree useful.

Table 153, which has been prepared by Dr. Traube of Berlin and deals with the indices of refraction of isotropic substances, contains many additions which will be hailed with satisfaction. This table now contains, for example, the results of Kundt, Drude, and others for the indices of refraction of the metals. The results of Rubens and Snow for the dispersion of fluor-spar in the infra-red are given in detail, and on account of the many advantages which this substance has been shown to possess for work in the invisible spectrum, will doubtless prove valuable for reference. Determinations by Langley, Rubens, and Snow, and others of the dispersion of rock salt, have also been added.

In the tables dealing with electrical data the improvement due to revision is not so marked as in other parts of the book. The tables of electrical conductivity, although comprehensive, will be found inconvenient. The tables, with one exception, refer to conductivity, and not to resistance. Yet in nine cases out of ten it is the resistance which is required in computation. Again, all results are expressed in terms of the conductivity of mercury. A troublesome computation is therefore necessary in order to reduce to absolute or practical units, — a computation which is rendered more annoying by the fact that the absolute conductivity of mercury is nowhere conspicuously stated. The reader must either choose a value from those incidentally mentioned in the foot-notes, or be content with value of the absolute *resistance*, which is given to three places in Table 194.

An idea of the character of the additions that have been made in the present edition may be obtained from the following greatly abbreviated list of new tables: 12, Influence of *g* on Barometer Readings; 20–24, Constants of Capillarity for Various Liquids; 39–41, Melting and Boiling Points of Condensed Gases; 54, Temperature of Maximum Density for Various Salt Solutions; 87, Boiling Points of Solutions; 100, Various Constants connected with Electricity (*e.g.* Modulus and Coefficient of Elasticity, Modulus of Torsion, Elastic Limits, etc.); 126, Velocity, Free Path, etc., of the Molecules of Various Gases; 176, Constants of Electromagnetic Rotation; 188, 189, Molecular Conductivity of Solutions and Mixtures; 196, Specific Inductive Capacity. The last table has the disadvantage that the reader is left in doubt as to whether the results refer to rapid or to slow changes of charge. The table would be improved by a brief statement, after each result, of the method employed in obtaining it.

In considering the book as a whole, I think that most readers will agree that its chief fault — in fact, almost its only fault — lies in the arbitrary character of the units employed. In the preparation of the second edition the editors have had an excellent opportunity to adopt a uniform and logical system of units, as has so admirably been done in Everett's little book on "the C.G.S. System." This opportunity has, however, been neglected, and units of all sorts and kinds appear in a most arbitrary way. The use of the C.G.S. system is, in fact, the exception rather than the rule. That in some cases the choice of units other than those of the absolute system is of advantage, cannot be denied. Yet when it is remembered that the book before us is intended for the laboratory rather than for the engineer, the number of such cases becomes very small. Attention has already been called to the annoyance that will be caused by the choice of units in the case of the tables dealing with electrical conductivity. Similar remarks may properly be made with regard to the tables on thermal conductivity, on elasticity, and in fact in connection with a large number of tables throughout the book. What possible advantage, for example, can be gained by expressing thermal conductivity in terms of millimeters and milligrams? In Table 100 we find the kilogram used as the unit of force, and results are in many cases carried to five places. Yet the value of g is not given. Is not this in the highest degree misleading and unscientific?

The results of this lack of uniformity are fortunately in large part counterbalanced by the clearness with which the units employed are defined. No uncertainty can exist as to the exact meaning of each number given in the tables. The importance of this statement can scarcely be overestimated, and that with many books it cannot truthfully be made is unfortunately too well known. Almost every one who has had to do with the properties of solutions has had occasion to puzzle over the expression "a 10 per cent solution." Does this mean 1 part salt to 9 parts water, or 1 part salt to 10 parts water? Can it be that the writer intended to say that the solution was one-tenth saturated? or is a 10 per cent normal solution referred to? The only way out of the difficulty is often either a happy guess, or an annoying search for the original authority. No difficulty of this kind will, however, be met with in Landolt and Börnstein's *Tables*. The arbitrary choice of units may often cause annoyance on account of the necessity of reducing to the absolute system, but no uncertainty can arise as to the meaning of the data given. A statement, in each case, of the factor by which the numbers given must be multiplied, in order to reduce to absolute units, would, however, be a great convenience.

In conclusion it is perhaps unnecessary to say that the book will be found useful to a degree which can hardly be estimated, and that to the editors, as well as to the numerous scientists who have assisted them in their arduous task, are due the thanks alike of physicists and chemists.

ERNEST MERRITT.

An Elementary Treatise on Fourier's Series, and Spherical, Cylindrical, and Ellipsoidal Harmonics, with Applications to Problems in Mathematical Physics. WILLIAM ELWOOD BYERLY, Ph.D., Professor of Mathematics in Harvard University. 8vo, pp. 287. Boston, Ginn & Co., 1893.

The following is a brief outline of the contents: Chap. I., pp. 1–29, Introduction; Chap. II., pp. 30–54, Development in Trigonometric Series; Chap. III., pp. 55–68, Convergence of Fourier's Series; Chap. IV., pp. 69–143, Solutions of Problems in Physics by the Aid of Fourier's Integrals and Fourier's Series, and Miscellaneous Problems; Chap. V., pp. 144–194, Zonal Harmonics; Chap. VI., pp. 195–218, Spherical Harmonics; Chap. VII., pp. 219–237, Cylindrical Harmonics; Chap. VIII., pp. 238–266, Laplace's Equation in Curvilinear Co-ordinates, Ellipsoidal Harmonics; Chap. IX., pp. 267–275, Historical Summary; Appendix, pp. 277–287, Tables. Chapter I. is rather uncommon in its style as an introduction to a mathematical treatise. It is, however, an exact reproduction[1] of the lectures given by Professor Byerly, as introductory to the course. It gives a list of the differential equations whose solutions are afterwards to be discussed, and brief statements of the general properties of homogeneous, linear, partial differential equations — particularly of the principle of superposition. By means of a few simple typical problems, which are discussed at some length, the student is fairly insinuated into the spirit of the methods for finding particular solutions, and for combining them by superposition so as to satisfy given conditions. It is a very unpretentious and exceedingly effective introduction.

Tentative methods only are given for finding particular solutions. This might be slightly improved by the statement, of which the proof is simple, that any homogeneous, linear, partial differential equation with constant coefficients, and which is free from terms of the form $\frac{d^n \cdot d^m}{dx^n \cdot dy^m}$, has particular solutions which are products of functions of the separate independent variables, and that such partial differential equations can always be degraded, for such solutions, to a number of ordinary differential equations from which the respective factors may be determined. It would be easy to show whether or not such method for finding particular solutions is generally applicable to this class of partial differential equations when transformed to orthogonal curvilinear co-ordinates. It seems that the method would be found general; and if so, we would thus arrive at the most general possible conception of the genesis of harmonic functions.

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[1] The writer is so fortunate as to have heard part of the course in 1891–92, at which time Professor Byerly's notes were mimeographed for the use of the class.

The work as a whole is simple and straightforward. in style, and easily within the reach of any student who has a moderate knowledge of differential and integral calculus. The simplicity of treatment is not of the degrading kind, however; and those students who would be likely under any circumstances to pursue a subject further than is possible in an elementary course will no doubt be left fresh and clear after a careful study of Professor Byerly's work.

Dr. Maxime Bôcher in Chapter IX. (historical outline), has given a set of choice references. Any references naturally lead the student into that branch of the subject which is of greatest mathematical interest; namely, into the formal study of the various harmonic functions. The conception of these various functions as actual distributions of potential, due to certain aggregations of positive and negative attracting points and lines, is almost as great a help to the physicist as is the conception of the various terms of Fourier's series considered as the different simple modes of vibration of a stretched string. For this reason, Maxwell's chapter on Spherical Harmonics is of particular interest to students of physics.

Professor Byerly's course is, on the whole, admirable, if we may by thus ignoring the title (treatise) emphasize the true nature of the work. We disagree with the author, however, in considering the work as introductory to mathematical physics. An introduction to that subject, indeed the subject itself, must eventually be, in essence at least, a discussion more or less philosophical of the origin and significance of physical quantity, of physical measurements, of the general theory of numerical specification, and of the transformation of physical definitions into differential equations and of physical laws into functional forms, leading to more or less complete methods for obtaining numerical data and for expressing mathematically the physical conditions they imply. Physical quantity, indeed, takes its origin from the application of differential equations and functional form, so that the above outline would seem to be not strictly systematic: but the order is chosen for the sake of emphasis. We cannot refrain from urging that mathematical physics, properly so called, is the only (formal) physics; that all problem work is essentially extraneous to it, and tolerated only because we know so imperfectly how to teach, and because students know so imperfectly how to understand. Mathematical physics, often so called, would be better characterized as physical mathematics.

W. S. FRANKLIN.

An Elementary Treatise on Theoretical Mechanics. By A. Ziwet. Part I., Kinematics. 8vo., pp. 8 + 181. Part II. Introduction to Dynamics: Statics. 8vo., pp. 8 + 183. New York, Macmillan & Co., 1893.

A text-book of elementary mechanics in three volumes octavo is something of a novelty. The custom of publishing books in parts is more common in Germany than in this country, and has some advantages.

The first two parts of the book have now appeared; the third part, devoted to kinetics, is promised for the fall of 1894.

The book is intended as an introduction to the Science of Theoretical Mechanics as such, but the author has constantly kept in mind the wants of engineering students more particularly. It assumes, on the part of the student, a good working knowledge of elementary mathematics, and of analytical geometry and the differential and integral calculus. An acquaintance with determinants, elliptic integrals, and the elements of quaternions is also required.

The arrangement of the matter is severely logical, all kinematical questions being disposed of before dynamics is taken up. The whole of Part I. is devoted to kinematics.

Part II. begins with a short introduction to dynamics, but the bulk of the volume is devoted to an exposition of the subject of statics. In the treatment of statics, much attention is given to geometrical methods and graphical constructions, and this volume is, on the whole, more easily read than the first.

Throughout, the language is clear and precise, and the number of illustrative examples, more especially in the elementary parts, large and well chosen. Answers to the examples are given with here and there a hint for solution. Appended to the various chapters are references to English, French, and German text-books for the convenience of students desiring to pursue farther any matter brought under discussion. This is an excellent feature, but in some cases, as, for example, after "Motion in a twisted curve" (Part I.), or "Constraints" (Part II.), it would have been more to the student's advantage to have given a few examples. In fact, it is to books such as this that we look for illustrations tending to clear up the more difficult parts of the subject.

The most novel part, perhaps, to American readers is the manner of bridging the gulf between kinematics and dynamics. It is done in this way: "The motion of a point moving uniformly from rest in a straight line is determined by the equations

$$v = at,\ s = at^2/2,\ v^2/2 = as, \qquad (1)$$

where s is the distance passed over in the time t, v the velocity, and a the acceleration at the time t.

"If now for the single point we substitute an m-tuple point, *i.e.* if we endow our point with the mass m, and thus make it a particle, the equations (1) must be multiplied by m, and we obtain

$$mv = mat,\ ms = mat^2/2,\ mv^2/2 = mas.$$

"The product mv is called *momentum;* the product ma is called *force;* and the product $mv^2/2$ is called the *kinetic energy* of the particle."

This cuts the knot at once, and will probably be as satisfactory to the student as the labored pages of explanation usually given in connection with Newton's laws of motion. Newton's laws indeed are referred to, but only incidentally, as forming the assumptions on which the application of dynamics to natural phenomena is founded.

Few changes from the usual notation are made. This is a matter for congratulation. One innovation, however, that seems quite uncalled for is the substitution of j for a as the symbol of acceleration. The letter j being appropriated for Joule's Equivalent, it is surely better to retain the old symbol a for acceleration.

A uniform system of notation in mechanics is very desirable, and the recommendations of the International Electrical Congress recently held at Chicago should be followed as closely as possible by writers and teachers. The final report is not yet published, but a preliminary report is given in the *Electrical World* of September 22, 1893.

An important feature of a book on mechanics is that the units that enter be carefully explained. This is done with great clearness in these volumes, but in some cases rather succinctly. All dynamical formulas are expressed in absolute units, and the relations of the British and metric units pointed out. The only slip we have noticed is on p. 7, where the U.S. yard is defined as "the distance between the twenty-seventh and sixty-third divisions of the brass standard kept in the Bureau of Weights and Measures at Washington." This brass bar, by reason of its faulty construction, was never much in favor, and for a long time has been of historical interest only. The Bureau of Weights and Measures regard the U.S. yard as identical with the British yard, or else define the yard in terms of the new International Meter by the relation

$$1 \text{ yard} = 3600/3937 \text{ meter}.$$

The only system of weights and measures that has received the sanction of Congress is the metric system.

The publishers have done their part with great care and taste. Indeed, it is seldom that one sees in a text-book so fine a display of the printer's art.

T. W. WRIGHT.

Volume I. *May–June, 1894.* *Number 6.*

THE
PHYSICAL REVIEW.

AN EXPERIMENTAL RESEARCH ON THE LONGITUDINAL AND TORSIONAL ELASTIC FATIGUE.

By Louis W. Austin.

THE fact that, when an elastic body is subjected to a strain, there is a deformation of the body, which does not entirely disappear for some time after the strain is removed, even when the limit of elasticity is not passed, was first noticed by W. Weber,[1] who named the phenomenon the *Elastische Nachwirkung* (elastic fatigue). Weber's work was done on silk fibers. Later, Kohlrausch[2] investigated glass, rubber, and wires of different material, and proved that the elastic fatigue follows the same laws in all these substances at least for torsional strain, and in the case of rubber also for stretching and bending. I give below the principal results of his investigation.

I. If a body is subjected to a strain, the resulting fatigue is nearly proportional to that strain.

II. If the strains continue for different lengths of time, the fatigue is proportional to a power of the time.

III. The fatigue increases uniformly with the temperature.

IV. The curves, which represent the return of a body to its original shape after being subjected to different strains for equal lengths of time, are similar.

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[1] W. Weber: Pogg. Ann., XXXIV., p. 247; LIV., p. 1.

[2] F. Kohlrausch: Pogg. Ann., CXIX., p. 337; CXXVIII., pp. 7–207–399; CLVIII., p. 337; CLX., p. 225.

V. In all these cases the fatigue may be represented by the following formulæ: —

1. $$-\frac{dx}{dt}=\alpha\frac{x}{t^n}, \qquad x=Ce^{-at^m},$$

where $\alpha=am$, and $n=1-m$.

When the time of strain did not exceed three minutes, the more simple form ($n=1$) could be used: —

2. $$-\frac{dx}{dt}=\alpha\frac{x}{t}, \qquad x=\frac{c}{t^\alpha}.$$

Here t is the time which has passed since the release from strain. The distance from the final position of equilibrium is represented by x. n, c, C, and α are constants.

A theoretical formula for the fatigue was derived by Boltzmann, who assumed that a strain, δ, which had continued during a time, $d\theta$, left behind it a fatigue that was proportional to $\delta d\theta$, and a function of the time, θ, which had passed since the release from strain.

One form of the hypothesis developed by Boltzmann[1] supposes that the fatigue is proportional to $\delta\frac{d\theta}{\theta}$. That is, if a body has undergone a strain during a time t, after a time t, counting from the moment of release from strain, there still exists a deformation, x, which, if p is a constant of the material, equals

$$x=p\delta\int_t^{t+T}\frac{d\theta}{\theta}=p\delta\log\frac{t+T}{t}.$$

Observations on the longitudinal fatigue of metals have been made by Miller,[2] but not in such a manner as to allow the study of the whole course of the phenomenon. I have therefore thought it of interest to carry out a series of observations on this form of fatigue on different kinds of wires, and also to investigate their torsional fatigue.

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[1] Boltzmann: Pogg. Ann. Erg., Bd. VII., p. 624.

[2] Miller: Sitz. Berichte des K. bayr. Akademie, 1878.

The chief questions are as follows: —

I. Does the longitudinal fatigue follow the same laws as the torsional?

II. How does the velocity of return to the position of equilibrium depend on the duration of the original strain?

III. Are the curves, which represent the course of the fatigue during strain and after release from strain, congruent?

IV. What relation exists between the longitudinal and torsional fatigue?

APPARATUS.

The portion of the investigation on the longitudinal fatigue was carried out in the tower of the Physical Laboratory in Strasburg, which is high enough to allow wires 23 m. long to be used, and as it lies on the north side of the building, and is built with very heavy stone walls, the temperature retains a very admirable degree of constancy. The tower contains four piers, entirely free from the floors, and extending nearly to its top. The wires used were fastened by iron clamps to a beam 25 cm. square, which was laid across two of these, so as to form a perfectly solid support. To be sure that the wires were held fast, a microscope was placed on a second beam alongside of the first, through which a mark on the wire was frequently observed.

To eliminate the effects of changes of temperature two wires of the same material were used. One of these bore a constant weight and also the mark, which was taken as the zero in the observations. Close to this standard wire, and running through the hollow weight, which was suspended from it, was hung the wire on which the fatigue was observed. On the bottom of this wire was fastened the pan for the weights, and below this was an ordinary large water damper. To prevent any tendency to revolve, each wire was provided with a light arm, which played with great freedom between two glass rods fastened to the pier. The observations were taken by means of a microscope provided with a micrometer scale in the eye-piece, which permitted readings to be taken to 0.001 mm. The edges of glass fibers fastened to the wires with sealing wax were used as marks. To prevent lateral motion, a little below the marks, each wire was allowed to rest without observable friction in the angle formed by two stiff glass fibers, which were fastened into a frame attached to the pier. The temperature of the tower was determined by observing the expansion and contraction of the standard wire through a second microscope.

The investigation on the torsional fatigue was carried out in another room of the laboratory. The wire hung in a case provided with a graduated torsion head, and bore at its lower end a mirror and a small stretching weight, to which was attached a water damper. An arm extended out from the weight between two vertical wires, thus preventing too great amplitude of swinging, and holding the lower end of the wire in place, when the torsion was applied. The observations were carried out in the ordinary way with telescope and scale.

GENERAL METHOD OF THE RESEARCH.

The Longitudinal Fatigue. — After each wire was hung in its place, it was stretched for twenty-four hours by a weight twice as great as any used in the experiments, and was then left for from three days to a week, after which the observations were begun. The first observation of each series was taken one-half minute after the removal of the weight; but as the wire had often not become perfectly steady at that time, too great weight is not to be attached to these values, and on this account they were not used in the calculation of the constants.

The Torsional Fatigue. — The wires for the torsional observations were hung and subjected to large torsions, with periods of rest between, for several days, and in some cases weeks, before they were finally used in the investigation. The one-half minute observations were here also not satisfactory, as the mirror was always in motion at that time. After each series of observations, no new ones were begun until the old fatigue had entirely disappeared, or until there had been no apparent movement of the wire for a time equal to that required for the coming series.

The meaning of the symbols used in this paper are as follows:—

τ denotes the temperature.
W " the stretching weight.
L " the resulting elongation.
ϕ " the angle of torsion.
T " the time of strain.
t " the time, counting from the release from strain.
x " the fatigue.
x_1 " the fatigue for $(t = 1)$.[1]
Δ " x observed $-$ x calculated.

λ equals $\frac{x_1}{L}$ for the elongation.

λ' equals $\frac{x_1}{\phi}$ for the torsion.[2]

BRASS WIRE.

Longitudinal Fatigue.

Length	23 m.
Diameter	0.30 mm.
Breaking weight	4.7 kg.
Constant stretching weight . . .	0.65 kg.
Elongation for 1 kg.	36 mm.

Table I. represents a number of series of observations, showing the fatigue resulting from different degrees of tension for equal lengths of time, and also that of equal tensions for different times. The unit chosen in the case of the longitudinal fatigue is 0.01 mm. The first three series show the fatigue due to a load of 0.5 kg., 0.7 kg., and 1 kg. applied for three minutes.

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[1] The calculated value of x_1 equals the constant c of the formula.

[2] x_1 and L, x_1 and ϕ are of course expressed respectively in the same units.

TABLE I.

$T = 3$ min.

$W = 0.5$ kg. $L = 1800$ $\tau = 20^\circ.7$

$W = 0.7$ kg. $L = 2520$ $\tau = 19^\circ.3$

$W = 1$ kg. $L = 3600$ $\tau = 19^\circ.7$

t	x		Δ	x		Δ	x		Δ
Min.	Obs.	Cal.		Obs.	Cal.		Obs.	Cal.	
0.5	2.0	2.16	−0.16	3.2	3.43	−0.23	4.6	4.85	−0.25
1.0	1.6	1.55	+0.05	2.4	2.46	−0.06	3.2	3.48	−0.28
2.0	1.2	1.11	+0.09	1.8	1.76	+0.04	2.6	2.50	+0.10
3.0	1.0	1.91	+0.09	1.5	1.45	+0.05	2.2	2.05	+0.15
5.0	0.7	0.72	−0.02	1.2	1.14	+0.06	1.8	1.61	+0.19
7.0	0.5	0.61	−0.11	1.0	0.97	+0.03	1.4	1.37	+0.03
10.0	0.3	0.51	−0.21	0.8	0.81	−0.01	1.2	1.15	+0.05
15.0				0.6	0.67	−0.07	0.9	0.95	−0.05
20.0				0.5	0.58	−0.08	0.8	0.83	−0.03

$c = 1.55$ $c = 2.46$ $c = 3.48$

$a = 0.48$ $x = \frac{c}{t^a}$

$W = 1$ kg. $L = 3600$

$T = 27$ min. $\tau = 20^\circ$

$T = 81$ min. $\tau = 19^\circ.7$

t	x		Δ	x		Δ
Min.	Obs.	Cal.		Obs.	Cal.	
1	10.2	10.13	+0.07	16.0	15.89	+0.11
2	8.8	8.78	+0.02	13.8	14.06	−0.26
3	7.8	7.96	−0.16	12.8	12.92	−0.12
5	6.8	6.91	−0.11	11.4	11.44	−0.04
7	6.1	6.21	−0.11	10.6	10.44	+0.16
10	5.4	5.48	−0.08	9.2	9.38	−0.18
15	4.6	4.67	−0.07	8.2	8.18	+0.02
20	4.0	4.12	−0.12	7.4	7.34	+0.06
30	3.4	3.39	+0.01	6.4	6.21	+0.19
40	3.2	2.90	+0.30			
50	2.8	2.55	+0.25	5.0	4.87	+0.13
70	2.0	2.06	−0.06			
80				3.8	3.75	+0.05
110				3.2	3.08	+0.12

$C = 18.80$ $a = 0.618$

$C = 27$ $a = 0.530$

$m = 0.3$ $x = Ce^{-at^m}$

The calculated[1] values were derived according to Kohlrausch's formula 2 (p. 1), which he found to apply in the case of torsion, when the strain had not lasted more than three minutes. The calculated curves are exactly similar, being calculated with the same value of α in all three series. The similarity of the observed curves is in general good, though from the values of Δ it is seen that for a greater load the curve descends a little more slowly than for a smaller one. For greater lengths of time of strain, formula 2 no longer applies, and the general formula must be used. In all these points, the longitudinal fatigue agrees with the torsional, as studied by Kohlrausch.

Kohlrausch found further that for a given temperature the value of x_1 is nearly proportional to the deforming force, and also to a power of the time T; or,

$$x_1 = KWT^{\beta},$$

when W is the deforming force, and K and β are constants. To prove this formula for the longitudinal fatigue, I have taken the observations from Table I., and also some others which are not elsewhere given, and have collected them in Table II. All the observations are reduced to a temperature of 20° (see p. 8).

If we let

$$K = 2.13, \beta = 0.46,$$

the formula becomes

$$x_1 = 2.13 \cdot W \cdot T^{0.46}.$$

TABLE II.

W	T	x_1		Δ
		Obs.	Cal.	
1 kg.	1 min.	2.14	2.13	+0.01
1 "	3 "	3.23	3.53	−0.30
1 "	9 "	5.80	5.85	−0.05
1 "	27 "	10.20	9.70	+0.50
1 "	81 "	16.12	16.08	+0.04
0.7	3	2.44	2.47	−0.03
0.5	3	1.58	1.76	−0.18

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[1] The constants were derived by the method of least squares

The elongation of the wire for a weight of 1 kg. is 36 mm.

If we replace the weight W by the elongation L, and remember that x_1 is expressed in 0.01 mm. for a temperature of 20°, and $T = 1$ min.,

$$\frac{x_1}{L} = 0.00059 = \lambda.$$

Torsional Fatigue.

I give only a few observations of the torsional fatigue, as the subject is already well understood. Table III. shows the results of three series of observations on the course of the fatigue for different degrees of torsion. ϕ denotes the angle of torsion, otherwise the symbols are the same as in Table I.

TABLE III.

BRASS TORSION.

$T = 2$ min.

	$\phi = 30^\circ$ $\tau = 23^\circ.3$			$\phi = 60^\circ$ $\tau = 23^\circ 2$			$\phi = 90^\circ$ $\tau = 23^\circ 6$		
t	x		Δ	x		Δ	x		Δ
	Obs.	Cal.		Obs.	Cal.		Obs.	Cal.	
0.5	4.0	4.50	−0.50	8.0	8.56	−0.56	12.8	13.23	−0.43
1.0	2.9	2.99	−0.09	5.5	5.69	−0.19	8.5	8.79	−0.29
2.0	2.1	1.99	+0.11	4.0	3.78	+0.22	6.0	5.84	+0.16
3.0	1.8	1.56	+0.24	3.1	2.98	+0.12	4.8	4.60	+0.20
5.0	1.0	1.16	−0.16	2.4	2.20	+0.20	3.6	3.40	+0.20
7.0	0.9	0.95	−0.05	2.0	1.81	+0.19	2.9	2.79	+0.11
10.0	0.8	0.77	+0.03	1.3	1.46	−0.16	2.1	2.26	−0.16
15.0				1.1	1.15	−0.05	1.8	1.78	+0.02
20.0				0.9	0.97	−0.07	1.5	1.50	±0.00

$c = 2.99$ $c = 5.69$ $c = 8.79$

$a = 0.59$ $x = \frac{c}{t^a}$

The similarity of the curves is here evident, and the agreement between the observed and calculated values is, with the exception of those for $t = \frac{1}{2}$ min., fairly good. x is here given in scale divisions which are equal to 0.018° in circular measure.

The value of x_1 for $T=1$ min. and $\tau=20°$ expressed in terms of the angle of torsion,

$$0.018\frac{x_1}{\phi}=0.0011=\lambda'.$$

Comparing this with the corresponding result for the longitudinal fatigue, we find

$$\lambda'=1.9\lambda.$$

The Influence of the Temperature.

The rôle played by the temperature is an important one, and it is to be regretted that it was found impossible to make a complete series of observations of its effects in the case of the longitudinal fatigue, but as there is no way of warming the tower, and the change from summer to winter temperature came while I was away from Strasburg, I was only able to make observations at these two temperatures. However, these two indicate that the temperature effect is nearly the same as in the case of the torsion.

Table IV. gives the values of x_1 for $\phi=120°$ and $T=1$ min. at four, and also for $W=1$ kg. and $T=3$ min. at two different temperatures.

TABLE IV.

Torsion		**Tension**	
τ	x_1	τ	x_1
23°	7.8	20°	3.3
19°	7.1		
16°	6.7		
11°	5.8	10°	2.4

The observations show the value of the torsion temperature coefficient, or increase of fatigue for 1° of temperature, in the neighborhood of 15°, to be

$$x=0.029.$$

The value for the longitudinal coefficient from these observations is 0.037. The observations in the last case are perhaps not exact enough to prove that the difference really exists.

The influence of change of temperature on the constant α, if it exists at all, is extremely small, but some of my observations would indicate that α decreases slightly with increasing temperature.

Fatigue of Long-continued Longitudinal Strain.

The accurate observation of the fatigue of this sort is extremely difficult, since, contrary to the usual views on elasticity, even a small weight, long enough applied, seems sufficient to produce a permanent set.

From this cause any positive conclusions in regard to the similarity of the curves for different degrees of strain, or during strain and after release from strain, is impossible.

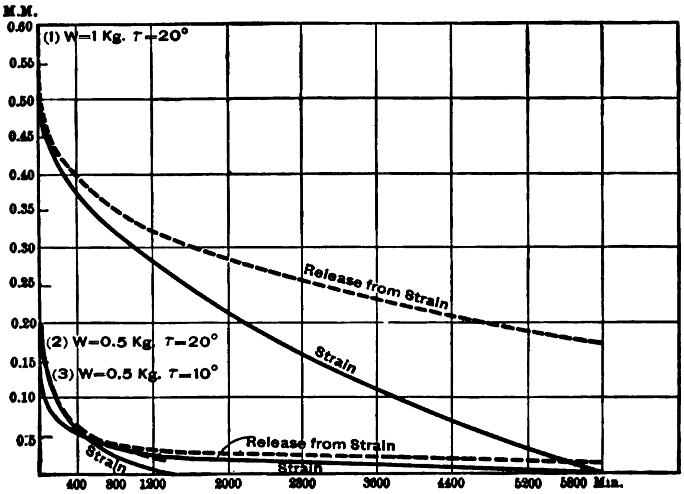

Fig. 1.

Kohlrausch[1] found during his observations on rubber, that in this last case the curves became more and more nearly congruent, as the stretching weights were made smaller, so that it seemed probable that for infinitely small strains they would really be of the same form.

In Fig. I. are given three pairs of curves, the solid line in each case representing the course of the fatigue during strain, and the dotted line after the release from strain.

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[1] F. Kohlrausch, Pogg. Ann., Bd. CLVIII., p. 337.

The curves (1) and (2) were taken at a temperature of 20°, while (3) was at 10°. The differences in the two parts of (1) can undoubtedly be in part explained by the permanent set of the wire.

The similarity in the parts of (2) and (3) is much greater, and it seems quite probable that, if there were no permanent set, the two parts would coincide in both cases.

COPPER WIRE.

Longitudinal Fatigue.

Length	23	m.
Diameter	0.29	mm.
Breaking weight	2.3	kg.
Constant stretching weight . . .	0.44	kg.
Elongation for 1 kg.	28	mm.

Though copper gives a greater amount of fatigue for a given strain than the other metals investigated, its low limit of elasticity made it necessary to use smaller weights, and therefore the actual results are in general less.

TABLE V.

$T = 0.5$ min.

	$W = 0.25$ kg. $L = 700$ $\tau = 18^\circ$			$W = 0.5$ kg. $L = 1400$ $\tau = 20^\circ.3$			$W = 0.6$ kg. $L = 1680$ $\tau = 18^\circ.6$		
t	x		Δ	x		Δ	x		Δ
	Obs.	Cal.		Obs.	Cal.		Obs.	Cal.	
0.5	1.2	0.90	+0.30	2.8	2.53	+0.27	4.0	3.47	+0.53
1.0	0.8	0.71	+0.09	2.0	2.00	±0.00	2.6	2.74	−0.14
2.0	0.6	0.56	+0.04	1.6	1.58	+0.02	2.2	2.16	+0.04
3.0	0.5	0.49	+0.01	1.4	1.38	+0.02	2.0	1.88	+0.12
5.0	0.3	0.41	−0.11	1.2	1.16	+0.04	1.6	1.58	+0.02
7.0				1.0	1.03	−0.03	1.4	1.41	−0.01
10.0	0.2	0.33	−0.13	0.9	0.92	−0.02	1.3	1.25	+0.05
15.0				0.8	0.80	±0.00	1.1	1.09	+0.01
20.0				0.7	0.74	−0.04	1.0	1.01	−0.01

$c = 0.713$ $c = 2.002$ $c = 2.737$

$a = 0.34$

$$x = \frac{c}{t^a}$$

TABLE V. (*continued.*)

$W = 0.5$ kg. $L = 1400$

$T = 3$ min. $\tau = 10^\circ$ $T = 20$ min. $\tau = 10^\circ.1$

t	x		Δ	x		Δ
	Obs.	Cal.		Obs.	Cal.	
1	3.6	3.67	−0.07	7.7	7.67	+0.03
2	3.2	3.10	+0.10	6.7	6.61	+0.09
3	2.8	2.76	+0.04	6.0	5.99	+0.01
5	2.3	2.33	−0.03	5.3	5.21	+0.09
7	2.0	2.05	−0.05	4.6	4.69	−0.09
10	1.8	1.75	+0.05	4.0	4.15	−0.15
15	1.4	1.42	−0.02	3.5	3.53	−0.03
20	1.1	1.18	−0.08	3.0	3.09	−0.09
30	0.9	0.84	+0.06	2.5	2.47	+0.03
50				1.7	1.69	+0.01
70				1.3	1.17	+0.13

$c = 3.67$ $c = 7.67$

$p = 0.001370$ $p = 0.002515$

$$x = c - p\,L \log t$$

Table V. shows the fatigue resulting for weights of 0.25 kg., 0.5 kg., and 0.6 kg., applied for 0.5 min., and also for 0.5 kg. for 3 min. and 20 min. The first three series are calculated from Kohlrausch's formula 2, and the agreement between the observed and calculated values leaves nothing to be desired. α is here somewhat smaller than in brass.

The fatigue in copper increases more rapidly in proportion than the weight W; this relation may be expressed for $T = 0.5$ min., and $\tau = 20^\circ$ by the equation:

$$\begin{aligned} x_1 &= 2.62\,W + 2.97\,W^2 \\ &= 0.00094\,L + 0.00000038\,L^2. \end{aligned}$$

From this we get

TABLE VI. *a*.

W	x^1		Δ
	Obs.	Cal.	
0.25	0.84	0.84	±0.00
0.50	1.99	2.05	−0.06
0.60	2.68	2.64	+0.04

The variations of x_1 for $W = 0.5$ kg. and $\tau = 20°$, due to different durations of strain, are given in the following table: —

TABLE VI. *b*.

T	x_1		Δ
	Obs.	Cal.	
0.5	1.99	2.00	−0.01
1.0	2.70	2.71	−0.01
3.0	4.68	4.39	+0.29
20.0	10.01	10.11	−0.10

Here $x_1 = 2.71\, T^{0.44}$

Connecting the results of Tables VI. *a* and VI. *b* as a general expression, we may write

$$x_1 = (0.00127\, L + 0.0000005\, L^2)\, T^{0.44}.$$

The last two series of Table V. I have attempted to calculate according to Boltzmann's formula:

$$x = pL \log \frac{t + T}{t}.$$

This, however, did not give satisfactory results. I then tried the more simple formula,

$$x = c - pL \log t.$$

With this the agreement between the observed and calculated values proved to be good.

Torsional Fatigue.

Table VII.

COPPER TORSION.

$T = 1$ min.

$\phi = 20^\circ$ $\tau = 20^\circ.2$ $\phi = 30^\circ$ $\tau = 19^\circ.9$ $\phi = 40$ $\tau = 18^\circ 6$

t	x		Δ	x		Δ	x		Δ
	Obs.	**Cal.**		**Obs.**	**Cal.**		**Obs.**	**Cal.**	
0.5	11.5	11.19	+0.31	18.0	17.57	+0.43	25.5	24.79	+0.71
1.0	7.9	7.86	+0.04	12.5	12.34	+0.16	17.6	17.41	+0.19
2.0	5.4	5.52	−0.12	8.8	8.66	+0.14	12.5	12.23	+0.27
3.0	4.4	4.49	−0.09	6.9	7.04	−0.14	9.8	9.94	−0.14
5.0	3.4	3.46	−0.06	5.1	5.43	−0.33	7.4	7.66	−0.26
7.0	2.9	2.91	−0.01	4.4	4.57	−0.17	6.2	6.45	−0.25
10.0	2.5	2.43	+0.07	3.8	3.81	−0.01	5.2	5.38	−0.18
15.0	2.1	1.98	+0.12	3.2	3.10	+0.10	4.3	4.37	−0.07
20.0	1.9	1.71	+0.19	2.8	2.68	+0.12	3.5	3.78	−0.28

$c = 7.86$ $c = 12.34$ $c = 17.41$

$a = 0.51$

$x = \frac{c}{t^a}$

Table VII., while showing that the curves of the fatigue are nearly similar, indicates that for larger torsions the return to equilibrium is slightly more rapid than for smaller.

The value for a is larger than in the longitudinal fatigue, but not as large as the corresponding a of brass. x_1 increases more rapidly in proportion than the angle of torsion, and in a quadratic expression its value is

$$x_1 = 0.324\,\phi + 0.0033\,\phi^2;$$

or, expressed in degrees, each scale division being equal to 0.014°,

$$0.014\,x_1 = 0.00454\,\phi + 0.000046\,\phi^2.$$

For small torsions applied for 1 min., at a temperature of 20°,

$$\lambda' = 0.014\frac{x_1}{\phi} = 0.0045.$$

For the longitudinal fatigue the corresponding value of λ was 0.00127; then

$$\lambda' = 3.6\,\lambda.$$

The fact that the fatigue does not increase at the same rate as the strain which produces it in copper, makes this last comparison unsatisfactory.

The Influence of the Temperature.

The values of x_1 for $\phi = 20°$ and $T = 1$ min., and also for $W = 0.5$ kg. and $T = 0.5$ min., are given in Table VIII.

TABLE VIII.

τ	x_1	
	Torsion.	**Tension.**
20°	7.8	2.0
15°	6.8	—
12°	6.2	—
10°	—	1.6
	$\gamma = 0.034$	$\gamma = 0.025$

Fatigue of Long-continued Longitudinal Strain.

Copper, from its extreme liability to take a permanent set during long-continued tension, was even more difficult to investigate than brass; but the conclusions which I am able to draw from the observations are somewhat more satisfactory.

Fig. II. shows two pairs of curves, the solid line, as before, representing the course of the fatigue during strain, and the dotted line after release from strain.

No. 1 ($W = 0.3$ kg.; $T = 5500$ min.; average temperature, $\tau = 10°.2$) shows a permanent set equal to nearly a quarter of the height of the whole curve, and it is manifestly impossible to make any comparison between the two parts.

No. 2 ($W = 0.15$ kg.; $T = 2800$ min.; average temperature, $\tau = 10°.5$) has a permanent set of about one-sixteenth of its total height. That this is really a set, and not due to a slow rate of return to a position of equilibrium, I am sure, as I observed the wire for more than a week after the last-recorded observation was taken; and the mark never varied by as much as 0.001 mm. This is also true for curve No. 1, though in this case the observations were only continued for three days. I judge it probable, therefore, that, if there had been no set, the two curves would coincide. *It is, at any rate, certain that the fatigue resulting from long periods of tension disappears in about the same time as that which is required to produce it.*

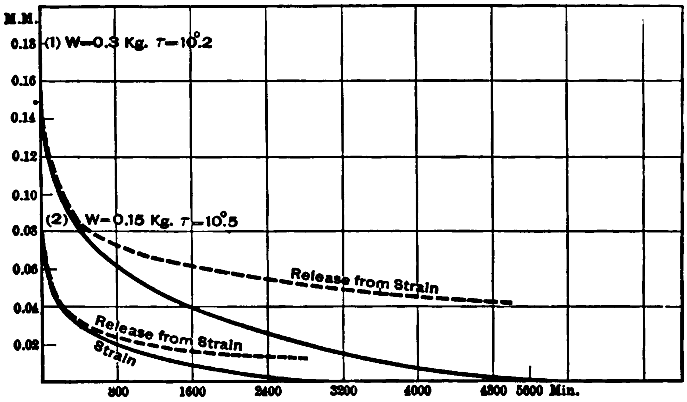

Fig. II.

SILVER WIRE.

Longitudinal Fatigue.

Length	23 m.
Diameter	0.32 mm.
Breaking weight	3.8 kg.
Constant stretching weight .	0.65 kg.
Elongation for 1 kg.	38 mm.

Table IX. contains the results of six series of observations. The first four are calculated according to formula 2. The constant α, which is common to all four, is somewhat smaller than in the corresponding cases of brass and copper.

Table IX.

SILVER.

	$T = 1$ min.												$W = 0.6$ kg. $L = 2280$.					
	$W = 0.4$ kg. $L = 1520$		$\tau = 10^\circ$	$W = 0.6$ kg. $L = 2280$		$\tau = 9^\circ.8$	$W = 0.8$ kg. $L = 3040$		$\tau = 9^\circ$	$W = 1$ kg. $L = 3800$		$\tau = 9^\circ.9$	$T = 10$ min.		$\tau = 9^\circ.0$	$T = 30$ min.		$\tau = 7^\circ.8$
t	x		Δ	x		Δ	x		Δ	x		Δ	x		Δ	x		Δ
	Obs.	Cal.		Obs.	Cal.		Obs.	Cal.		Obs.	Cal.		Obs.	Cal.		Obs.	Cal.	
0.5	1.6	1.31	+0.29	2.3	2.08	+0.22	2.7	2.37	+0.33	3.8	3.56	+0.24						
1.0	1.1	1.09	+0.01	1.7	1.72	−0.02	2.0	1.97	+0.03	3.0	2.95	+0.05	3.4	3.48	−0.08	6.2	6.28	−0.08
2.0	0.9	0.90	±0.00	1.4	1.43	−0.03	1.7	1.63	+0.07	2.6	2.45	+0.15	3.0	2.95	+0.05	5.5	5.54	−0.04
3.0	0.8	0.81	−0.01	1.3	1.28	+0.02	1.5	1.46	+0.04	2.3	2.19	+0.11	2.7	2.64	+0.06	5.2	5.10	+0.10
5.0	0.7	0.71	−0.01	1.2	1.12	+0.08	1.2	1.28	−0.08	2.0	1.91	+0.09	2.2	2.25	−0.05	4.6	4.56	+0.04
7.0				1.0	1.02	−0.02	1.1	1.16	−0.06	1.7	1.74	−0.04	2.0	1.99	+0.01	4.2	4.20	±0.00
10.0	0.6	0.59	+0.01	0.9	0.93	−0.03	1.0	1.06	−0.06	1.5	1.58	−0.08	1.8	1.71	+0.09	3.8	3.81	−0.01
15.0										1.2	1.42	−0.22	1.4	1.40	±0.00	3.4	3.38	+0.02
20.0										1.0	1.31	−0.31	1.2	1.18	+0.02	3.2	3.07	+0.13
40.0													0.6	0.65	−0.05	2.3	2.33	−0.03
60.0																1.9	1.89	+0.01
80.0																1.5	1.58	−0.08
	$c = 1.090$ $a = 0.27$			$c = 1.722$ $a = 0.27$			$c = 1.968$ $a = 0.27$			$c = 2.951$ $a = 0.27$			$c = 3.48$ $p = 0.000776$			$c = 6.28$ $p = 0.001082$		

$$x = \frac{c}{t^a}$$

$$x = c - pL \log t.$$

The agreement between the observed and calculated values in the first three is exceedingly good, where Δ is, with the exception of the rather uncertain observations for $t=\frac{1}{2}$ min., in every case less than 0.001 min. The fourth series ($W=1$ kg.) is not as good, and I think that perhaps in this case the strain was a little too near the limits of elasticity, where my observations have led me to believe that the fatigue becomes irregular.

TABLE X.

T	x_1		Δ
	Obs.	Cal.	
0.5	1.56	1.71	−0.15
1.0	2.23	2.23	±0.00
3.0	3.72	3.39	+0.33
10.0	4.52	5.35	−0.83
30.0	8.56	8.12	+0.44

The above table gives the values of x_1 for $W=0.6$ kg. and $\tau=20°$ for different values of T. The calculations were made from the formula

$$x_1=2.23\cdot T^{0.38}.$$

x_1 varies nearly proportionally with W, and as a general expression at a temperature of 20° we may write: —

$$x_1=3.74\cdot W\cdot T^{0.38},$$

or in terms of the elongation,

$$x_1=0.00098\,L\cdot T^{0.38}$$

and

$$\lambda=0.00098.$$

The series representing the fatigue for larger values of T are calculated according to the formula

$$x=c-pL\log t,$$

and the agreement is seen to be exceedingly good.

TABLE XI.

SILVER TORSION.

$T = 1$ min.

$\phi = 60^\circ$ $\tau = 18^\circ.2$ $\phi = 90^\circ$ $\tau = 17^\circ.2$ $\phi = 150^\circ$ $\tau = 20^\circ$

s	x		Δ	x		Δ	x		Δ
	Obs.	Cal.		Obs.	Cal.		Obs.	Cal.	
0.5	9.5	10.28	−0.78	14.5	14.79	−0.29	28.0	28.05	−0.05
1.0	7.2	7.32	−0.12	10.2	10.53	−0.33	19.8	19.97	−0.17
2.0	5.4	5.21	+0.19	7.8	7.50	+0.30	14.3	14.22	+0.08
3.0	4.4	4.28	+0.12	6.1	6.16	−0.06	11.1	11.67	−0.57
5.0	3.3	3.33	−0.03	4.9	4.79	+0.11	9.4	9.08	+0.32
7.0	2.7	2.82	−0.12	4.2	4.06	+0.14	8.0	7.70	+0.30
10.0	2.3	2.37	−0.07	3.6	3.41	+0.19	6.8	6.46	+0.34
15.0							5.4	5.30	+0.10
20.0							4.6	4.60	±0.00
40.0							3.6	3.28	+0.32

$c = 7.32$ $c = 10.53$ $c = 19.97$

$a = 0.49$

$$x = \frac{c}{t^a}$$

Torsional Fatigue.

The results of three series of observations for $T = 1$ min. are contained in Table XI. The theoretical values are calculated according to formula 2 with the constant $a = 0.49$ common to all the series. x_1 is here nearly proportional to ϕ, and for $\tau = 20^\circ$

$$x_1 = 0.129\,\phi,$$

or by multiplying by 0.014 (the value of one scale division in circular measure), and dividing by ϕ, we get

$$\lambda' = 0.014\frac{x_1}{\phi} = 0.0018,$$

from which $$\lambda' = 1.84\,\lambda.$$

Kohlrausch[1] found, in his investigation on the torsional fatigue of a silver wire having a diameter of 0.092 mm. for a torsion of 1 min. at a temperature of 20°, $\alpha=0.39$ and $\lambda'=0.00102$.

This led me to suspect that perhaps the diameter of the wire might affect the fatigue. I therefore drew out a piece of the same wire used in getting the above results, until its diameter was only about half its former size. For the first few days, probably as a result of the drawing, the fatigue was larger than before, and somewhat irregular, but after a week had passed, I again found the same values for α and λ' as before. However, as my wire was pure silver and Professor Kohlrausch's, as he himself has told me, was not, it is probable that this is the cause of the difference in the results.

The Influence of the Temperature.

The wire for the longitudinal fatigue was not placed in position before the winter set in. On this account I was unable to make any observations on its temperature coefficient.

In Table XII. are given the values of x_1 for four temperatures.

TABLE XII.

τ	x_1 (Torsion)
20°	15.8
16°	14.4
12°	12.3
8°	11.1
$\phi = 120°$	$T = 1$ min.

From this, the value of the temperature coefficient,

$$\gamma=0.036,$$

which is about the same as for brass and copper.

Fatigue of Long-continued Tension.

Fig. III. shows two curves illustrating the course of the fatigue in silver during strain and after release from strain. For both of these the time of strain was 2880 min., and the stretching weights were respectively 0.4 kg. and 0.2 kg. The results are a little better than in brass and copper, but still far from satisfactory. As in the other cases, the silver wire seems to recover from a long-continued fatigue in about the same time as that required to produce it.

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[1] Kohlrausch: Pogg. Ann., 1876, Bd. CLVIII., p. 342.

In order to allow a more exact comparison than Fig. III. offers in Table XIII., the observed values of x are given for curve No. 1, to which are added the calculated values for the part of the curve showing the fatigue after release from strain. The calculations were made from the formula

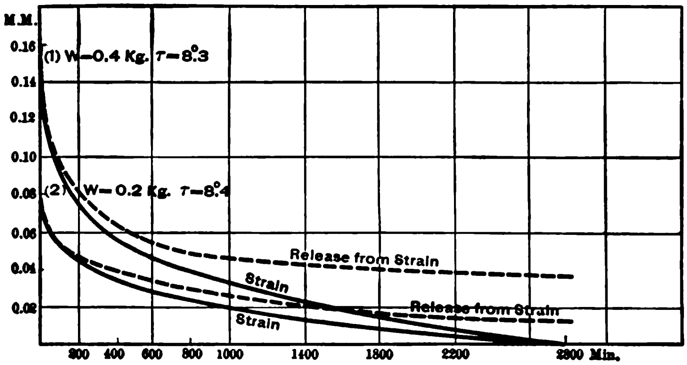

Fig. III.

$$x = c - pL \log t.$$

The two curves of the fatigue, after release from strain, seem to be fairly similar and nearly proportional to the producing strain, but the similarity and proportionality are both far from perfect. The agreement between the observed and calculated values in No. 1 is also poor.

The Formula, $x = c - pL \log t$.

It was found, on trial, that the formula $x = c - pL \log t$ also represents the course of the fatigue for short periods of strain. Table XIV. shows the first three series of Table IX. calculated according to it.

The agreement between the observed and calculated values seems to be quite as good as in the case where Kohlrausch's

TABLE XIII.

$T = 2880$ min.

$W = 0.4$ kg. $L = 1520$

t	Strain.		After Release from Strain.		
	x		x		Δ
	Obs.		Obs.	Cal.	
1	15.0		15.0	15.8	−0.8
2	14.4		14.4	14.8	−0.4
3	14.0		14.0	14.2	−0.2
5	13.6		13.7	13.4	+0.3
10	12.4		13.2	12.4	+0.8
20	11.6		12.0	11.3	+0.7
40	10.6		11.0	10.3	+0.7
60	9.6		10.1	9.7	+0.4
120	8.4		9.3	8.6	+0.7
180	7.4		8.2	8.0	+0.2
300	6.3		6.8	7.3	−0.5
1340	2.4		4.2	5.0	−0.8
2880	0.0		3.4	3.9	−0.5

$\tau = 8^{\circ}.3$

$c = 15.8$
$p = 0.00227$
$\tau = 8.5$
$x = c - pL \log t$

TABLE XIV.

$T = 1$ min.

$W = 0.4$ kg. $L = 1520$ $\tau = 10^{\circ}$

$W = 0.6$ kg. $L = 2280$ $\tau = 9^{\circ}.8$

$W = 0.8$ kg. $L = 3040$ $\tau = 9^{\circ}$

t	x		Δ	x		Δ	x		Δ
	Obs.	Cal.		Obs.	Cal.		Obs.	Cal.	
1	1.1	1.07	+0.03	1.7	1.68	+0.02	2.0	1.97	+0.03
2	0.9	0.92	−0.02	1.4	1.45	−0.05	1.7	1.67	+0.03
3	0.8	0.83	−0.03	1.3	1.32	−0.02	1.5	1.49	+0.01
5	0.7	0.72	−0.02	1.2	1.16	+0.04	1.2	1.27	−0.07
7				1.1	1.05	+0.05	1.1	1.12	−0.02
10	0.6	0.57	+0.03	0.9	0.93	−0.03	1.0	0.97	+0.03

$c = 1.07$

$c = 1.68$
$p = 0.000329$
$x = c - pL \log t$

$c = 1.97$

formula No. 2 was used. I have also tried to apply my formula to the calculation of the torsional fatigue, but while it represents it fairly well, its accuracy is not as great as Kohlrausch's. Perhaps this last is not to be wondered at, as the molecular motions in torsion are probably more complicated than in stretching.

According to this formula, the change of shape of a body which has been strained is proportional to the log of the time since the strain was removed; then, by differentiation, the velocity of this change is inversely proportional to that time, or

$$-\frac{dx}{dt}=\frac{pL}{t}.$$

SUMMARY OF THE RESULTS.

The longitudinal fatigue in general seems to follow the same laws as the torsional; it is nearly proportional to the deforming strain; the curves, which represent its disappearance, are similar, so long as the time of strain is short; the curve sinks more slowly as this time increases.

The fatigue increases with the temperature, and within the limits observed this increase is proportional to the increase of temperature. The temperature has very little influence on the velocity of disappearance of the fatigue, but apparently for lower temperatures this velocity is slightly increased.

The course of the longitudinal fatigue may be represented by Kohlrausch's formulæ 1 and 2, but it also seems possible to represent it by the more simple formula

$$x=c-pL\log t.$$

In Table XV. are given the values of λ' and λ for a time of strain of 1 min. and a temperature of 20°, and also the temperature coefficients for torsion of all three metals investigated.

The fatigue in copper, silver, and brass are to each other nearly as 7 : 3 : 2 for the torsion, and as 4 : 3 : 2 for the tension. I do not think that the values of λ' and λ in copper can be properly compared, on account of the lack of proportionality between the fatigue and the deforming strain, and I think it probable that in general the relation between the fatigue of torsion and tension is about as 2 is to 1.

The temperature coefficient both for torsion and tension in all three metals seems to be not far from $\frac{1}{80}$.

The results in regard to the course of the fatigue during long-continued strain, and after release from strain, are unsatisfactory, as all the wires seemed to take a permanent set if the strain lasted more than a few hours. The most that can be said is, that for small strains lasting for several days, the fatigue apparently disappears in about the same length of time as that required to produce it; and, if the strain is very small, the two curves take nearly the same course.

TABLE XV.

	λ'	λ	$\frac{\lambda'}{\lambda}$	γ
	Torsion.	Tension.		
Brass	0.0011	0.00059	1.9	0.029
Copper	0.0045	0.0013	3.6	0.034
Silver	0.0018	0.00098	1.8	0.036

From the observations on the silver wire it seems that in general the fatigue is entirely independent of the diameter of the wire.

The poor agreement of the observed and calculated values in the case of the long-continued strain of silver (Table XIII.), together with some other observations made shortly after the wires had been strained beyond the limits of elasticity, has led me to believe that probably the passage of this limit is accompanied by such a disturbance of the molecules of the body that it requires several days before it again assumes its normal condition, and that during this time the fatigue is somewhat irregular.

I hope within a short time to publish further results in regard to the effects of temperature on the fatigue, and also on a possible relation between the elastic limit and the time of strain.

I wish to take this opportunity of thanking Professor Kohlrausch, in whose laboratory this research was carried out, for his great kindness and many valuable suggestions during the progress of the work.

Physical Laboratory of the University of Wisconsin,
December, 1893.

ON THE CONDITION OF THE ETHER SURROUNDING A MOVING BODY.

BY WILLIAM S. FRANKLIN AND EDWARD L. NICHOLS.

THE problem with which this paper deals is really that of the existence of terms involving products of currents and ordinary velocities in the general expression for the kinetic energy of an electric current. It is our purpose to describe an experiment bearing upon this question, and to give the theory.

I.

Introductory. — The only rapid motion which is experimentally attainable is rotary motion. Such motion is cyclic[1] when the rotating body possesses circular symmetry with respect to the axis of rotation.

The expression for the kinetic energy of a system in cyclic motion[2] does not involve the co-ordinate of which the given velocity is the rate, nor the rates, when they change, of the geometric parameters. If a rotating body is not symmetrical, the motion is not strictly cyclic, for the energy of the system will depend upon the value of the cyclic variable if the non-symmetrical rotating body produces waves of any kind in the surrounding medium. If no waves are produced, then the rotation of a non-symmetrical body may be treated as cyclic.[3]

In this paper an hypothetical motion of the ether in the neighborhood of a moving body is assumed as the basis for the existence of kinetic energy terms involving products of velocity and current, so that it will not be allowable for our purpose to consider the rotation of a non-symmetrical body as cyclic. The theoretical and experimental parts of the paper will be limited to the case of circularly symmetrical coils rotating about their axes of symmetry. Electric circuits moving otherwise than cyclically may present no phenomena essentially different from the phenomena due to cyclic motion, except in the production of electro-magnetic waves due to the moving current, and perhaps ether-waves of some kind due to the moving matter. Kinetic energy terms involving products of current and velocity have nothing to do with this production of waves, so that the study of cyclic motion of circuits may lead to results of more general significance than might at first be anticipated, considering the restricted nature of cyclic motion.

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[1] See von Helmholtz, "Principien der Statik monocyklischer Systeme," Crelle's Journal, Vol. 97, p. 111 and p. 317.

[2] The characteristic of cyclic motion is that it can be completely specified by a number of constant or slowly varying geometric parameters and a single constant, or slowly varying, generalized velocity.

[3] See Boltzmann, Vorlesungen über Maxwell's Theorie der Electricität und des Lichtes, Erster Theil.

II.

Monocyclic System. — Let x_1, x_2, x_3, ... be geometric parameters which determine the character of a monocyclic motion; let y_1 be the cyclic variable, and $\dot{y}_1$ its rate. Let v be the velocity of the moving substance at the point p; v is a vector function of the position of the point p, involving the x's and $\dot{y}_1$, and it is linear[1] with respect to y_1; so that we may write

$$v=\dot{y}_1 fi+\dot{y}_1 gj+\dot{y}_1 hk, \qquad (1)$$

where ijk are rectangular unit vectors, and f, g, and h are determinate scalar functions of the x's and of the co-ordinates of the point p. The form of f, g, and h depends upon the nature of the cyclic motion. If the moving mechanism has finite rigid parts, f, g, and h will be discontinuous functions. In moving matter, f, g, and h may be two-valued, one value referring to the moving matter, the other to the ether which may be carried with moving matter at a distinct velocity.[2]

Equation (1) and all that follow apply to any monocyclic or bicyclic systems whatever. We shall, however, keep in mind only motions of rotation, including any possible associated motion of the ether, and the cyclic motion which constitutes the electric current.

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[1] See von Helmholtz, *loc. cit.*, pp. 319, 320.

[2] See J. J. Thomson, Recent Researches in Electricity and Magnetism, p. 544.

The total kinetic energy of the monocyclic system is

$$\tfrac{1}{2}L_1\dot{y}_1^2 = \frac{\dot{y}_1^2}{2}\int\int\int(f^2+g^2+h^2)\rho\, dx dy dz, \tag{2}$$

in which ρ is the density of the moving substance. The integration is to be extended throughout all space, or where v is other than zero. If v is other than a linear velocity, then ρ cannot be so simply defined. If v is an angular velocity, and if the energy due to it is finite, then v must be at each point of the same order of magnitude as the reciprocal of the linear dimensions of the ultimate rotating parts of the moving substance, and ρ will be the moment of inertia of one of these parts multiplied by the number of them in unit volume. See IV. *b*.

The quantity

$$L_1 = \int\int\int(f^2+g^2+h^2)\rho\, dx dy dz \tag{3}$$

may be called the inertia of the monocyclic system.

In case $\dot{y}_1$ is an electric current, then $(f^2+g^2+h^2)\rho$ becomes $(\alpha^2+\beta^2+\gamma^2)\frac{\mu}{4\pi}$, where α, β, and γ are the components of the magnetic field at p, and μ is the permeability of the medium. We cannot yet separate $(\alpha^2+\beta^2+\gamma^2)\frac{\mu}{4\pi}$ into its two factors v^2 and ρ; when we can do this we shall be able to specify strength of magnetic field as a velocity and permeability as some kind of moment of inertia per unit volume. Equation (3) leads to an expression for the coefficient of self-induction in a circuit as $\frac{1}{4\pi}$ into the volume integral of the square at each point of the magnetic field per unit current in the coil.

III.

Bicyclic System. — Let y_1 and y_2 be the cyclic variables concerned in a bicyclic motion, and $\dot{y}_1$ and $\dot{y}_2$ their respective rates. Let $x_1, x_2, x_3, \cdots$ and $z_1, z_2, z_3, \cdots$ be the geometric parameters which determine the character of the respective cycles. The velocity of the moving substance at the point p will now, in general, be the superposition of two vector parts linear with respect $\dot{y}_1$ and $\dot{y}_2$ to respectively. Let these parts be $\dot{y}_1 v_1$ and $\dot{y}_2 v_2$. As in equation (1) we have

$$\dot{y}_1 v_1 = \dot{y}_1 f_1 i + \dot{y}_1 g_1 j + \dot{y}_1 h_1 k.$$

$$\dot{y}_2 v_2 = \dot{y}_2 f_2 i + \dot{y}_2 g_2 j + \dot{y}_2 h_2 k. \tag{4}$$

The total velocity at the point p is $\dot{y}_1 v_1 + \dot{y}_2 v_2$. There is nothing inconsistent in thus indicating the sum of the vectors $y_1 v_1$ and $y_2 v_2$, even if they are velocities of different kinds, and for the moment we shall consider them to be alike in kind so that the energy per unit volume at the point p is $\frac{1}{2}(\dot{y}_1 v_1 + \dot{y}_2 v_2)^2 \rho$, or $\frac{1}{2}\dot{y}_1^2 v_1^2 \rho + \dot{y}_1 \dot{y}_2 \rho S \cdot v_1 v_2 + \frac{1}{2}\dot{y}_2^2 v_2^2 \rho$, and the total kinetic energy of the system embraces three terms which are the volume integrals of the respective terms in this expression; that is,

$$W = \tfrac{1}{2} L_1 \dot{y}_1^2 + M \dot{y}_1 \dot{y}_2 + \tfrac{1}{2} L_2 \dot{y}_2^2, \tag{5}$$

where

$$L_1 = \int\!\!\int\!\!\int (f_1^2 + g_1^2 + h_1^2)\rho\, dx\, dy\, dz = \int\!\!\int\!\!\int v_1^2 \rho\, dx\, dy\, dz,$$

$$L_2 = \int\!\!\int\!\!\int (f_2^2 + g_2^2 + h_2^2)\rho\, dx\, dy\, dz = \int\!\!\int\!\!\int v_2^2 \rho\, dx\, dy\, dz, \tag{6}$$

$$M = \int\!\!\int\!\!\int (f_1 f_2 + g_1 g_2 + h_1 h_2)\rho\, dx\, dy\, dz = \int\!\!\int\!\!\int S v_1 v_2 \cdot \rho\, dx\, dy\, dz. \tag{7}$$

We shall call the quantity M the *concurrence* of the two cyclic motions, since it is a generalization of that which for simple vectors involves the cosine of an included angle. M is evidently a function of the x's, of the z's, and of the co-ordinates which determine the relative positions of the cycles. When M is a maximum, the two cycles are said to be *concurrent.* If v_1 and v_2 are everywhere parallel, $V \cdot v_1 v_2 = 0$, and the two cycles are said to be *identically* concurrent. If $M = 0$, the two cycles are said to be *orthogonal;* if $S \cdot v_1 v_2 = 0$ everywhere, that is, if v_1 and v_2 are everywhere mutually perpendicular, then the two cycles are said to be *identically* orthogonal. It is worthy of note that a cycle has in a general way many *vector* properties which depend upon a number of independent numerical specifications. If $\dot{y}_1 v_1$ and

$\dot{y}_2 v_2$ are velocities of different kinds, angular and linear velocities, for example, then the quantity M may have no physical significance. Equation (7) will, however, be retained as its mathematical definition; ρ being merely some scalar function of position. The reason for thus retaining a conception of M is that we do not wish to assume that no part of the energy of a current in cyclic motion can depend upon this motion when the motion produced in the ether by the moving coil is of a different kind from the motion produced in the ether by a current in the coil. (See VI.) Equation (7) is easily interpreted for the case of two electric circuits. It leads to an expression for the coefficient of mutual induction of two coils as $\frac{1}{4\pi}$ into the volume integral of the scalar product at each point of the magnetic fields per unit current in the respective coils.

IV.

The Motion of the Ether near Moving Bodies. — In the expression for the kinetic energy of a system it is scarcely conceivable that terms involving the product of generalized velocities can occur unless parts of the moving system depend for their actual motion upon both these generalized velocities.[1] Hence we must consider the possible motion of the ether in the neighborhood of moving bodies as the basis for the existence of mutual energy terms in a bicycle consisting of a rotating coil.

We shall consider two possibilities in the motion of the ether: —

(*a*) That the ether in the neighborhood of a moving body moves as a viscous fluid.[2]

(*b*) That the ether may be cellular in structure, adjacent cells being positive and negative respectively.[3] Such an ether might be capable of moving as a viscous fluid, as in (*a*), without the finite angular velocity, due to the curl, if it exists, of the linear velocity, contributing sensibly to the energy of the motion (see II.). Such an ether would also be capable of a violent rotary motion of its ultimate parts or cells, independently of any linear velocity, and without breaking down the connecting mechanism. If moving matter should have a stronger hold upon one set of these cells than upon the other, this violent rotary motion might be set up in the neighborhood.

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[1] See Maxwell's Treatise, Vol. II., p. 199, 2d edition.

[2] See J. J. Thomson, Recent Researches in Electricity and Magnetism, p. 543.

[3] See Oliver Lodge, Modern Views on Electricity.

The lines of flow of a viscous fluid in the neighborhood of a rotating body are circles coaxial with the rotating body. Hence a bicycle consisting of a current in a rotating circular coil would be identically orthogonal if the ether move as a viscous fluid. A bicycle consisting of a current in a rotating uniformly wound Gramme ring would be identically concurrent if the ether move as a viscous fluid.

The vortex lines in the ether, moving as described in (*b*), in the neighborhood of a long rotating cylinder would be straight lines parallel to the axis. For a short cylinder the vortex lines would approximate to the magnetic lines due to a current in a circular coil. Hence a bicycle consisting of a current in a rotating circular coil would be concurrent if the ether move as described in (*b*). A bicycle consisting of a current in a rotating Gramme ring would be identically orthogonal for this case of ether motion.

If the quantity M be found to have physical existence for a rotating circular coil, the case (*b*) of ether motion would be, to some extent, verified. If the quantity M be found to have physical existence for a rotating Gramme ring, the case (*a*) of ether motion would be verified. If M exists for both coils, then the ether could perhaps be considered to move in both ways in the neighborhood of a moving body.

V.

The phenomena which depend upon the mutual action of two cycles during a permanent or varying state of the bicyclic motion.

(A) *Permanent State.* — Let X ... and Z ... represent the forces which tend to change the respective x's and z's, or any of the co-ordinates which specify the relative position of the cycles, in a bicyclic motion. (X ... and Z ... being understood to mean only those portions of these forces which depend upon the mutual action of the cycles.) Then

$$X = -\frac{dM}{dx}\dot{y}_1\dot{y}_2,$$

$$Z = -\frac{dM}{dz}\dot{y}_1\dot{y}_2, \text{ etc.,} \tag{8}$$

and from Lagrange's equation, total force $X = \frac{d}{dt}\frac{dW}{d\dot{x}} - \frac{dW}{dx}$; in which W is the total kinetic energy (see equation (5)), and is independent of $\dot{x}$, which is, in fact, equal to zero. The force due to mutual action depends only upon the mutual energy term.

(B) *Varying State.* — When the state of a cyclic motion is changing, it is not strictly cyclic. In this case the expression (1) for the velocity of the moving substance at each point fails. Indeed, equation (1) is in many respects the most evident characteristic of a cyclic motion. If the change of state takes place slowly, the motion may be treated as cyclic. The requisite slowness of change depends upon the quickness with which any change of state is established throughout the moving mechanism, and upon the degree of approximation required in equation (1). A rotating rigid body may change its angular velocity with great rapidity without losing the character of cyclic motion. If the ether is involved in such motion, then the angular velocity must not change so rapidly that the ether motion is not sensibly conformable at each instant to the motion of the body. An electric current, however, may change with great rapidity without becoming essentially non-cyclic in character; for the state of motion of the ether in the neighborhood of a current is propagated with the velocity of light, and the greater part of the energy is in the immediate neighborhood of the coil. In the ordinary treatment of alternating currents, the kinetic energy of the current is assumed proportional to the square of the current at each instant, which by equation (2), a result of (1), amounts to assuming the current to be sensibly a cyclic motion at each instant.

Let Y_1 and Y_2 be the forces, due to mutual action of the cycles, and depending upon a changing state, which tend to change the cyclic rates $\dot{y}_1$ and $\dot{y}_2$ respectively; then by Lagrange's equation,

$$Y_1 = M\frac{d\dot{y}_2}{dt}, \tag{9}$$

$$Y_2 = M\frac{d\dot{y}_1}{dt}. \tag{10}$$

The impulses of these forces, Y_1 and Y_2, for any integral changes, $\dot{y}_1$ and $\dot{y}_2$, of these cyclic rates, are:

$$\text{Impulse of } Y_1 = M\dot{y}_2,$$
$$\text{Impulse of } Y_2 = M\dot{y}_1. \tag{11}$$

There are then three methods depending upon the mutual action of two cycles by which the mutual energy term of a bicycle consisting of a rotating coil may be determined. (*a*) By observing the effects of the forces X or Z. This requires the observation of the effect of a force upon a rapidly moving rigid body. [The experiment described by Maxwell on page 203 of the second volume of his Treatise is an attempt to detect one of these forces, X, since the angle θ in his discussion is one of the parameters of the motion, which, inasmuch as he has ignored electro-magnetic and other ether-waves, may be considered to be cyclic.] (*b*) By observing the effect of the impulsive torque $M\dot{y}_2$, due to the establishment of a current $\dot{y}_2$ in the coil.[1] (*c*) By observing the effects of the impulsive electromotive force $M\dot{y}_1$, due to a change, $\dot{y}_1$, in the angular velocity of the coil.

Let a be the observed angular velocity produced in a coil by the establishment of a current $\dot{y}_2$, and let K be the moment of inertia of the coil; then

$$Ka = M\dot{y}_2. \tag{12}$$

This equation will enable us to calculate the value of M when K, a, and $\dot{y}_2$ are known, or to assign an upper limit to M when we have estimated from the conditions of an experiment the least observable angular velocity, and have failed to detect any angular velocity when the current is changed.

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[1] See Maxwell's Treatise, Vol. II., p. 201.

Let k be the reduction factor of a ballistic galvanometer, r the combined resistance of the galvanometer and rotating coil, and d the deflection produced by the impulsive electromotive force $M\dot{y}_1$;

then
$$\frac{M\dot{y}_1}{r} = kd, \tag{13}$$

from which inferences, similar to those deduced from equation 12, may be drawn, provided that d has been observed; or, in case of no deflection, when the least observable d has been estimated.

VI.

Another Means for the Detection of the Quantity M. — If it be assumed that M must depend not only upon motion at each point due to both cycles, but also that these two motions must be alike in kind (see III.), then the existence of the term $M\dot{y}_1\dot{y}_2$ would require the disturbance of the ether in the neighborhood of a moving body to be, partly at least, of the nature of magnetic field, so that a suspended magnet would be affected by the motion. In this case, the numerical value of M cannot be rigorously calculated from an observed deflection, nor can the upper limit be easily assigned. A roughly approximate, and perhaps questionable, reduction formula may be derived as follows: assume the motion and current to be identically concurrent, so that v_1 and v_2 are everywhere parallel, and assume the ratio $\frac{v_1}{v_2} = a$ to have everywhere the same value. Substitute the value of $v_1 = av_2$ in equation (7); then

$$M = \int\int\int S \cdot v_1 v_2 \rho dx dy dz = a\int\int\int v_2^2 \rho dx dy dz = aL_2. \tag{14}$$

Let d_1 be the observed deflection produced upon the suspended magnet by an angular velocity $\dot{y}_1$ of the coil, and let d_2 be the observed deflection produced by a known current $\dot{y}_2$ in the coil.

Then
$$a = \frac{d_1\dot{y}_2}{d_2\dot{y}_1}, \tag{15}$$

from which expression, when the coefficient of self-induction, L_2, of the coil is known, $M = aL_2$ (14) gives the value of M.

The existence of the quantity M, depending as it does upon motion produced in the ether by a rotating body, requires part of the energy of any rotation to be due to the motion of the ether. It is interesting to notice that the amount of this energy may be estimated when $\dot{y}_1 v_1$ and $\dot{y}_2 v_2$ are of the same nature. Let the coil rotate at angular velocity $\dot{y}_1$, producing at each point a magnetic field $\dot{y}_1 v_1$, which is equal to $\dot{y}_1 a v_2$. Then the field $\dot{y}_1 a v_2$ is that which would be produced by a current $\dot{y}_2 = \dot{y}_1 a$ in the coil of which the kinetic energy is $\frac{1}{2} L_2 \dot{y}_1^2 a^2$. Since $\frac{1}{2} K \dot{y}_1^2$ is the total energy of the rotation, the ratio of energy of the motion of the ether to the total energy is

$$\frac{L_2 a^2}{K}. \qquad (16)$$

VII.

Apparatus and Experimental Results. — The apparatus used was designed for carrying out method (*c*) of V. and the method described in VI.; the method (*c*) of V. being applied to both coils *A* and *B* (see below), and the method of VI. being applied only to coil *A*.

Two coils, *A* and *B*, were constructed, each one foot in diameter and five inches thick. They were wound upon cores of well-seasoned pine (Fig. 1), with hollow brass trunnions securely fastened to the core by means of brass discs five inches in diameter. The hubs of these trunnions were about two inches in diameter, and were grooved to receive a driving cord. The total weight of each coil was about 15 pounds. Coil *A* consisted of 61 turns of No. 18 B. & S.

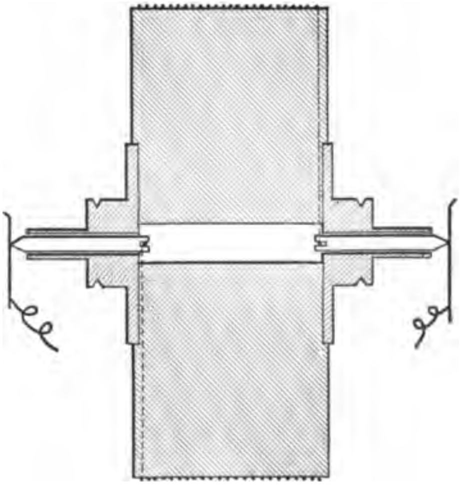


Fig. 1.

bare, hard-drawn, copper wire, laid in a thread turned upon the periphery of the core. It had a resistance of about 1.3 ohms.

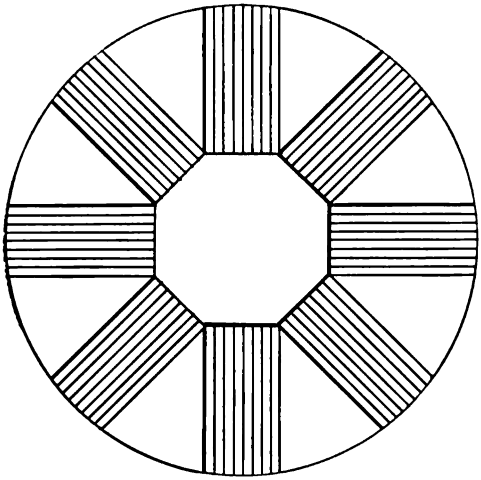


Fig. 2.

Coil *B* (Figs. 2 and 3) consisted of a Gramme ring wound in eight sections of 36 turns each, of double cotton-covered No. 20 B. & S. copper wire, connected continuously and bound with a band of brass wire. It was necessary to wind coil *B* thus in sections, leaving spaces on each side in which wooden lugs were placed, forming faces against which circular boards of ash, each one foot in diameter, were screwed, giving smooth surfaces against which a brake could be applied. Fig. 2 shows the hollow wooden drum upon which coil *B* was wound. The resistance of *B* was 3.2 ohms. The coils were carried on hard-wood bearings, in a heavy wooden frame. Their terminals were connected, without solder, to insulated copper rods held in the hollow trunnions and ending in carefully centered points, against which pressed copper springs (see Figs. 1 and 3). The brake, which was supported in the same frame, consisted of a pair of hinged wooden jaws, which could be pressed with equal force against the two ends

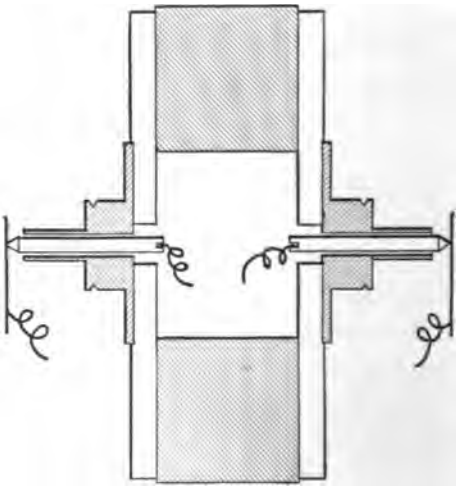


Fig. 3.

of the rotating cylindrical coil by means of a system of cords leading to a strong pedal. The rubbing surfaces of the brake consisted of two leather-covered blocks, securely fastened to each of the jaws so as to rub at points diametrically opposite. With this arrangement the bearings were not subject to any great end or side thrust during the application of the brake. The brake was thoroughly effective, bringing either coil from a speed of 60 revolutions per second to a dead stop in about a second or a second and a half.

The galvanometer used was constructed in the shop of the physical laboratory at Ames, Iowa. The elements of the astatic pair contained four magnets each. They were very nearly equal in strength, and were only two inches apart, — the mirror being placed below the coils instead of between them. By this arrangement the galvanometer was rendered comparatively insensible to magnetic disturbances. The galvanometer was provided with freshly made magnets just before being used, — a precaution which should always be taken in preparing for any important work requiring the last degree of sensitiveness, and care was taken to send no currents of any ordinary strength through the instrument. In the preparation of the magnets the following precautions were taken. Piano wire ¼ mm. in diameter was straightened by subjecting it to slight tension at a low red heat, and was cut into two-inch lengths. These were placed, two or three at a time, in an acute V-shaped iron trough, and after being heated uniformly to a cherry-red heat (800° C.) in a Bunsen flame, were quickly dropped into cool water. Two small pieces of exactly the same length were then cut from the central portion of each, and magnetized under similar conditions. The cutting was done by placing the hardened wire upon a smooth block of hard wood, and pressing an edged tool against it. If this procedure be carefully followed a highly astatic pair may always be obtained.

The two small magnets thus made from each piece were used, one in each of the elements of the astatic system. The galvanometer had a resistance of 15 ohms with its coils in series. When so arranged, and with a half-period of seven seconds, and scale distance of 120 cm., a deflection of 1 mm. corresponded to

6×10^{-10} amperes. During the course of the experiments the coils of the galvanometer were connected in multiple so as to have a total resistance of about one ohm.

It was found necessary to make the circuit external to the galvanometer, which was housed in a box, entirely of copper, and without soldered joints. The key for opening and closing the circuit was made by scraping the ends of the copper wires and bending them into hooks, one of which was fastened to a small weight, and the other to the edge of the table. Even with such precautions it was found necessary to include in the circuit about three feet of a heavy copper rod, No. 0000 B. & S., through which a current from a gravity cell could be passed, so as to balance any existing thermo-electromotive force.

The suspended needle for carrying out the method of VI. consisted of a widely separated astatic pair, each element consisting of two magnets about 6 mm. long, and $\frac{1}{3}$ mm. in diameter, carrying a small mirror, and suspended in a brass tube which was supported above the coil by a scaffold fastened to the brick walls of the room.

The coils were driven by a cord from a small electric motor, placed 75 feet away from the coil. The galvanometer was conveniently near the coil. Coil *B* was driven, during a trial run, at a speed which approached 100 revolutions per second. While taking observations, however, the speed was always approximately 60 revolutions per second.

The extreme sensitiveness of such a low-resistance galvanometer to electromotive force is very striking. The movement of an oil can, having an iron snout, used in oiling the bearings of the coil, for example, would produce a quick motion of the galvanometer needle through several centimeters of deflection. One of the connecting wires, about eight feet long, happened to be stretched horizontally east and west. It performed about twelve half-vibrations per second when disturbed. The electromotive force induced in this wire by the earth's field while vibrating through an amplitude of about an inch was found to force a synchronous vibration upon the galvanometer needle with an amplitude of about a centimeter on the scale, the proper half-period of the galvanometer needle being about seven seconds. Placing the finger on one side of the hooked junction of the key gave a deflection of three centimeters; placing the finger on the back of one of the commutator springs gave 6 cm. deflection. During some preliminary experiments, when the wires leading to the galvanometer were soldered to the ends of these springs, the effect of touching the finger for a second or two to the other end of the spring was to throw the galvanometer off the scale, although the springs were thin, and about four inches long. The copper rod shunt, already described, was used principally to balance electromotive forces produced by the warming of the bearings. Obscure magnetic disturbances in the neighborhood of the coil — especially when coil *A* was in circuit — gave troublesome irregular movements of the galvanometer. With this coil, *A*, it was found that there was enough undetachable iron about the person of the assistant manipulating the brake, even when he wore slippers containing no iron nails, to produce a throw on the galvanometer for every movement of his body. This was shown to be due to inductive action on the coil, since it did not occur with open circuit. It was found that the movement necessary in applying the brake produced from two to five millimeters of throw, the direction of which was always the same, but the magnitude of which could not be entirely controlled, even when care was taken to perform the movement always in the same way. The motion of the assistant in walking up to the coil after starting the motor (or after going through the motions of starting the motor) produced always, with coil *A*, about 20 mm. deflection, "making ready" for applying the brake, 2 mm. The signal "now," for the application of brake, was followed by 2 to 5 mm. deflection, when the assistant went through the motions of braking with the coil stationary. With coil *B* no serious trouble of this kind was experienced. The effect of the magnetism of the motor — the field being continually excited — was slightly noticeable with coil *A*, and was eliminated by reversing the connections of the motor after several runs. This was perhaps a needless precaution, as the coil was driven alternately in opposite directions by crossing the driving cord. While the coils were in motion, the galvanometer was generally unsteady, and it was always necessary to await a favorable moment before giving the signal for brakes.

The result of thirty runs with coil *A*, of which naturally a few of the later ones were more reliable than all the others, failed to show any distinct deflection due to the stopping of the coil. Under the conditions of the experiment a deflection of 2 mm. due to this cause could certainly have been detected, and even 1 mm. seems a reasonable upper limit. The galvanometer during these observations gave 1 mm. throw for 8 c.g.s. units of electromotive impulse with the coil in circuit. Hence kr in equation (13) has a numerical value 8. $\dot{y}_1$ was equal to 375 radians per second; so that

$$M < 0.021 \text{ c.g.s. for coil } A.$$

The extreme smallness of this number is best exhibited by comparing it with the values of L_1 and L_2. L_1 is the moment of inertia of the rotating body. As it seems reasonable to ascribe as much importance to the moving wood as to the moving wire of the coil, its value is $L_1 = 700000$ and $L_2 = 1500000$ (L_2 being the coefficient of self-induction of the coil). With the above limiting value for M, it is found that $M\dot{y}_1\dot{y}_2$ is less than $\frac{1}{55000000}$ of the total energy of the bicyclic system when $\dot{y}_1$ and $\dot{y}_2$ are numerically equal.

With coil *B* the impulsive electromotive force produced by stopping was certainly not large enough to give a deflection of $\frac{1}{2}$ mm. The sensitiveness of the galvanometer to impulsive electromotive force was somewhat less with coil *B* in circuit than with coil *A*, on account of the increased resistance; but 10 c.g.s. units of electromotive impulse gave 1 mm. deflection, so that kr in equation (13) is equal to 10. The value of $\dot{y}_1$ being 375 as before, we find

$$M < 0.013,$$

for coil *B*, $L_1 = 700000$ g.cm.^{2} as before, and $L_2 = 1000000$ (approximately); so that when $\dot{y}_1$ and $\dot{y}_2$ are equal, $M\dot{y}_1\dot{y}_2$ is less than $\frac{1}{600000000}$ of the total energy of the system.

The method of VI. was applied to coil *A* with the following results: The suspended needle itself was quite steady during the motion, the greatest trouble arising from an unconquerable vibratory motion of the telescope stand, which was tall and unstable. It was, however, certain that d_1 in equation (15) was less than 1 mm., the speed being 375 radians per second. A current $\frac{1}{90000}$ c.g.s. in A gave a deflection of 16 mm. $= d_2$, whence from (15)

$$a < \frac{1}{5400000000}.$$

Since $L_2 = 1500000$, equation (14) gives $M < 0.003$. The expression (16) has the approximate value 10^{-17}, so that if the assumptions of VI. hold true, less than one hundred thousand million millionth part of the energy of the rotating dead coil depends upon the motion in the ether produced by its rotation.

ITHACA, N.Y., January, 1894.

THREE PROBLEMS IN FORCED VIBRATION.

By William S. Franklin.

THE following is a discussion of three problems in forced vibration. The third problem is simple and well known, and as Helmholtz has said, it alone completely answers the problem of forced vibration. It is given here for the sake of the graphical solution which, though certainly not new, seems never to have been mentioned in this connection.

Problem I. — *An unknown periodic force F acts upon a body, the motion of which is observed. It is required to find the harmonic components of F.*

We have

$$F = \sum_m A_m \operatorname{Cos} mqt + \sum_m B_m \operatorname{Sin} mqt, \tag{1}$$

in which the A's and B's are the quantities to be determined.

Let x be the variable co-ordinate of the body; then from the observed motion we have

$$x = \sum_m C_m \operatorname{Cos} mq(t+\delta) + \sum_m D_m \operatorname{Sin} mq(t+\delta), \tag{2}$$

in which everything is known except δ, a time interval introduced in order that t may be the same in equations (1) and (2). To determine the A's and B's, the motion due to F must be calculated and compared with the observed motion. The total motion due to F is the superposition of the various simply harmonic motions due to the respective terms of (1) separately.

Consider the equation (compare equation (25))

$$\frac{d^2x}{dt^2} + \beta\frac{dx}{dt} + \gamma x - A\epsilon^{imqt} = 0, \tag{3}$$

in which β is the coefficient of damping; $\gamma = \frac{4\pi^2}{\tau^2}$, where τ is the proper period of vibration of the body, the mass of which is taken as unity for simplicity; x is the displacement, complex of course; and $i=\sqrt{-1}$. Equation (3) can be broken up into two ordinary equations, viz.:

$$\frac{d^2x}{dt^2}+\beta\frac{dx}{dt}+\gamma x-A\,\mathrm{Sin}\,mqt=0, \tag{4}$$

in which ix is understood to be the imaginary part of x in (3), the i being cancelled; and

$$\frac{d^2x}{dt^2}+\beta\frac{dx}{dt}+\gamma x-A\,\mathrm{Cos}\,mqt=0, \tag{5}$$

in which x is understood to be the real part of x in (3). Hence if we obtain the solution of (3), the real part of this solution will be the solution of (5); *i.e.* the motion due to the force $A\,\mathrm{Cos}\,mqt$. The imaginary part of this solution, dropping i, will be the motion due to the force $A\,\mathrm{Sin}\,mqt$. The solution of (3), for our purpose, is

$$x=Be^{imqt}, \tag{6}$$

in which B is unknown. Substituting x, $\frac{dx}{dt}$, and $\frac{d^2x}{dt^2}$ from (6) in (3), we obtain an equation, free from t, from which B may be determined. Substituting this value of B in (6), and breaking up the resulting equation into its real and imaginary parts, we have, writing A_m and B_m in turn for A,

$$x=\frac{A_m(\gamma-m^2q^2)}{(\gamma-m^2q^2)^2+m^2q^2\beta^2}\mathrm{Cos}\,mqt+\frac{A_m mq\beta}{(\gamma-m^2q^2)^2+m^2q^2\beta^2}\mathrm{Sin}\,mqt.$$

This expression gives the simple harmonic motion maintained by the force $A_m\,\mathrm{Cos}\,mqt$; and

$$x=\frac{B_m(\gamma-m^2q^2)}{(\gamma-m^2q^2)^2+m^2q^2\beta^2}\mathrm{Sin}\,mqt-\frac{B_m mq\beta}{(\gamma-m^2q^2)^2+m^2q^2\beta^2}\mathrm{Cos}\,mqt$$

gives the simply harmonic motion maintained by the force $B_m\,\mathrm{Sin}\,mqt$. The total motion maintained by F is then

$$\begin{aligned}x=&\sum_m\left[\frac{A_m(\gamma-m^2q^2)}{(\gamma-m^2q^2)^2+m^2q^2\beta^2}-\frac{B_m mq\beta}{(\gamma-m^2q^2)^2+m^2q^2\beta^2}\right]\mathrm{Cos}\,mqt\\&+\sum_m\left[\frac{A_m mq\beta}{(\gamma-m^2q^2)^2+m^2q^2\beta^2}+\frac{B_m(\gamma-m^2q^2)}{(\gamma-m^2q^2)^2+m^2q^2\beta^2}\right]\mathrm{Sin}\,mqt.\end{aligned} \tag{7}$$

Expanding $\operatorname{Cos} mq(t+\delta)$ and $\operatorname{Sin} mq(t+\delta)$ in equation (2), we have for the observed motion

$$x=\sum_m [C_m \operatorname{Cos} mq\delta + D_m \operatorname{Sin} mq\delta] \operatorname{Cos} mqt$$
$$+\sum_m [D_m \operatorname{Cos} mq\delta - C_m \operatorname{Sin} mq\delta] \operatorname{Sin} mqt. \tag{8}$$

Since the calculated and observed motions are identical, all assumed conditions being realized, we obtain, by equating the coefficients of $\operatorname{Sin} mqt$ and $\operatorname{Cos} mqt$ respectively in (7) and (8), a set of equations, the number of which is equal to twice the largest value of m. From these equations, γ and β being known, δ may be eliminated, and all the A's and B's calculated in terms of one of them.

The ratio of the amplitude of the mth harmonic of the impressed force to the amplitude of the mth harmonic of the motion approaches zero as m increases. When m is large so that equation (2) must be considered as a finite series, small errors in the determination of the motion introduce very large errors into the values of A_m and B_m. Similar remarks apply to problem II. in regard to the summation with respect to m.

If the motion due to the force F be observed before it becomes steady, then in addition to the above motion there will be a superposed vibratory motion of the body with its *proper* period and damping, such that at the instant at which F begins to act, the velocity and displacement pertaining to this proper motion will be equal and opposite to the velocity and displacement pertaining to the maintained vibrations each to each. In that case also the problem is easily solved.

The motion produced by a periodic force is *similar* to the force when corresponding harmonics of the motion and of the force have their amplitudes in a constant ratio and their phase identical. By comparing (1) and (7), this is seen to require that

$$\frac{\gamma-m^2q^2}{(\gamma-m^2q^2)^2+m^2q^2\beta^2}$$

be independent of m, and that

$$\frac{mq\beta}{(\gamma-m^2q^2)^2+m^2q^2\beta^2}$$

be equal to zero. These two conditions are satisfied if γ is very large, *i.e.* if the proper period of the body is very short. Slight differences in the phase of corresponding harmonics have less effect than differences in amplitude in producing dissimilarity. It follows, therefore, that for a given large value of γ the similarity may be made more nearly exact by so choosing $\beta(=\sqrt{2\gamma})$ that the middle term in $(\gamma-m^2q^2)^2$ is annulled by $m^2q^2\beta^2$. This is easily seen to be true by arranging (7) thus:

$$x=\sum_m[X\,\mathrm{Cos}\,(mqt-\theta)+Y\,\mathrm{Sin}\,(mqt-\theta)],$$

in which

$$X=\frac{A_m}{\sqrt{(\gamma-m^2q^2)^2+m^2q^2\beta^2}},\quad Y=\frac{B_m}{\sqrt{(\gamma-m^2q^2)^2+m^2q^2\beta^2}},$$

and

$$\mathrm{Sin}\,\theta=\frac{mq\beta}{\sqrt{(\gamma-m^2q^2)^2+m^2q^2\beta^2}}.$$

The point in question is to make X and Y independent of m with little reference to the value assumed by θ. However, $\beta\frac{dx}{dt}$ (see equation (3)), is ordinarily a roughly approximate expression for damping force, so that to choose β large may seriously complicate the motion. It is best to select a small value for β in any case.

Problem II. — *An alternating current flows in a wire stretched between the poles of a constant electro-magnet. The motion of a point of the wire is observed; required the harmonic components of the current.*

Let

$$f=\sum_n A_n\,\mathrm{Sin}\,npx \tag{9}$$

be the magnetic field at the point x of the wire, in which the A's are known, $p=\frac{2\pi}{l}$, and l is the length of the wire. The origin is to be taken at one end of the wire, the undisturbed wire being chosen as X-axis, and f being everywhere parallel to the Z-axis. Let

$$I=\sum_m[B_m\,\mathrm{Sin}\,mqt+C_m\,\mathrm{Cos}\,mqt] \tag{10}$$

be the alternating current, in which $q=\frac{2\pi}{\tau}$, τ being the period of the alternations. The B's and C's are required.

The force F per unit length per unit mass of wire is proportional to fI. Ignoring the proportionality factor, we have

$$F=\sum_m\sum_n A_nB_m \operatorname{Sin} npx \operatorname{Sin} mqt+\sum_m\sum_n A_nC_m \operatorname{Sin} npx \operatorname{Cos} mqt. \qquad (11)$$

The total motion maintained by F is obtained by the superposition of the motions maintained by the separate terms. These separate motions are to be found by solving the equations

$$\frac{d^2y}{dt^2}+\beta\frac{dy}{dt}-c^2\frac{d^2y}{dx^2}-A_nB_m \operatorname{Sin} npx \operatorname{Sin} mqt=0, \qquad (12)$$

and

$$\frac{d^2y}{dt^2}+\beta\frac{dy}{dt}-c^2\frac{d^2y}{dx^2}-A_nC_m \operatorname{Sin} npx \operatorname{Cos} mqt=0, \qquad (13)$$

in which β is the damping coefficient of the wire per unit length, and c^2 is a constant depending upon the tension and mass of wire.

These equations do not admit of any great simplification by the introduction of complex quantities as in problem I.

The maintained vibrations depending upon (12) are of form[1]

$$y=D_{mn} \operatorname{Sin} npx \operatorname{Sin} mqt+E_{mn} \operatorname{Sin} npx \operatorname{Cos} mqt. \qquad (14)$$

If we substitute $\frac{dy}{dt}$, $\frac{d^2y}{dt^2}$, and $\frac{d^2y}{dx^2}$ from (14) in (12) and place, in the resulting equation, the coefficients of Sin mqt and Cos mqt respectively, equal to zero, we have two equations from which we find

$$D_{mn}=\frac{(c^2n^2p^2-m^2q^2)A_nB_m}{(c^2n^2p^2-m^2q^2)^2+m^2q^2\beta^2}, \qquad (15)$$

$$E_{mn}=-\frac{mq\beta A_nB_m}{(c^2n^2p^2-m^2q^2)^2+m^2q^2\beta^2}. \qquad (16)$$

---

[1] The motion of any system having one degree of freedom, maintained by a simply periodic force, is of the form $A \operatorname{Sin} qt + B \operatorname{Cos} qt$, in which $q=\frac{2\pi}{\tau}$, τ being the period of the force. The wire may be considered as a system having one degree of freedom so far as any one of the terms of (11) is concerned. Indeed, any system may be considered as having one degree of freedom so far as concerns the action of an impressed distributed force depending upon a single variable, provided that we ignore such motion as is *proper* to the system, such, that is, as is independent of impressed force.

Similarly, the maintained vibrations depending upon (13) are of the form

$$y = D'_{mn} \operatorname{Sin} npx \operatorname{Sin} mqt + E'_{mn} \operatorname{Sin} npx \operatorname{Cos} mqt, \tag{17}$$

in which

$$D'_{mn} = \frac{mq\beta A_n C_m}{(c^2n^2p^2 - m^2q^2)^2 + m^2q^2\beta^2}, \tag{18}$$

and

$$E'_{mn} = \frac{(c^2n^2p^2 - m^2q^2)A_n C_m}{(c^2n^2p^2 - m^2q^2)^2 + m^2q^2\beta^2}. \tag{19}$$

Summing (14) and (17) for all values of m and n, we have for the total motion maintained by (11)

$$\begin{aligned} y = &\sum_m\sum_n [D_{mn} \operatorname{Sin} npx + D'_{mn} \operatorname{Sin} npx] \operatorname{Sin} mqt \\ &+ \sum_m\sum_n [E_{mn} \operatorname{Sin} npx + E'_{mn} \operatorname{Sin} npx] \operatorname{Cos} mqt. \end{aligned} \tag{20}$$

Let the motion of the wire be observed at the point x, so that for this point

$$y = \Sigma[G_m \operatorname{Sin} mq(t+\alpha) + H_m \operatorname{Cos} mq(t+\alpha)], \tag{21}$$

in which the G's and H's are known, and α is an unknown time interval. By expansion of $\operatorname{Sin} mq(t+\alpha)$ and $\operatorname{Cos} mq(t+\alpha)$, (21) becomes

$$\begin{aligned} y = &\sum_m [G_m \operatorname{Cos} mq\alpha - H_m \operatorname{Sin} mq\alpha] \operatorname{Sin} mqt \\ &+ \sum_m [G_m \operatorname{Sin} mq\alpha + H_m \operatorname{Cos} mq\alpha] \operatorname{Cos} mqt. \end{aligned} \tag{22}$$

This must be identical with (20), whence by equating coefficients, we have equations of the form

$$\sum_n [D_{mn} \operatorname{Sin} npx + D'_{mn} \operatorname{Sin} npx] = G_m \operatorname{Cos} mq\alpha - H_m \operatorname{Sin} mq\alpha, \tag{23}$$

$$\sum_m [E_{mn} \operatorname{Sin} npx + E'_{mn} \operatorname{Sin} npx] = G_m \operatorname{Sin} mq\alpha + H_m \operatorname{Cos} mq\alpha. \tag{24}$$

The number of such equations is equal to twice the largest value of m. The quantities c^2, l, and β being known, equations (23) and (24) involve the B's and C's and α as the only unknowns. We may, therefore, eliminate α, and calculate all the B's and C's in terms of one of them.

If the motion of the wire be observed before it becomes steady, then there will be, in addition to the above motion, a superposed proper motion of the wire, such that at the instant the alternating circuit is closed, the velocity and displacement of each point of the wire pertaining to this proper motion will be equal and opposite to the velocity and displacement pertaining to the maintained motion (considered as being already under way). In this case also the problem is easily solved.

PROBLEM III. — *A body of unit mass is maintained in vibration by a simply periodic force,* $F\,\mathrm{Sin}\,qt$; *required its amplitude and phase.*

Let $-\gamma x$ be the force, due to elasticity, tending to bring the body from a displacement x to its position of equilibrium, and let $-\beta\frac{dx}{dt}$ be the damping force which acts upon the body. Consider a fictitious force $-\frac{d^2x}{dt^2}$ equal and opposite to that required to accelerate the body in its motion. Then the four forces,

$$F\,\mathrm{Sin}\,qt,\quad -\gamma x,\quad -\beta\frac{dx}{dt},\ \text{and}\ -\frac{d^2x}{dt^2},$$

are in equilibrium, and

$$\frac{d^2x}{dt^2}+\beta\frac{dx}{dt}+\gamma x-F\,\mathrm{Sin}\,qt=0. \qquad (25)$$

For the graphical solution we assume that vibrations maintained by a simply periodic force are also simply periodic and of the same period as the force. Any consistent result to which this assumption leads will be a legitimate solution of the problem. Let X (unknown) be the maximum value of x; x being simply periodic; so also are γx, $\beta\frac{dx}{dt}$, and $\frac{d^2x}{dt^2}$ with maximum values γX, $q\beta X$, and q^2X respectively. The instantaneous values of x, $-\gamma x$, $-\beta\frac{dx}{dt}$, $-\frac{d^2x}{dt^2}$, and F Sin qt will be represented by the respective projections, on any stationary line, of a rigid system of lines representing X, γX, $q\beta X$, q^2X, and F to scale, and rotating at angular velocity q in the diagram, Fig. 1. These lines, γX, $q\beta X$,

q^2X, and F are respectively π, $\frac{3\pi}{2}$, 0, and θ (unknown) ahead of the line X. In order that the sum of the projections of γX, $q\beta X$, q^2X, and F may always be zero, the vector sum of these lines must be zero, so that $-F$ is the diagonal of the rectangle shown.

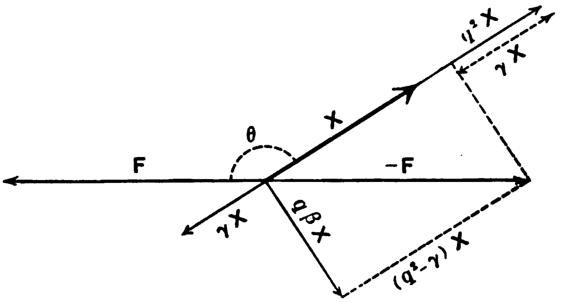


Fig. 1.

From this diagram we have

$$F^2 = q^2\beta^2X^2 + (q^2-\gamma)^2X^2,$$

or

$$X = \frac{F}{\sqrt{(q^2-\gamma)^2+q^2\beta^2}}, \tag{26}$$

and

$$\tan\theta = \frac{q\beta}{q^2-\gamma}. \tag{27}$$

These two equations determine the amplitude X of the vibrations, and the phase difference θ between the force and the vibrations. It is well to notice, what the diagram shows with great clearness, that any simply periodic force which maintains a body in vibration is broken up into two parts, one of which balances damping, and the other of which combines with elastic forces and determines the period of the vibrations.

In conclusion, we would call the reader's attention to the building-up process employed in problems I. and II., to the note to problem II., and to von Helmholtz's words concerning this third problem: "Hierdurch ist das Problem der erzwungenen Schwingungen gänzlich beantwortet." Perhaps the only case of maintained vibration which is not built up from this problem III.

is that of a string, forced by a tuning fork, which produces a periodic variation in the tension of the string — Melde's experiment. For a discussion of this problem see Rayleigh, *Philosophical Magazine* (5), Vol. 15, p. 229. Vibrations forced in a string by lateral *motion* of one of its ends also does not clearly fall in line with the above. For very good discussion of such, see A. Elsas, *Wiedemann's Annalen*, 23, pp. 161–173.

MINOR CONTRIBUTIONS.

THE THOMPSON PHYSICAL LABORATORY AT WILLIAMS COLLEGE.

By Henry Lefavour.

THE Thompson Physical Laboratory, in addition to two other buildings for the chemical and biological departments respectively, was built and presented to Williams College by Frederick F. Thompson, Esq., of New York City. The architect was Mr. Francis R. Allen, of Boston, and the principal contractors were Messrs. Haskell Dodge & Co., also of Boston.

The buildings are constructed of "rain-struck" brick, trimmed with Warsaw bluestone, their front elevations of the same height, and planned with reference to a symmetrical arrangement. The physical laboratory (Fig. 1) is the easternmost of the three, and is separated from the second building, the chemical laboratory, by an interval of about 50 feet. It has a frontage of 80 feet on the north, a depth of 60 feet on the west, while

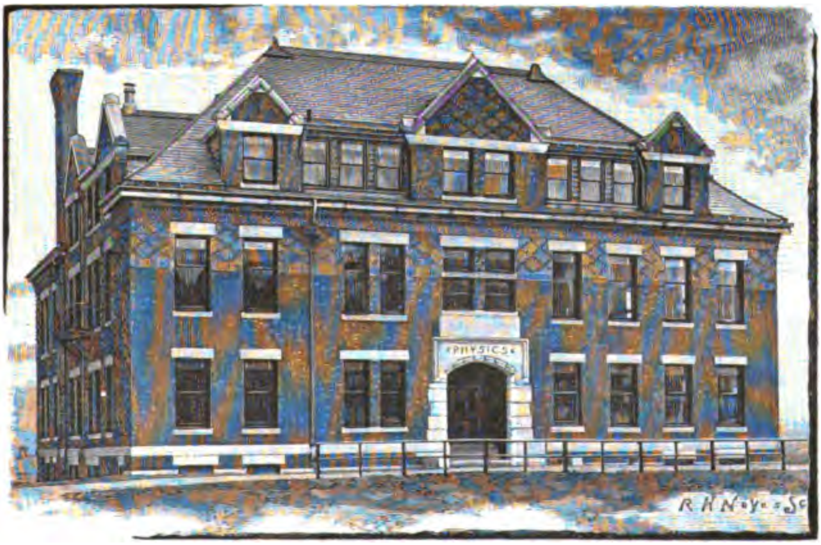
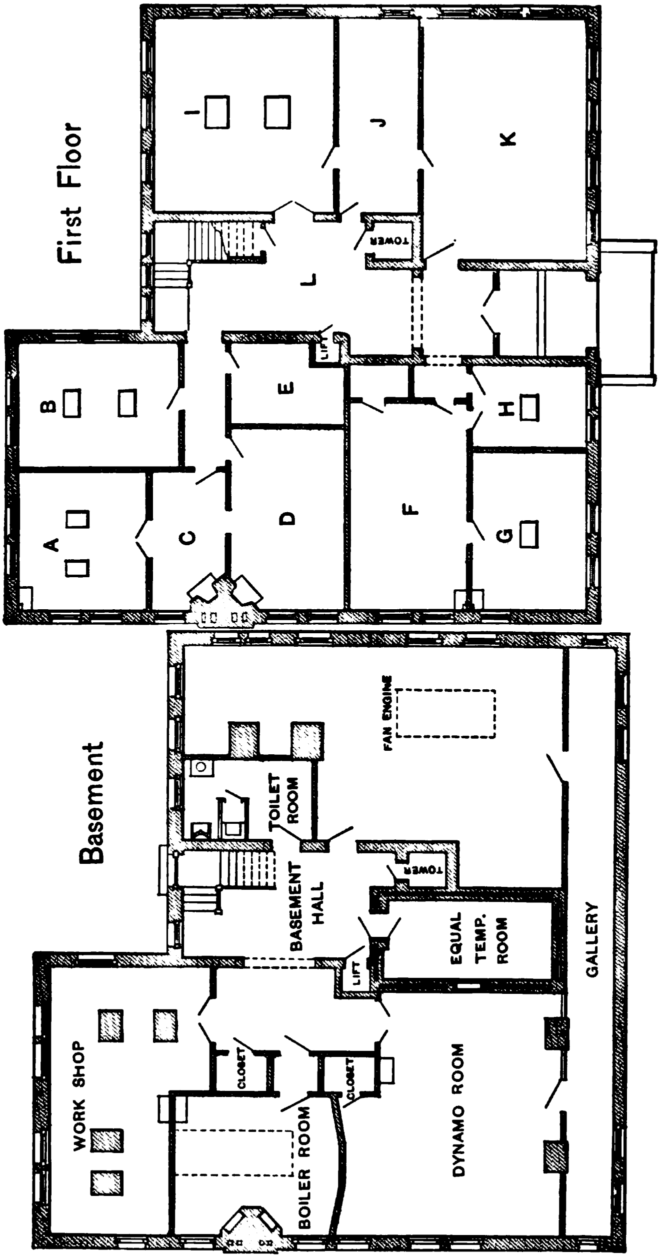


Fig. 1.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Basement
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EQUAL TEMP. ROOM
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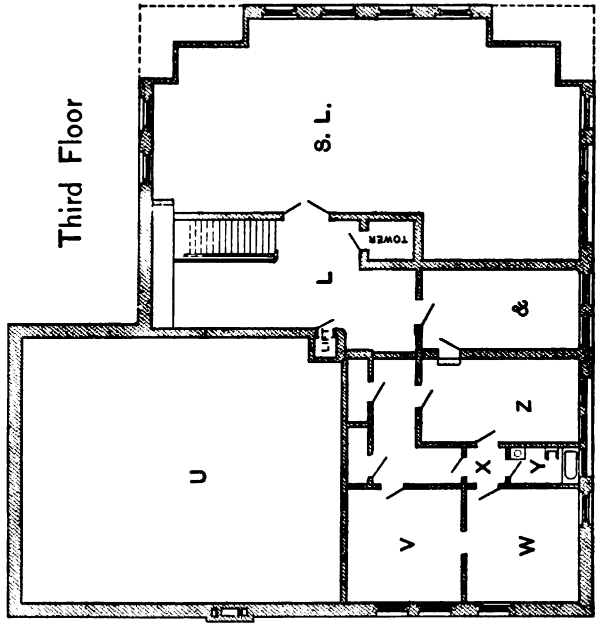
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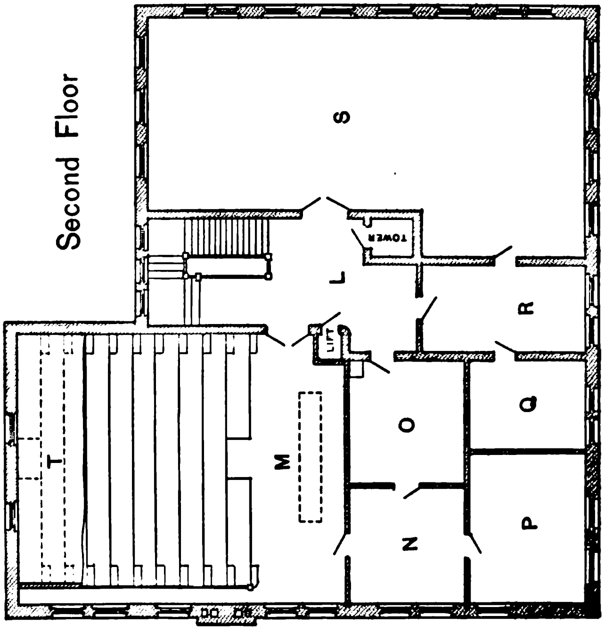
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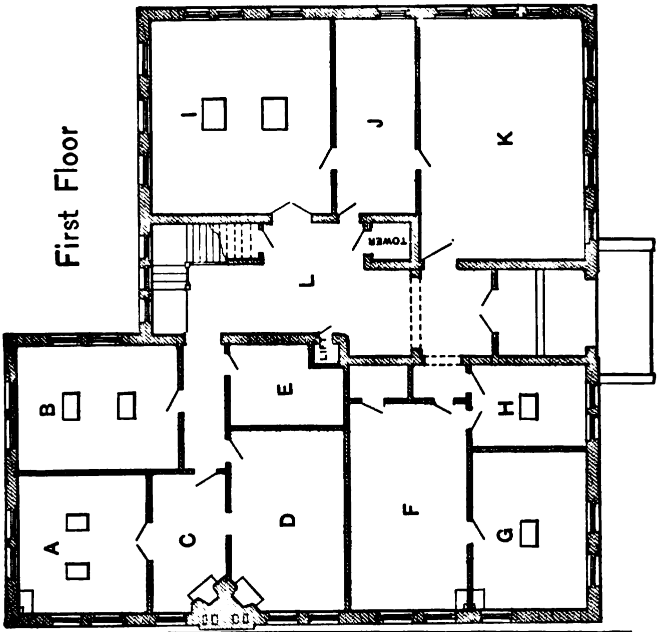
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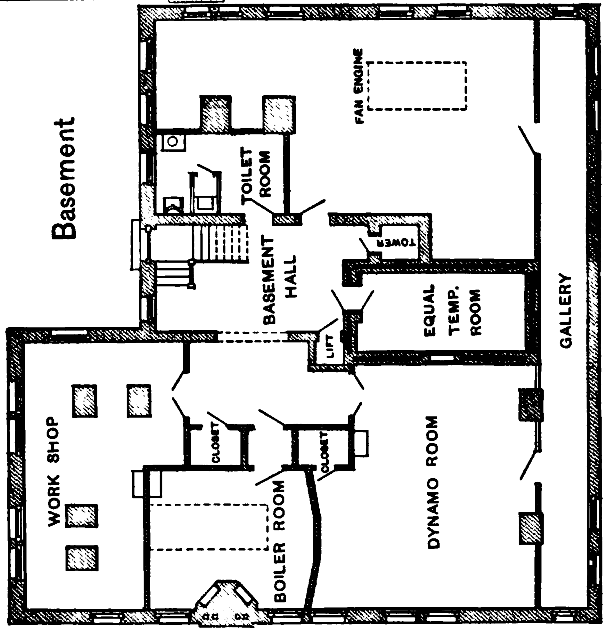
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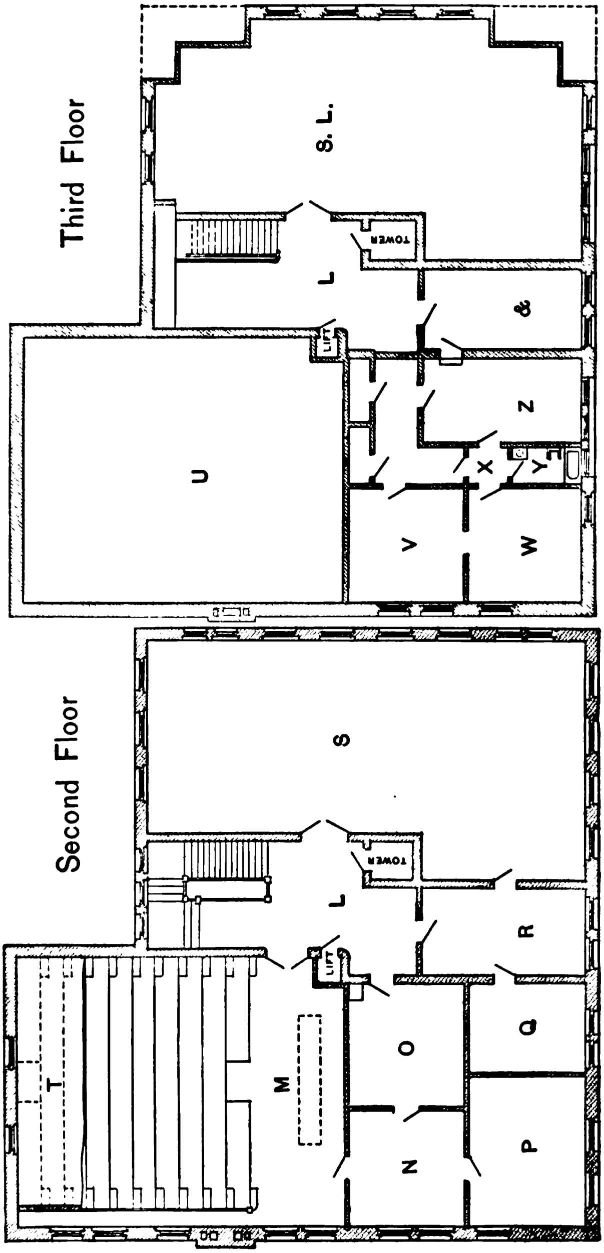
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the eastern half is carried back by a projection to a total depth of 77 feet. The heights of the different floors vary, that of the basement being 8 feet in the clear; the first and third floors, 10 feet and 6 inches; and the second floor, 12 feet and 10 inches. The lecture room has the height of two floors for the most part, but all the rooms of the third floor lose in height near the walls because of the shape of the roof.

The interior is finished in hard pine, the walls and ceilings are plastered with King's Windsor cement, and are tinted with a light cream color. The building is divided into three sections by brick partitions, which enclose the hallways, and the floor timbers rest on iron girders, which in turn are supported by these walls. Eight brick piers with stone caps rise from the foundations into various rooms of the first story, and all workrooms are amply provided with slate shelves imbedded in the brick walls.

The laboratory is heated by direct radiation and by forced draught; the former method being employed in the small rooms, the latter in the larger and more crowded rooms, thus securing thorough ventilation at the same time. The steam is brought from the chemical laboratory at high pressure, and is distributed to the engine which operates the Sturtevant blower, and to a reducing-valve, from which it is carried at the ordinary low pressure to the radiating coils. Each workroom is supplied with hoods, water, gas for illuminating and heating purposes, electric current, and with cases for apparatus.

The instruction in physics in the college contemplates a required course in general physics for the sophomores, to be carried on by lectures, recitations, and elementary laboratory work, a year of more advanced general physical measurements (which is elective to the juniors), and a third year's course in special problems or thesis work for those of the senior class who may have developed a special interest in the subject. The arrangement of the rooms, therefore, includes a lecture room and the accessory apparatus rooms, an elementary laboratory, a general laboratory, and a few smaller rooms to be equipped for special investigations and measurements, as well as for the use of professors and assistants. The accompanying floor plans show the situation of the different rooms.

The basement contains a workshop, for which power is to be supplied by water and electric motors, a boiler room containing a 20 h.-p. boiler, used exclusively to furnish power for the engine, a toilet room, a room for the Sturtevant blower and the heating coils, the room to be used also for unpacking and storing cases, a room with double walls, in which the temperature can be maintained fairly constant, a long gallery extending the length of the building, for experiments requiring a long uninterrupted space, closets for chemicals and the storage battery, and a dynamo room. The dynamo room is equipped with a 15 h. p. Westinghouse engine, a

5 k. w. incandescent dynamo, two experimental motors, and the usual regulating and measuring apparatus.

On the first floor are the professor's office, *C*, with his private laboratory, *A*, on the one side, and the library of the laboratory, *D*, on the other. By permission of the Trustees, the books and periodicals that are of especial interest to the several scientific departments are now kept in the respective laboratories, and it has already proved to be of the greatest advantage to have the books so accessible. *B*, *H*, *F*, and *I* are the rooms to be devoted to advanced physical work, *B* to optical or spectrometric measurements, *F* to photography and chemical physics, and *I* to electrical testing. In the last room and in its vicinity an effort has been made to avoid the use of iron in any way that would make the magnetic field unusually variable. The small room *H* is reserved for any general research work. A recitation room, *K*, with seats for fifty, is at the right of the entrance, and next to it is the coat room, *J*. *E* is a storeroom for laboratory supplies, and *G* is the assistant's private laboratory.

On the second floor, *S* is the general laboratory for the juniors. It is furnished with movable tables with slate tops, with wall shelves, a bookcase, a case for chemicals, and a set of lockers for the students. Leading from this is the laboratory apparatus room, *R*, which also serves as the assistant's office. *Q* is used for acoustical measurements or for any work that requires isolation. Next to this is the photometric room, *O*, without windows and with blackened walls. The lecture room, *M*, will seat about 140. It is furnished with the Heywood chair, with tablet, arm, and hat rack. The tiers rise with a general slope of one foot in three. The lecture table has the usual conveniences of water, gas, and electricity, and in addition there are pipes for bringing compressed air, oxygen, and hydrogen from the basement. There are places for the projecting lantern in the rear of the room and in front of the lecture table, and for both positions there are permanent plaster screens on the wall behind the table. The room is lighted by eight windows, and these may be darkened by black, opaque shades, reinforced by shutters when there is need of perfect darkness. These shades can be operated from three points, but it is possible to have them all connected with a water cylinder. For the present, whenever there is need of interrupting the projections, the room will be lighted by electricity. The sun's rays may be brought into the room through either the east or the south windows. The room is connected with the dynamo room by a speaking-tube, and the whole building is well supplied with electric bells. Adjoining the lecture room are the apparatus rooms, *N* and *P*, and the space under the seats, *T*, is also used for storing apparatus.

On the third floor is the elementary laboratory, *S.L.*, of nearly the same size as the general laboratory below it, but it is more simply equipped.

The remainder of the floor is taken up with two suites of assistants' rooms, *V–W* and *Z–&*, the bath room, *Y*, and the upper part of the lecture room, *U*. Provision in case of fire is made by having the stairways of iron with stone treads, and by placing two fire escapes on the outside. A brick-walled shaft, marked "Tower" on the plans, extends from the basement to the roof, and affords facilities for pendulum and other experiments requiring height or depth, and a small lift serves to transport apparatus from the basement to the upper floors.

As is frequently the case, there have been restrictions which prevented the fullest realization of the original plans, and larger experience always suggests improvements, but thus far the building has shown itself in its six months of use to be admirably adapted to the demands made upon it.

REACTION VELOCITY AND BOILING-POINT.

By J. E. Trevor and F. L. Kortright.

It is well known that the action of acids upon aqueous solutions of cane sugar consists in a transformation of the cane sugar as such into a mixture of dextrose and levulose, the so-called invert sugar, a transformation represented by the chemical equation

$$C_{12}H_{22}O_{11} + H_2O = C_6H_{12}O_6 + C_6H_{12}O_6.$$

The acid present remains entirely unaffected. During this process the molecular concentration of the solution, *i.e.* the number of gram-molecules of dissolved substance per liter, suffers a continuous change, increasing to twice its initial value as the chemical transformation approaches completion.

In consequence of this the change, with the time, of any property of the solution which is dependent upon the molecular concentration, furnishes a ready means for direct observation of the velocity with which the chemical reaction in question takes place. The decrease of the vapor pressure of the solution, the depression of its freezing-point below that of water, the rise of its boiling-point, and the solubility of its solvent in some second liquid are such properties, each being directly proportional to the molecular concentration, or very nearly so. It follows, then, for example, that the boiling temperature of a sugar solution must rise as the inversion proceeds, and that the velocity with which the boiling-point is displaced measures the velocity of the reaction. This furnishes a very convenient method for observing the latter.

From the classical researches of Wilhelmy[1] and of Ostwald,[2] it is known that the velocity with which cane sugar is inverted by an aqueous acid is, at any moment, proportional to the concentration of the sugar remaining unchanged, and to the acidity of the acid; *i.e.* to the concentration of its electrolytically dissociated, free hydrogen ions. For an acid of a constant concentration this latter quantity remains constant. The relations obtaining are expressed by the differential equation

$$\frac{dx}{dt} = k \cdot a(s - x),$$

in which x represents the amount of sugar which is inverted at the time t, s the initial concentration of the sugar, and a that of the acid; k is the proportionality factor. For s is to be substituted the rise of boiling-point of the original solution, and for x the temperature increase at the time t over that due to the sugar and acid employed. Integrating from 0 to t, and noting that for $t = 0$, $s - x = s$, there results

$$\frac{1}{t} \cdot \log \frac{s}{s - x} = k \cdot a.$$

For the experimental determination of this constant ka, a small quantity (0.220 grams) of succinic acid was added to a boiling solution of cane sugar contained in a Beckmann boiling-apparatus,[3] and successive readings of the boiling temperature of the liquid were taken. The sugar solution contained 33 grams of granulated sugar in 70 grams of a very pure distilled water, and was, therefore, approximately a normal solution. The thermometer employed was graduated to hundredths of a degree, and was read with a lens. The results obtained are as tabulated:

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[1] Poggendorf's Annalen, 81, p. 413 (1850).

[2] Journal für praktische Chemie, 29, p. 385 (1884).

[3] Zeitschrift für physikalische Chemie, 4, 532 (1889).

TABLE.

$s = 0^\circ.759$. $a = 0^\circ.008$.

t	x	$\frac{1}{t}\log_{10}\frac{s}{s-x}$
6 minutes	$0^\circ.114$	0.0118
9	$0^\circ.171$	122
16	$0^\circ.275$	122
19	$0^\circ.311$	121
22	$0^\circ.342$	118
25	$0^\circ.383$	122
28	$0^\circ.412$	121
31	$0^\circ.442$	122
34	$0^\circ.463$	120
37	$0^\circ.485$	120
40	$0^\circ.506$	119
43	$0^\circ.523$	118
85	$0^\circ.683$	118
Mean		120

The observed constancy of the characteristic logarithmic expression is as satisfactory as might have been anticipated, especially since no correction was made for the influence of fluctuations in the barometric pressure. During the experiment the barometer fortunately remained very nearly constant at 754.5 mm. At the last reading, after the lapse of an hour and a half, the table shows $\frac{683}{759}$, or 90 per cent, of the total sugar present to have been inverted.

The accuracy of the method might be materially increased by carefully connecting the boiling apparatus with a large, empty bottle, and closing all connection with the outside atmosphere at the beginning of each experiment. In this way the work would be conducted under an unvarying pressure, and from simultaneous barometric readings the observed boiling-points could be reduced to a normal condition of pressure. By making the experiments with, perhaps, three different concentrations of the sugar solution, and extrapolating for the velocity constant at zero concentration, the effect of the sugar in altering the electrolytic dissociation of the acid, and consequently the velocity of inversion, would be very largely eliminated. The ratio of each constant so obtained to that found with a very dilute, *i.e.* completely dissociated, solution of hydrochloric acid would then give in each case the relative concentration of the free hydrogen ions of the acid employed.[1]

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[1] See Trevor, Zeitschrift für physikalische Chemie, 10, 321 (1892).

In this way it would become possible to detect, with a very considerable accuracy, the degree of electrolytic dissociation of the acids at high temperatures, and from the data so secured, by a comparison with the very extended and accurate determinations made by Ostwald[1] at 25°, the change of this dissociation with changing temperature would become very much better known than is at present the case. This knowledge would render accessible, by direct calculation, that important and but slightly known quantity, the heat of electrolytic dissociation, which determines the influence of temperature change upon the dissociation of electrolytes.

CORNELL UNIVERSITY, January, 1894.

DYNAMICS OF THE ETHER.

BY A. E. KENNELLY AND REGINALD A. FESSENDEN.

THE fundamental equation in the dynamical theory of the propagation of disturbances in an infinite isotropic medium such as the ether may be, is

$$\rho\ddot{v} = F\rho + \nabla P,$$

where v is the vector velocity, ρ the density, F any external force subject to a potential, and P the impressed force per unit area.

The existing dynamical theory of ether, therefore, assumes that ether possesses inertia like material media, and the same equation is also fundamental in determining the propagation of disturbances in material media such as liquids or gases.

But it is possible that the inertia, which is the attribute of matter, is really an effect due to the ether pervading matter, and not in the matter itself, or in other words, it is possible that if a body could be completely divested of its internal ether, and be then placed in a space devoid of ether, it would possess no inertia.

If this should be the case, the property of inertia could not be assumed to be possessed by the ether, itself considered as matter, without further justification, and the present fundamental dynamical equation could no longer be claimed.

The conclusions which follow from this equation, when applied to the ether, are known to involve the existence of two independent systems of waves from any source of disturbance: the first, a wave of distortion and rotation without compression, moving with velocity $\sqrt{(\nu/\rho)}$; and the second, a wave of simple compression like a sound wave in air, proceeding with a velocity $\sqrt{(\sigma + \frac{4}{3}\nu)/\rho}$, where σ is the resistance to compression, and ν the rigidity. The former accords with the behavior of light, while there is experimental evidence that the latter wave is either totally absent or has only a very diminutive existence. In order to account for the absence of this sound-like wave, it has been suggested that its velocity of propagation is either enormously great, or exceedingly small, which requires that the resistance σ of the ether to compression should be either very large, compared to its resistance to shearing ν, or should have a certain negative value; *i.e.* that the medium would have to be prevented from exploding.

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[1] Zeitschrift für physikalische Chemie, 3, 170 (1889).

It appears, therefore, that both these difficult suppositions can be avoided if the inertia possessed by matter is due to reaction from distortion of the ether within that matter, for in that case the equations of motion hitherto assumed for ether would need modification.

That such distortion takes place in the ether within a moving body, seems indicated by the experiments of Fizeau and Michelson; that it is limited, at least sensibly, to the interior of the body, appears to be shown from the recent experiments of Lodge, while that reaction follows the distortion of ether is evidenced by the phenomena of light.

THE LEYDEN JAR AS A STORAGE BATTERY.

BY S. T. MORELAND.

FOR some years I have been in the habit of showing the following experiment to my classes. Take two large Leyden jars and connect their inner coatings respectively to the two poles of a ten-inch Toepler-Holtz electrical machine, their outer coatings being connected to each other. Separate the poles of the machine so as to get as long a spark as possible, and charge the jars nearly to the point of discharging. Now remove the belt and give the plate a slight backwards motion, and it will continue to revolve backwards for several seconds, and the jars will gradually lose their charge, the energy being used of course to drive the plate. For success in the experiment, it is necessary to have the bearings of the plate nicely cleaned and oiled, and to see that the brushes do not cause too much friction by rubbing against the buttons on the plate.

WASHINGTON AND LEE UNIVERSITY,
Oct. 5, 1893.

The Times of Descent along Chords of a Circle.

By James S. Stevens.

THIS note deals with a simple experiment to show that the times of descent down the chords of a circle are equal to each other and to the time of falling through the vertical diameter. I have devised a simple method of demonstrating the above proposition by the use of two pieces of lath joined together by a hinge. As it gives uniformly good results, I submit it to physical laboratory work.

AB and AC (Fig. 1) are two pieces of wood of equal length hinged at A, so that C can trace the curve CD.

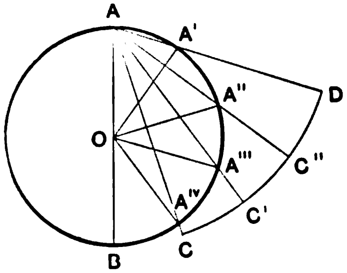

Fig. 1.

BAC can be made any angle up to 90°; and if a string equal to the radius AO be fastened at O and the other extremity be joined to the moving bar when at AC'' (say), then AA'' will be a chord of the circle. Now if the upper surface of AC'' is grooved so that a marble can roll down, and if a tin box be fastened at A'', marbles dropped through AB and rolled through AA'' will reach their destination simultaneously.

This will, of course, be true when the point of contact between the string and bar is at A', A''', or A^{iv}.

Maine State College, Dec. 2, 1893.

NEW BOOKS.

The Todhunter-Pearson History of Elasticity.[1]

Dr. Isaac Todhunter, at his death in 1884, at the age of sixty-four, left, in an unfinished manuscript, the fourth of his celebrated Mathematical Histories, — "The History of the Theory of Elasticity." This manuscript consists of two parts: the first is a history of the theory of elasticity; and the second, a mathematical treatise on the theory of the "perfect" elastic solid, is supplementary thereto. This manuscript contains analyses of most of the important mathematical memoirs up to 1870; it had been once rewritten, and then again revised by Dr. Todhunter. It was sent to Professor Cayley[2] for examination, and was pronounced of the same grade of excellence as the previous histories, and fairly complete. The task of completing and editing was soon afterwards entrusted, at the suggestion of Dr. Routh, to Professor Pearson by the Syndics of the University Press.

Volume I.,[3] pp. 924, Gallilei to Saint-Venant, 1639 to 1850, appeared in 1886. In the preparation of this volume Professor Pearson, "from his own account, devoted sufficient labor to have enabled him to complete it *ab initio*," without much exceeding the strict duties of editor. An extract from Vol. II. appeared in 1889 under the title "The Elastical Researches of Barré de Saint-Venant,"[4] pp. 286, of which Professor Greenhill says, it is "a monument of painstaking energy and enthusiasm!" The History now reaches completion by the appearance (1893) of Vol. II., Parts I. and II., pp. 762 and 558. This carries the analysis of individual memoirs completely to the year 1860, and the work of Saint-Venant and Boussinesq, Rankine and Lord Kelvin, F. Neumann, Kirchhoff, and Clebsch, up to the present date. The plan of the History has been considerably modified by Professor Pearson. It was decided to print only the historical part of Dr. Todhunter's manuscript, so that the gaps thus left had to be filled; Professor Pearson undertook — wisely, it seems — to widen the scope of the

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[1] *A History of the Theory of Elasticity and of the Strength of Materials* from Gallilei to the present time. By the late Isaac Todhunter, D.Sc., F.R.S. Edited and completed for the Syndics of the University. Press by Karl Pearson, M.A., Professor of Applied Mathematics, University College, London. Cambridge, 1886 and 1893.

[2] See obituary notice of Dr. Todhunter. Proc. R. Soc., Vol. 37, p. xxvii.

[3] Review by Professor A. G. Greenhill, Nature, Feb. 3, 1887.

[4] Review by Professor A. G. Greenhill, Nature, March 20, 1890.

History so as to include physical and technical elasticity; and the adoption of uniform terminology and notation became an evident necessity during the progress of Vol. I., which was printed in instalments. This uniformity is found in all but the first three chapters and part of Chapter IV., in which, indeed, terminology and notation are matters of some indifference. The changes thus introduced into Vol. I. are slight; in Vol. II., however, but little of Dr. Todhunter's manuscript remains, although the substance of most of it is no doubt incorporated. The History, judging mainly from the chapter (XIV.) devoted to Lord Kelvin and P. G. Tait, is thoroughly and well done, and constitutes indeed a "*Repertorium* of elasticity rather than an *Historique Abrégé.*" It is pleasing to note the extent to which Professor Pearson has been encouraged and assisted in the execution of the work; in the way of careful verification of the analysis throughout (not mere proof-reading); the correction and revision of certain chapters by those in position to make their revision more or less authoritative; and by friendly aid and suggestion from several hands. This is an important feature of the work, and must go far towards giving permanent value to the enormous amount of labor which Dr. Todhunter and Professor Pearson have devoted to the History.

The general value of such compendious writings is no doubt in proportion to the labor they involve, but to us Americans in particular their value cannot be overestimated, since they enable us to outlive, in a measure, the inconvenience of new and incomplete libraries. Yet many of us who study to teach will be slow to feel the benefit. We should keep in mind the "true ends of knowledge and endeavor, not after it for curiosity, contention, or the sake of despising others, nor yet for profit, reputation, power, or any such minor consideration, but solely for the occasions and uses of life." Study is the cosmic thread of action, otherwise dark, frantic confusion and chaos without end.

W. S. FRANKLIN.

On Vision.[1] By WILLIAM FERREL.

No one ever associates the name of the late Professor Ferrel with physiological optics. He was one of those who never attacked a subject except by methods exclusively his own; and never was it left without the attainment of truth new to himself. His modesty was such as might almost be deemed blamable; and the present essay would have remained forgotten but for the faithfulness of a scientific friend, who will incorporate it in a forthcoming volume of collected memoirs. It was published in 1855 in the Nashville *Journal of Medicine and Surgery*, and quite naturally attracted but scant attention at that time, for the subject was one of little interest to practitioners of the art of healing. It was one of the first papers ever published by its author, whose subsequent work established his position as a mathematician and meteorologist of the highest rank.

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[1] To be republished in a forthcoming collection of memoirs.

To appreciate this research it is necessary to know something of Ferrel's early life. Amid humble surroundings his opportunities for systematic education had been slender, and there was no access to scientific libraries or apparatus. Throughout a period of twenty years, beginning with 1838, he earned a livelihood by teaching school, for the most part in small western towns. His residence in Nashville was included between 1854 and 1858. This fact accounts for his recital of a few discoveries which were none the less his own, even if he was anticipated by others whose writings had not been accessible to him.

Ferrel's treatment of the subject of vision is plainly that of one accustomed to mathematical rather than metaphysical methods. He indulges in no speculation, and apparently consults no authorities until after he has reasoned out his problem analytically and tested his conclusions by experiment. On his first page he refers to the possibility of having been anticipated to some extent by Wheatstone, whose papers he had been unable to see. Wheatstone's communication on the "Physiology of Vision" had been given to the Royal Society in 1838, but no general impulse to the study of this subject was received until 1849, when Brewster's refracting stereoscope was brought before the public, while the new art of photography was rapidly developing so as to make it possible to construct stereographs more accurately and easily than by hand. Ferrel was evidently unaware of the warm controversy between these two eminent physicists on the question of priority regarding the invention of the stereoscope, and his investigation was undertaken independently of what they had done.

The discussion is begun with a consideration of the law of visible direction for a single eye. The theory of Brewster, then generally accepted, was that we see every object in the direction of a perpendicular to the retinal point on which the object is depicted. Ferrel shows mathematically that this hypothesis is applicable only to a special case in which the visual line passes through both the optical center of the refracting combination and the geometric center of the eyeball, supposing the latter to be spherical. If the surface of the cornea be assumed spherical, its center of curvature does not coincide with either of the two centers just named. These facts being taken into account, it is shown by the laws of refraction that no object seen by indirect vision can appear in accordance with Brewster's hypothesis, which requires that the visual line shall pass through the center of the eyeball. On the contrary, an object must appear in the direction of the optical center of the refracting combination, which may be assumed without sensible error to be that of the crystalline lens. This direction is in general oblique to the retinal surface impressed. The following experiment, devised by Ferrel, serves as an excellent and decisive test, to show that by indirect vision the apparent direction of an object is nearer to the axis of vision than is its real direction. Let the eye be placed just above the corner of a rectangular table, and directed along the bisector of the right angle. Then by indirect vision the two adjacent sides next the eye appear to form with each other an angle considerably less than a right angle. The illusion is quite striking when perceived for the first time.

An inquiry is then made regarding the locus of distinct vision with a single eye; for example, in a horizontal plane. This is shown to be a circle passing through the optical center of the crystalline, but with variable radius, depending upon the distance for which the accommodation of the eye is adapted. To test this experimentally is not so easy as in the previous case, for not only does the accommodation involuntarily vary, but it is impossible to assign a definite limit at which distinct vision begins to become indistinct, especially when the vision is indirect.

The author next treats of the principles of binocular vision. The circle of distinct vision for a single eye must pass through both optic centers, since by experiment it is found that the accommodation of the two eyes is always the same at the same moment. As to the mechanism of accommodation no conjecture is offered. The now classic experiments of Donders and Helmholtz were at that time in progress, but nothing was known about them in America. The common axis of vision for the two eyes the author considers to be the bisector of the optic angle between the two visual lines which meet at the point of fixation. No mathematical reason for this is or can be assigned, but assuming as a fact that the apparent standpoint of the binocular observer is midway between the two eyes, and accepting the theory of corresponding retinal points, it is easily shown geometrically that all objects in a given field, when viewed with both eyes, must appear as if viewed with one, so far as position is concerned, and that any object within or beyond the circular locus of fixation must appear double. This subject is developed somewhat in detail, and a number of experiments are described, some of which demand more muscular training of the eyes than is possessed by the majority of readers, though it is not hard to acquire. The judgment of distance, whether with a single eye or both eyes, is in reality somewhat complex; and the apparent position of an object is often determined largely by expectant attention and imagination, when vision becomes in any way abnormal, so that two good observers may honestly disagree. The author expresses the conclusion that the mind, even in monocular vision, judges of distance by visual triangulation, the interocular distance serving as a base line. The uncertainty of judgment is, of course, less in binocular than monocular vision. A formula is deduced for the determination of the smallest space upon the retina sensible to the mind. From experimental data this diameter is computed to be 0.000048 of an inch, which corresponds to about 16″ of arc, taking the distance to optical center of crystalline as radius. This value is much smaller than that commonly accepted, 1′ of arc. Ferrel shows by application of his formula that at the distance of about a mile the apparent difference of distance between the sun or moon and an intervening cloud must vanish. If the ordinary value, 1′ of arc, be taken, the distance is less than a mile. In either case all judgment of distance vanishes when the observer is restricted to visual triangulation.

The principles set forth are now applied to vision with the stereoscope, and the apparent position of the stereoscopic image is determined by intersection of visual lines. It is shown that with the unaided eyes either orthoscopic or pseudoscopic effects may be obtained by varying properly the relation between the visual lines. In relation to the diminished image and pseudoscopic effect attained by cross-vision Ferrel remarks, "this, so far as I know, has never been noticed heretofore." The discovery was thus wholly original with him, though he had been anticipated by both Wheatstone and Brewster. The latter especially describes various phenomena of cross-vision, and exhibits much geometric ingenuity in the work of explanation, though he falls into error because dominated by geometric ideas which in reality cannot be fully applied to binocular phenomena. The present writer showed some years ago (1881) that these binocular effects are attainable with the visual lines either parallel or divergent, a condition that forbids all intersection of visual lines for the determination of the apparent position of stereoscopic images. When the visual lines are convergent there is generally a rough accordance between the appearance and the result attained by applying a mathematical formula. Indeed, it may be said that mathematics fails when applied to visual judgments, though fully applicable to the eye as an optical instrument.

Ferrel evolves analytically a method of making stereographs by use of rectangular co-ordinates, resulting in a general formula appropriate for either orthoscopic or pseudoscopic effects. Brewster had already discussed this subject geometrically, but his discussion lacked the simplicity and generality of Ferrel's.

The essay is closed with a consideration of the views on the physiology of vision held by such authorities as Carpenter and Berkeley. The author, expressing himself with due caution, inclines to the belief that the existence of corresponding retinal points cannot be owing to habit, but to some natural constitution of the eyes. He notices somewhat fully Berkeley's theory of vision, that all visual sensations are refinements of the sense of touch, and shows that it is untenable; that lines and angles have something to do with our visual perceptions, and that the place occupied by mathematics cannot be removed by psychological considerations from the philosophy of vision.

The entire investigation is worthy of the careful, exact, and thorough man of science, whose achievements in other fields made him an undisputed authority, respected and honored by all whose attainments entitled them to an opinion of his work.

W. Le Conte Stevens.

Inventions, Researches, and Writings of Nikola Tesla. By Thomas Comerford Martin. 8vo, 500 pp. The Electrical Engineer, New York, 1894.

This book for the first time affords the reader an opportunity to make a continuous and consecutive study of Mr. Tesla's work. Mr. Tesla has been more of a student and inventor than a writer and lecturer. He finds the laboratory more congenial than the lecture-room, and takes a much deeper interest in the prosecution of new lines of research and the development of new facts, than in the presentation to the public of the results of his work. As a consequence, Mr. Tesla's lectures and published papers have appeared at irregular and rather long intervals, and are scattered through the technical journals and in separate pamphlets.

Those interested in Mr. Tesla's work are greatly indebted to Mr. Martin for collecting as he has done into one volume all of Mr. Tesla's published papers, and especially for the connecting links and summaries that, as Mr. Tesla's intimate personal friend, he was so well qualified to supply.

The work begins with a brief biographical sketch, and we find that, as usual with great inventors, Mr. Tesla's fertile brain was even, while he was a student, developing new ideas, and it is interesting to note that the germ of the idea of the rotating field was conceived before the beginning of 1880.

Part I. deals with the rotating field apparatus, beginning with Mr. Tesla's paper read before the American Institute of Electrical Engineers in May, 1888. This paper was a revelation, to most of those present at that meeting, of an entirely new departure in alternating current work. The rotating field principle has since become familiar to electrical engineers through the published accounts of experiments conducted in this country and abroad, and numerous workers have entered the field. Simple as the principle seemed when first explained by Mr. Tesla, it was found when the attempt was made to apply it to the construction of large motors, that, contrary to the usual experience with continuous current machines, the large motors were less efficient than the small ones, especially when operated at high frequencies. With the talent brought to bear upon the subject, sources of loss have been discovered and eliminated, and the motors improved until now they stand nearly on a par in efficiency with continuous current motors.

The chapters which follow in Part I. describe a great variety of forms of rotary field motors. In some the difference of phase is obtained on branches from a single circuit by the proper proportioning of resistance and self-induction. In others, a "magnetic lag" in peculiarly formed polar-projections serves to produce the rotary effort by the use of a single alternating current. Many of these forms could hardly be developed into efficient commercial machines, but they are curious examples of the working of alternating currents, and are especially interesting here as showing the extreme fertility of Mr. Tesla's mind.

Part II. relates to Mr. Tesla's high frequency experiments. In it are published in full Mr. Tesla's three lectures delivered respectively, at New York, May 20, 1891; at London and Paris, 1892; and at Philadelphia and St. Louis in 1893. The lectures are preceded by a review by Mr. Martin.

It is to be regretted that this review is not more discriminating. Instead of pointing out what properly belongs to Mr. Tesla, and what had already been done by other experimenters, he leaves the reader to infer that everything described is Mr. Tesla's work. The claim for novelty of the experiments seems, in some cases, to be unduly exaggerated. The operation of electrical devices by means of one wire is spoken of as though it were something before unheard of; and yet, how does it differ in principle from the old method of measuring the capacity of a condenser by charging it through a galvanometer? One wire from the battery passes through the galvanometer and terminates in the condenser. Close the key, and a current rushes through the galvanometer coil into the condenser and swings the needle. Substitute an alternating potential for the constant potential of the battery, and an alternating current would flow through the galvanometer coil.

The writer more than ten years ago showed that the needle of a galvanometer would be deflected by the rush of electricity into the coil when a glass rod charged by rubbing with a silk handkerchief was brought to one terminal, the other terminal having no connection with anything whatever. Deflection in the opposite direction was caused by charging the coil through the other terminal. Deflection was also produced by discharging the coil by one of its terminals. The coil used in this experiment was very thoroughly insulated, and held the charge given to it for some time. This was a case of an electrical device actuated through one connection only, the generator being a glass rod and a silk handkerchief. The writer did not consider it anything wonderful; in fact, it was just what he expected would take place in a coil sufficiently well insulated, and having a sufficient number of convolutions.

How are we to understand the statement on page 120 that Mr. Tesla has demonstrated, that for the production of light-waves, "primarily," electrostatic effects must be brought into play? It does not appear that Mr. Tesla has produced light-waves *primarily*, in any way.

But the writer does not wish to be understood as finding nothing except for criticism in Mr. Martin's summary; far otherwise. The reader will find it an excellent *résumé* of the results portrayed in Mr. Tesla's lectures, and will find it very useful, even after having read the lectures themselves.

Referring to the experiments exhibited by Mr. Tesla in his three lectures, their novelty seems to the writer to consist more in the brilliancy and magnitude of the results than in the development of anything radically new. By this he does not mean in any way to belittle the value of Mr. Tesla's work. His experiments are no less a revelation if they only exhibit old phenomena in a new light, and demonstrate experimentally facts that were before only theoretically known. By means of a high frequency alternating machine, and still further by the extremely rapid Hertzian oscillations, the old induction coil is made to exhibit some wonderfully brilliant phenomena. Geissler tubes glow with unwonted brilliancy. In Crookes' tubes, in place of heating to redness only a tiny piece of platinum, a whole lamp filament or a block of carbon is rendered incandescent. In consequence of the high frequency, great differences in the phenomena are produced by exceedingly small changes in capacity, and many of the experiments strikingly illustrate this fact.

In the title to Mr. Tesla's New York lecture we find that he proposes to deal with high frequency phenomena "and their application to methods of artificial illumination." It is in this last that the practical engineer is specially interested; and the first question that occurs to him is, what is the efficiency of the methods hinted at by Mr. Tesla? [We do not find the methods more than hinted at in these lectures, and there is nowhere any light thrown upon the question of efficiency.] Knowing as we do the losses that accompany the use of high frequencies and high potentials, such as those due to disturbances in surrounding media, and currents in conductors due to their capacity, it would be interesting to learn what method Mr. Tesla may have in view for eliminating these. Mr. Tesla speaks of using an electrostatic screen for a conducting wire, and remarks that such a screen would have the same effect as removing all objects to an infinite distance. The writer does not know of any such screen.

A metallic sheath would serve as a screen only if its own potential were maintained constant, and that could only be if it were in perfect conducting communication with the earth. The effect of any such sheath, whether connected to earth or not, would increase the capacity of the conductor.

The scheme of lighting by means of isolated vacuum tubes in a rapidly alternating electrostatic field is an attractive one, but the plates forming the boundaries of such a field would have considerable capacity. To charge them to a potential of a hundred thousand volts would require a considerable quantity of electricity, and if this charge were reversed many thousand times per second, the transfer of that quantity of electricity would mean a considerable current, and this current would be continually flowing whether any work were being done or not.

To be efficient it would be necessary that the capacity of the system when the lights were not in use should be extremely small, and should be largely increased by bringing them into action. Is it likely that such a relation could be achieved?

In the single wire distribution about which so much is said in the lectures, the result is only reached by a very accurate adjustment of frequency, potential, and capacity. Supposing the question of efficiency settled, would not such a system still be inapplicable for an extended distribution where lamps are to be turned on and off, on account of the nicety of the adjustments required? Would not the balance be destroyed with every lamp turned off or on. Is it not true that in any system involving the use of very high potentials and high frequencies capacity becomes a most important factor, and that small changes in capacity may interfere seriously with the working of the system?

In closing his London lecture Mr. Tesla tells us that his screened cables will not be made, "for ere long intelligence — transmitted without wires — will throb through the earth like a pulse through a living organism." But how many people could use the earth at once? Would not the telephone girls' frequent answer, "line in use," have to be accepted in that case as at least plausible?

In the Franklin Institute lecture a number of pages are given to the subject of resonance. The writer is not sure that he understands fully the statements and illustrations in that part of the lecture, and will only refer to one or two points.

The statement that "pure" resonance effects are impossible in nature evidently means simply that motion as we have to deal with it cannot be maintained without the constant consumption of energy. This is a fact perfectly well known, and is, of course, true of vibratory motion as well as of any other.

A body acted on by a force ceases to increase in velocity when the resistances it meets with equal the force applied. A vibrating body kept in motion by successive small impulses derived from another body which vibrates in unison with it, which is all there is of resonance, ceases to increase in vibration when the resistances it meets with equal the impulses, or, in other words, when the loss of energy during each vibration is equal to the energy received during the same time. If the energy of the receiver is employed for some useful purpose, it must, of course, receive from the original vibrating body as much energy as is used; otherwise it will cease to vibrate.

When Mr. Tesla says that in "electrical vibration it is of enormous importance to arrange the conditions so that the vibration is effected with the greatest freedom," it is the same as saying that the waste of energy must be reduced to a minimum, and this is just as true when electrical energy is transferred in other ways.

It is also just as important that the transfer should be efficiently effected.

The case of the resonant tuning-forks, where one is sounded and imparts its vibrations to all the surrounding air, a small portion of which air imparts its vibration to the other fork, is an example of a very inefficient way of transferring energy. The energy of the first fork is mainly frittered away in imparting useless vibration to the surrounding air. If the energy of the first fork could be efficiently transferred, the vibration of the second might be maintained in mercury, and if agitation of the mercury were the object to be attained, it might be efficiently effected; and yet the means of transferring the energy from one fork to the other might be such that no effective vibration could be imparted unless the two forks were tuned to unison.

In short, when energy is to be transmitted by means of periodic motions, it may be desirable or convenient to use receivers having a definite vibration period of their own, in which case they must be adjusted to unison with the generator; but the use of such receivers does not increase or diminish the necessity of observing the other conditions of efficient transformation; viz. that as little energy as possible should be used outside of the receivers, and that in the receivers as little energy as possible should be consumed except for the useful purpose. In any case the use of resonant receivers introduces a new difficulty, that of adjusting them to unison, and maintaining unison under the varying demands for energy by the consumers.

Now it seems to the writer that in the transmission of energy electrically by means of high frequency and high potential currents the difficulties in preventing losses are greatly increased; and although, by the use of such high potentials, light may be obtained by means of discharges through rarefied gases without the accompaniment of so large a proportion of energy in the form of dark heat, yet the losses in the use of electrical energy in this form, and applied in this way, seem to much more than counterbalance the efficiency of the luminous source itself.

As has been already said, Mr. Tesla has devised some wonderfully brilliant and striking experiments illustrative of the action of high potential and high frequency currents, but when it comes to the application of these to artificial illumination he is silent upon the all-important question of efficiency, and from such information as we have, it would seem that very little progress had been made toward any reasonable result in that direction.

Part III. treats of miscellaneous inventions. These are of interest as showing the diversified character of Mr. Tesla's work. Some of them show great ingenuity, but probably there are few of them to which Mr. Tesla would to-day attach any practical value.

Part IV. relates to Mr. Tesla's World's Fair exhibit, and to the oscillator, which was made the subject of the lecture before the members of the Electrical Congress.

The personal exhibit was itself a very interesting one, showing as it did, the progress of Mr. Tesla's work, especially in the development of the rotating field apparatus. The reader will be glad to find embodied here brief but excellent descriptions of the separate pieces of apparatus there exhibited.

The oscillators shown by Mr. Tesla at the Congress lecture were very ingeniously designed, and would no doubt give a fairly uniform vibration under varying loads ; but the claim that the vibrations would be so uniform under all conditions, that a clock actuated by the apparatus "would keep absolutely correct time" is probably not intended to be taken literally. The apparatus appears to be an excellent one for the production of alternating currents whose frequency can be accurately known, and it may serve an excellent purpose in experiments and investigations, and perhaps for telegraphy, but the writer is unable to feel that confidence in the great future of this apparatus that he felt in that of the rotating field motors when they were first explained to him in Mr. Tesla's laboratory.

The chapter devoted to the oscillators in this volume is very brief, and does not describe very fully the oscillators themselves, or the apparatus used in connection with them.

The work of Mr. Tesla described in this volume covers a period of hardly eight years. In that time the rotating field principle was developed, and a multitude of different forms of motors embodying the principle were built. The high frequency alternators, induction coils, condensers, lamps, vacuum tubes, etc., etc., have all been developed through successive experimental failures, and the immense variety of apparatus shown in successful operation on the occasion of the three lectures on high frequency phenomena are only a tithe of the instruments and apparatus built. All this shows not only Mr. Tesla's wonderful fertility, but his industry and perseverance as well. The writer understands that it is wholly his own work; the few assistants employed by him in his laboratory being only mechanics who could give him no help in the development of the apparatus. Few men can point to more work accomplished in so short a time.

William A. Anthony.

Drum Armatures and Commutators (*Theory and Practice*). By F. Marten Weymouth. 8vo. 1 vol. London, The Electrician Publishing Co., 1894.

Mr. Weymouth has taken up, in the book before us, a specialized department of the subject of dynamo design and construction, and has treated it in a manner "entirely free from mathematics." It is to be questioned whether this method of treatment is a happy one in such a case as the one before us. A needless use of higher mathematics is certainly inadvisable, but the majority of men who are likely to read this book can readily understand and appreciate the comparatively simple formulas which may sometimes be used to replace sentences of explanatory matter. The author, in his introduction, calls this book an elementary one (or a beginner's book), but it is doubtful if a beginner could properly understand the theory of armatures as given by the text and figures of the first chapter, unless he had lost his title by previous careful study of the general theory of dynamos.

In the second and third chapters is given an explanation of the conditions existing in ordinary drum armatures, and the differences that mark windings for high and low pressure. Heavy windings are given much attention, and the difficulties met in winding armatures for large currents, with their attendant demand for compact and symmetrical end connections and good magnetic balance, are pointed out. These chapters are particularly full in the treatment of bar windings laid upon smooth drum armature cores. For the American reader this is somewhat unfortunate, since, in this country, bar windings are now commonly used only with toothed cores. The common surface winding made of wire receives comparatively little attention.

The next chapter is given up to a short discussion of ventilation and the prevention of Foucault or parasitic currents in bar conductors, but neither of these subjects is very fully treated. The usual methods of dividing the conductors and twisting them to prevent excessive parasitic currents are well set forth. The cause of these currents is also examined, and is shown to be the irregular density of the magnetism under the polar tips. To remove this cause, and thus its effect, the suggestion is offered to make the air space greater under the pole tips. This construction is largely used in this country but for the principal object of gaining sparkless collection at the commutator. It serves this purpose well and at the same time does reduce parasitic currents, though it can scarcely be said to entirely remove their cause.

In five short chapters various forms of bar and special windings with their end connections are described. None of these have any extended use in this country, and many of them are unknown to service here. The author shows a satisfactory appreciation of the advantages of lathe-wound coils which may be made up in forms and be separately applied to the armature body. This section of the book is notable for the indistinctness of its winding diagrams which surely cannot be intended for beginners in the art. The presentation of poorly drawn diagrams of windings is a common fault of books dealing with this subject, and it is a fault for which no adequate excuse can be offered. The common forms of winding diagrams, which are used in the winding rooms of manufacturers, may be made perfectly clear if they are properly engraved.

The author does not mention toothed drum armatures which are meeting with so much favor with American makers, since their advantages have become fully appreciated and the means of avoiding their difficulties have been learned. Neither do multipolar windings receive the attention which their importance warrants.

Four excellent chapters are devoted to commutators and their constructive details. This section is well illustrated by drawings of commutators taken from the best English designers. The discussion of commutator insulation would lead us to believe that English makers are only beginning to realize that vulcanized fiber is not a satisfactory insulator for commutators. Successful American manufacturers have long since abandoned its use for this purpose, owing to its poor insulating and mechanical qualities when subjected to an atmosphere of varying humidity. The later chapters of the section, which deal in an excellent manner with the constructive details of commutators, tend to remove the erroneous impression regarding the use of fiber.

The latter half of the book is devoted to a consideration of the causes and the prevention of sparking. This section contains a great deal of interesting and useful information, but it is marred by prolixity, and in some parts by obscurity. Much of the discussion also appears to be impractical. Fortunately two important chapters (those on carbon brushes and armature defects) stand out in contrast to the remainder of the book on account of their simple and practical explanations. In discussing carbon brushes the author limits the safe working current to thirty amperes per square inch. This will scarcely be concurred in by the majority of American manufacturers, who are in the habit of using densities of from forty to sixty amperes per square inch.[1] In his discussion of sparking and its prevention, the author properly considers the inductance of the armature coils, the electric pressure developed in each, and the magnetism cut by the short-circuited coils, as determining factors. In dealing with them as independent factors he falls into an error. It is now coming into general acknowledgment that the relations between these factors, rather than their actual proportions, determine the condition of sparkless operation.

Two chapters on armature reactions and their effects are based upon the well-known papers of Swinburne and Esson. Considerable later matter is also introduced from the familiar papers of Sayers and Fischer-Hinnen, but in no place is mentioned Ryan's arrangement of balancing coils for the prevention of field distortion.

Reviewed as a whole, this is a book that contains much of interest, but its value and interest to the majority of readers would have been increased by careful condensation of the less important sections. Unfortunately the book is not sufficiently thorough to give it the position amongst specialized technical works that its title might claim.

DUGALD C. JACKSON.

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[1] Jackson's text-book on Electromagnetism and the Construction of Dynamos, p. 193.

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